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Experimental FEM validation of the composite recurve bow

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Abstract. The main scope of this article is to present the results of several experiments that were performed on the developed home-made composite recurve bow fabricated from ash wood, GFRP and CFRP. The results are compared with validated FEM. The FEM is validated by comparison with: measured draw force, measured strain during the bow draw at two selected locations, measured signals at the same locations of the excited bow without the string from which eigen frequencies are evaluated using FFT. All measured data show good compliance with the validated FEM. The main advantage of those experiments is a possible alternative evaluation of stiffness orthotropic material properties without the need of dedicated test setup generally connected with extra cost.

Introduction

Bows are most commonly described by the maximum draw force (draw weight) and maximum draw displacement (draw length) because those parameters can be different for each person. From technical point of view there is another parameter though that is usually not specified for most of the bows: its energy. This energy can be calculated as an area beneath the curve draw force – displacement. In general, for all types of bows (excluding compound bows) non-linear behaviour between drawing force and deflection is characteristic. Historically, there has been significant progress in this area. Main goal has been to find optimized bow shape which would be able to store maximum energy. Primitive bows had been replaced by long bows and recurve bows. In the end, recurve bow became the winner of this optimization race that took centuries [1].

There still has been a great effort to improve energy of the recurve bows using different types of mathematical models [2], [3], [4]. Several non-linear problems, such as: large deflections, contact between the bow string and limbs, string pretension, complicate the applicability of those models. On top of that, most authors neglect the fact that recurve bows are fabricated from composite materials with orthotropic properties (wood, CFRP, GFRP).

The main scope of this article is to present the results of several experiments that have been performed on developed home-made composite recurve bow fabricated from ash wood, GFRP and CFRP. The results are compared with validated FEM. The main advantage of those experiments is a possible alternative evaluation of stiffness orthotropic properties without the need of dedicated test set-up connected with extra cost.

Materials and methods

Material properties. Thicknesses of glass plies (0.09 mm) and carbon plies (0.25 mm) without epoxy resin were measured. Thicknesses of fabricated recurve bow were measured at three specific locations where the lay-up was known.

Figure 1: Fabricated recurve bow

Optimisation in terms of minimising sum of relative errors between measured values was performed in software MS Excel while design variables were only CFRP/GFRP fibre volume ratios. Resulting fibre volume ratios were determined as shown in the Table 1. Comparison between calculated thickness values and measured values are herein reported in the Table 2 showing good compliance.

Table 1: Fabricated fibre reinforced polymers volume ratios

CFRP Fibre volume ratio [-]	GFRP Fibre volume ratio [-]
0.53	0.45

Table 2: Comparison of measured and calculated thicknesses at specific locations of the recurve bow

Nr. of CFRP plies [-]	Nr. of GFRP plies [-]	Measured thickness [mm]	Calculated thickness [mm]	Relative error [%]
6	10	4.88	4.83	0.85
6	9	4.70	4.63	1.42
6	7	4.13	4.23	2.63

The resulting fibre volume ratios were applied as inputs for determination of orthotropic stiffness properties for CFRP/GFRP plies considering properties from supplier datasheets applying rules of mixtures [5]. Orthotropic parameters of ash wood [6] which had been used for the riser were also considered as herein reported as shown in the Table 3.

Table 3: Linear orthotropic models of material used in FEM

	CFRP	GFRP	Ash wood
E_1 [GPa]	127.85	15.90	15.00
E_2 [GPa]	5.36	4.93	10.00
E_3 [GPa]	5.36	4.93	10.00
μ_{12} [-]	0.29	0.36	0.20
μ_{23} [-]	0.73	0.68	0.20
μ_{13} [-]	0.29	0.36	0.20
G_{12} [GPa]	2.08	1.72	6.00
G_{23} [GPa]	1.55	1.47	2.80
G_{13} [GPa]	2.08	1.72	6.00

Densities of glass plies, carbon plies and epoxy resin were taken from supplier datasheets. Two GFRP samples masses without 2 mm overall thicknesses were measured.

Table 4: Densities of raw materials

Glass density [kg/m^3]	Carbon density [kg/m^3]	Epoxy density [kg/m^3]
2090	1800	2090

Table 5: Comparison of measured and calculated GFRP sample masses

Measured Sample mass [g]	Measured Sample surface [mm^2]	Calculated mass [g]
22.0	5440	22.7
30.0	7298	30.5

Results showed good compliance between expected and measured masses. Thus, for CFRP plies same relations based on rules of mixtures were applied without additional sample manufacturing. The resulting plie densities reported in Table 6 were implemented into FEM.

