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Experimental verification of analytical model for deflection prediction of hybrid carbon fibre profile under three point bending tests

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Abstract

The aim of the article is an experimental verification of new analytical method for prediction of composite profile behaviour under bending and transverse shearing loading when testing the composite profiles for the production machine or automation industry. The article contains the analytic theory for the stiffness prediction and the experimental data from a standard and modified three point bending tests.

Introduction

The composite profiles usually offer a great bending stiffness and reduced mass in comparison with commonly used isotropic structural materials like steel, cast-iron or aluminium, see [1]. These benefits might be limited by a reduced stiffness in out-of plane laminate directions – mainly in transverse shearing.

Fig. 1: Bilsing Automation pressline manipulator based on hybrid carbon profiles

The aim of the article is an experimental verification of new analytical method in revise shear stiffness calculation. This verification was done for composite profile behaviour under bending and transverse shearing loading. The low transverse shearing stiffness might cause issues during the profile application as it might lead to a large global beam deformation or to large local deformation in places, where loading is introduced into the design. The effect of the local compliance, when testing the composite profiles in three-point bending tests and their influence to results is discussed. The experimental results with and without the local compliance effects are compared with developed analytic solution results and with finite element results.

Profile description

A hybrid composite profile of external dimensions 100x100x1000 mm with 8mm wall thickness was selected for the three-point bending test (in Figure 2 first on the left). The profile represents a typical beam used in the production machine, which typical high-stiffness – low-mass application shown in Figure 1. The profile lay-up was of a hybrid composition as it was made from ultra-high modulus carbon fibres (UHMC) and also from high-strength standard carbon fibres (HSC). A resin from epoxy group was used as a matrix system.

Fig. 2: Coupons of structural parts (all hybrid composite profile)

A comparison of material mechanical properties, which are used in production machines, is given in Table1. Properties of the HSC and UHMC laminate are for unidirectional composites with 50% fibre volume fraction in the laminate. Elastic modulus in the direction of fibres E_1, elastic modulus in the transversal direction E_2 and shear elastic modulus G_{12} are important inputs for calculation under bending and transverse shearing loading.

Table 1: Comparison of mechanical properties

Material	ρ [kg·m^{-3}]	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]
HSC C/E UD	1500	137	9	4
UHMC C/E UD	1750	380	5	3
Epoxy	1200	2.5-4.5	2.5-4.5	1.6
Steel	7800	210	210	80
Cast iron	7200	80-125	80-125	30-55

Analytical approach

Analytical model are really useful for their quick and simple usage. They are mostly used as quick design tools. For isotropic materials, Bernoulli beam theory is often used. However, for anisotropic materials the application of this method is limited. A low shear stiffness of carbon fibre profiles need to be taken into account and in calculation, see [2].

The Timoshenko beam theory works with bending and shear stiffness. Application of the Timoshenko theory to a cantilever beam is shown in Eq. (1),

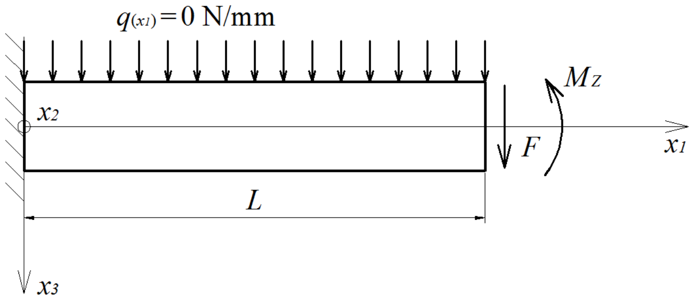

Fig. 3: Beam element

where L is length of a beam, T_{D2} is profile bending stiffness (x_2 is bending neutral axis), T_{A2} is a profile shear stiffness (x_2 is bending neutral axis).

$$w_{(x_1)} = \frac{-F}{6 \cdot T_{D2}} \cdot x_1^3 + \frac{F \cdot L - M_Z}{2 \cdot T_{D2}} \cdot x_1^2 + \frac{F}{\kappa \cdot T_{A2}} \cdot x_1 \tag{1}$$

The bending stiffness T_{D2} and shear stiffness T_{A2} are often evaluated as a sum of every layer stiffness as it is shown in Eq. (2) and Eq. (3), where A is profile cross section area, G_i is shear modulus of i-th layer, E_i is elastic modulus of i-th layer, n is number of layer, A_i is area of i-th layer, J_i is inertia moment of i-th layer and κ is shear revise coefficient.

$$T_{A2} = \int_{(A)} \mathrm{G}\, dA = \sum_i^n G_i \cdot A_i \tag{2}$$

$$T_{D2} = \int_{(A)} \mathrm{E} \cdot {x_3}^2\, dA = \sum_i^n E_i \cdot J_i \tag{3}$$

Direct approach to revise shear stiffness

The big advantage of the direct approach to revise shear stiffness is simple usage, especially for rectangular profiles. Explicit evaluation of the shear revise coefficient is not necessary, because the goal is revise shear stiffness, not the coefficient. Evaluation of shear revise coefficient is difficult for composite materials in general. Raman [3] express the shear revise coefficient as function of many variables (e.g. number of layer, orientation of layer, type of fibres, etc.) but it isn't applicable in analytical methods.