Table 6: Linear orthotropic models of material used in FEM

CFRP density [kg/m^3]	GFRP density [kg/m^3]	Ash wood density [kg/m^3]
1936	2090	710

Methods. Finite element modelling was chosen for the problem solution. Analyses were done in the student version of SW Ansys APDL. Created batch file based on parametric input variables was particularly useful during FEM validation loop described below. Bow symmetry was considered in the FEM and symmetric boundary conditions were defined at the bow symmetry axis. Linear SHELL181 elements, recommended for large deflections, were used with lay-up properties described above. Rigid string was modelled with one linear element LINK180 which was set up to be functional in tension only. Connection between LINK and SHELL elements was done by merging two coincident nodes which models the joint transferring only forces while rotations are free. Nodal displacement load in horizontal axis was applied at the second end of the string rigid element. All analyses were done with large displacements turned on which means that the load had been applied step by step and stiffness matrix was updated adequately. For the mode shape extraction Block Lanczos method was used. Same FEM and boundary conditions were applied excluding rigid string element and nodal displacement load.

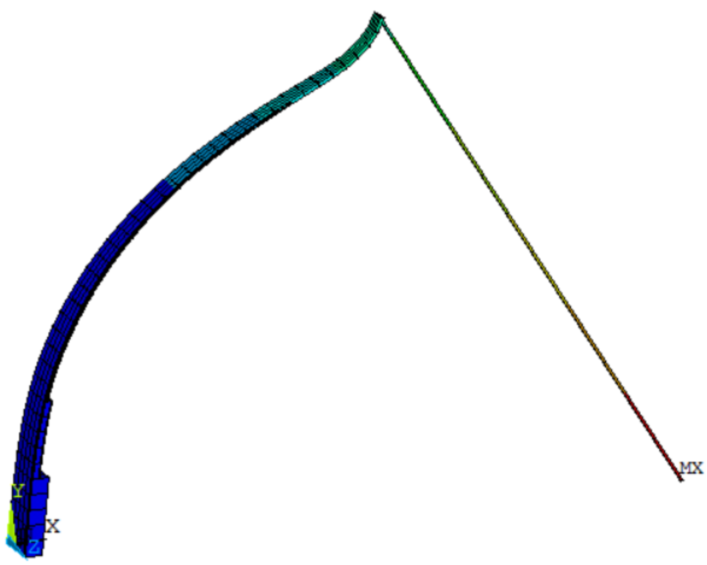


Figure 2: FEM deformed shape at maximum draw length

Figure 3: FEM modal shape of the system without the string at 15.80 Hz

Testing

For the testing of the recurve bow following devices were used:

- strain gauge at height 240 mm from the middle of the bow = "end of the limbs";
- strain gauge at height 470 mm from the middle of the bow = "middle of the limbs";
- load cell at the string symmetry axis for monitoring of the Draw Force.

Figure 4: Harness and strain gauge bonded on the recurve bow

Figure 5: Harness and strain gauges

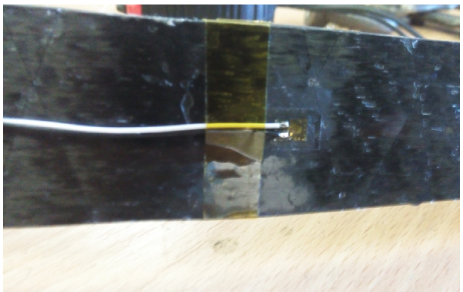


Figure 6: Detail of the bonded strain gauge 1

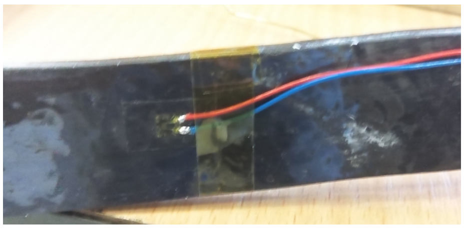


Figure 7: Detail of the bonded strain gauge 2

Following tests were performed using devices described above:

- static test - Strain measurements at the outer (external) plane in tension of the bow and Draw Force measurements during drawing of the bow;
- dynamic test - Resonance search of the bow without the string using strain gauges.

Static test was performed as follows:

1. load cell put between the string and the drawing hand;
2. bow supported (hold) at the riser only;
3. drawing of the bow to the predefined displacement values and sustaining at those values for a few seconds for clear data post processing (three measurements done).

Dynamic test was performed as follows:

1. bow supported (hold) at the riser only;
2. excitation of the upper bow limb.

Data processing

All data were processed in software MATLAB.

Static test. Test Data were processed as follows:

1. evaluation of Force-time and Strain-time curves;
2. determination of Force-Displacement and Strain-Displacement curves (relation between time-displacement was known as reported above);
3. averaging Force and Strain values from three measurements done.

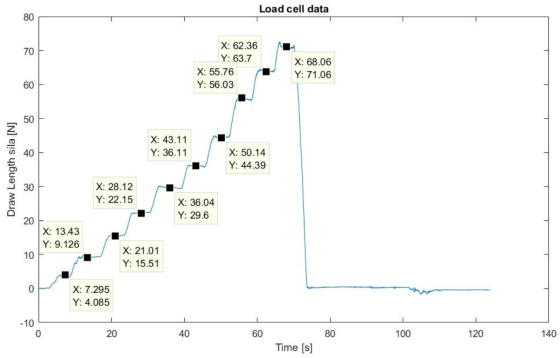


Figure 8: Example of Force evaluation

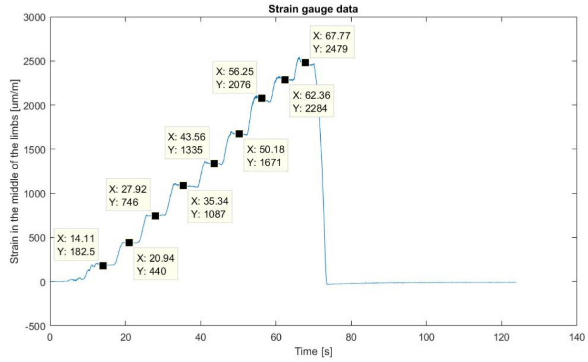


Figure 9: Example of Strain evaluation

Dynamic test. Test Data were processed as follows:

1. signal import and evaluation;
2. selection of only steady-state vibrations for FFT inputs;
3. FFT and resonance evaluation.

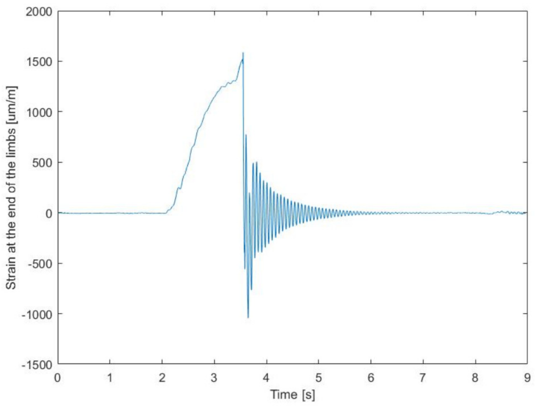


Figure 10: Imported Signal

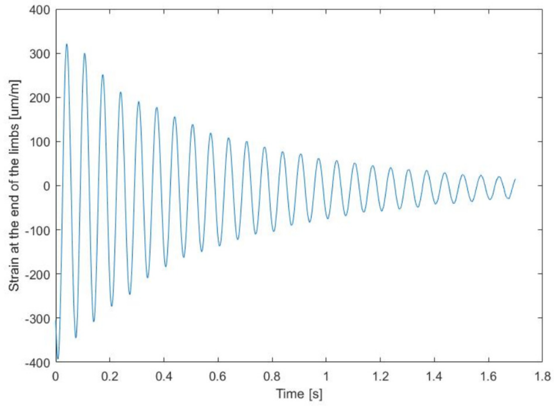


Figure 11: Selected steady-state oscillations for FFT

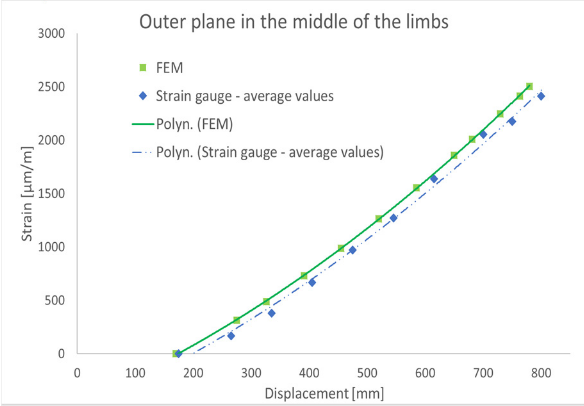
Experimental FEM validation

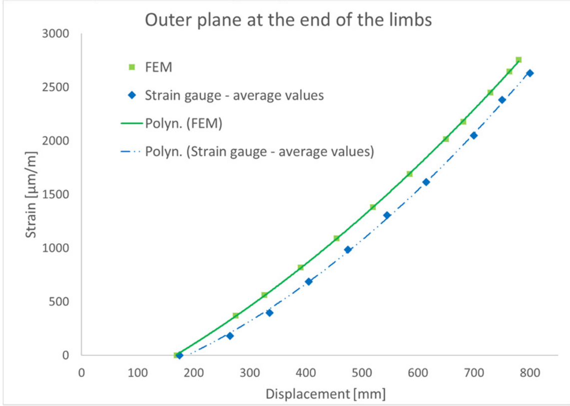
The main goal of this activity was to update FEM of the bow to be compliant with the test results. After loop of analyses varying different input parameters, it was found out that external CFRP plies in compression (inner plane closer to the archer) do not have expected stiffness evaluated from rules of mixtures compared to external CFRP plies on the opposite outer plane in tension. The stiffness in compression that well matched with all performed experiments was evaluated as 30 % of expected CFRP stiffness in fibre direction ($E1 = 38.36$ GPa). There was no other need to change other input parameter.

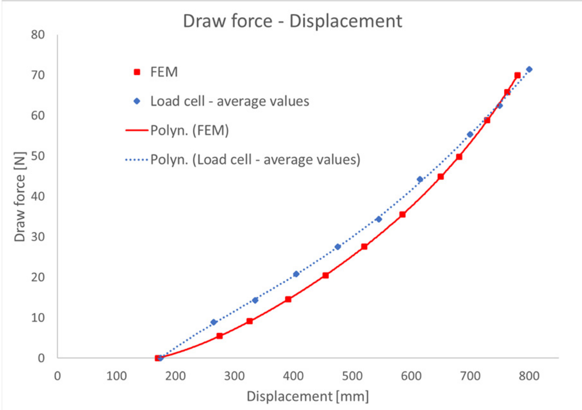
Rationale. Used fabrication process of the recurve bow (hand lay-up moulding excluding vacuum bagging and curing) is not standard fabrication process of FRP in aerospace industry. Thus, it is not surprising that the layer with reduced stiffness properties is the top layer which had been in contact with air during drying. It is possible that this layer had more air bubbles inside and worse properties than the layers beneath which were not in direct contact with air. This effect might be amplified by non-linear effects such as: large deflections and buckling on the fibre level since in FEM reduced stiffness had been applied only to the plies in compression.

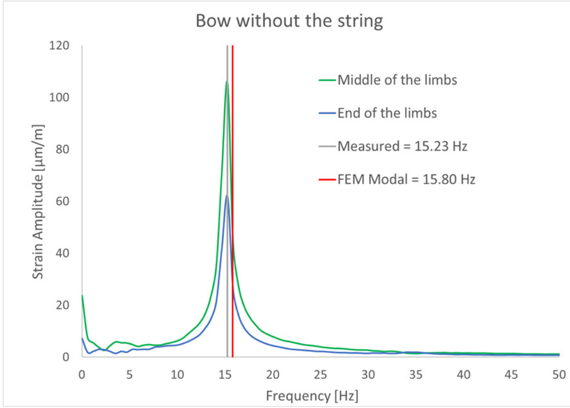
Results

The comparison of FEM and experimental results shows good compliance. Considering that modal analysis does not consider the damping of the real system, resonance peaks would be probably shifted to the lower frequencies and compliance would be even better if transient analysis was done in the time domain considering also system damping.









Conclusions

The results show good compliance between FEM prediction and experiment results. Used fabrication process of the recurve bow is characteristic by following material behaviour:

- 100 % of expected stiffness CFRP on the outer plane in tension, E_1 = 127.85 GPa
- 30 % of expected stiffness CFRP on the inner plane in compression, E_1 = 38.36 GPa

Composites in compression generally have lower strength but almost similar stiffness [7]. Thus, it is generally recommended to design composites to be in tension. From the FEM results and experiment results it seems though, that for a hand-layup technology excluding vacuum bagging and curing also stiffness may differ from expected values. This effect might be amplified by non-linear effects such as: large deflections and buckling on the fibre level.

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