This analytical method is based on energy equivalence between transversal force deformation energy and a shear stress deformation energy as it is shown in Eq. (4).

$$\frac{1}{2} \int_{(l)} \frac{F^2}{\kappa \cdot G \cdot A}\, dx = \frac{1}{2} \int_{(V)} \frac{\tau^2}{G}\, dV \tag{4}$$

Where, A is profile cross area, F is transversal force, l is beam length, V is beam volume and τ is shear stress. Revise shear stiffness $T_{A2\,direct}$ is shown in Eq. (5).

$$T_{A2\,direct} = \kappa \cdot G \cdot A \tag{5}$$

The shear stress is determined from Fig.3 by equilibrium equation into Eq. (6).

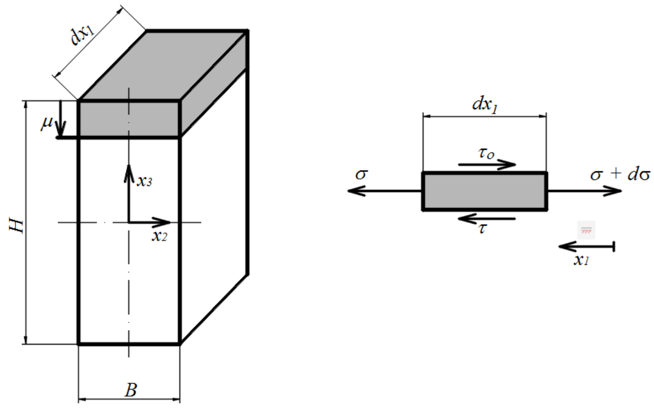

Fig. 4: Element to derive shear stress in cross section

Equilibrium equation is also known as Zhuravskii formulae where σ is axial stress.

$$\tau_{(\mu)} = \frac{d\sigma}{dx_1} \cdot \mu + \tau_o \tag{6}$$

Combine Eq. (6) with Schwedler equation, coordinates transformation and equilibrium equation for the axial stress is obtained Eq. (7) for a prismatic beam with a constant cross sections parameters through the length. Where H is height of web and τ_o is shear stress boundary condition.

$$\tau_{(\mu)} = \frac{\mathrm{F}}{T_{D2}} \cdot E_i \cdot \frac{H-\mu}{2} \cdot \mu + \tau_o \tag{7}$$

The boundary condition on top and bottom flange surface is obviously $\tau=0$. Because of this fact, we can neglect deformation energy in flange.

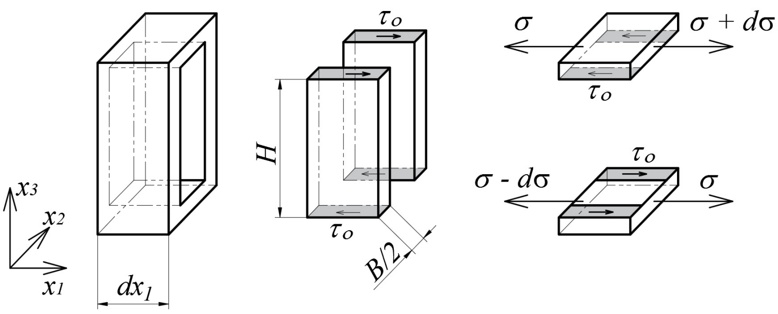

Fig. 5: Element to derive shear stress boundary condition

Shear stress τ_o is the boundary condition between the flange and web, which can be easily evaluated from the axial force in flange F_{axial} , where A_{fl} is flange area, σ_i axial stress of i-th layer, $B/2$ is thickness of web, σ_{mi} average axial stress of i-th layer and A_{if} is area of i-th layer in flange.

$$\tau_o = \frac{F_{axial}}{B \cdot dx_1} = \frac{1}{B \cdot dx_1} \cdot \int_{(A_{fl})} \sigma_i dA = \frac{1}{B \cdot dx_1} \cdot \sum_i^n \sigma_{mi} \cdot A_{if} \tag{6}$$

Three point bending test configurations

The verification of new calculation method required a modified experiment setup, where the beam deformation was captured using a line of displacement sensors at the beam top surface. As the line of displacement was in collision with the actuator, the actuator was moved to the bottom of beam. A modified actuator connection is based on a glued “U” shape clamp and the actuator is placed under the support beam with a roller. The first modified test was done using 600mm span between the supports for easy comparison of the standard 3-point bending test results. But from the results, it was obvious that a huge local deformation in the support areas occurred. The second modified test was done with reinforcement in the areas around the beam supports. Because the coupon length 1000mm, the span between supports was changed to 940mm (no cutting of coupon required by coupon owner). Because of the span change, the results are not directly comparable, but all data are easy comparable with analytical model prediction.

A standard three-point bending configuration was tested using the force actuator on the top surface between the supports of the beam, see Fig. 6 The results look good and compliance 2,53e-02 mm/kN was evaluated.

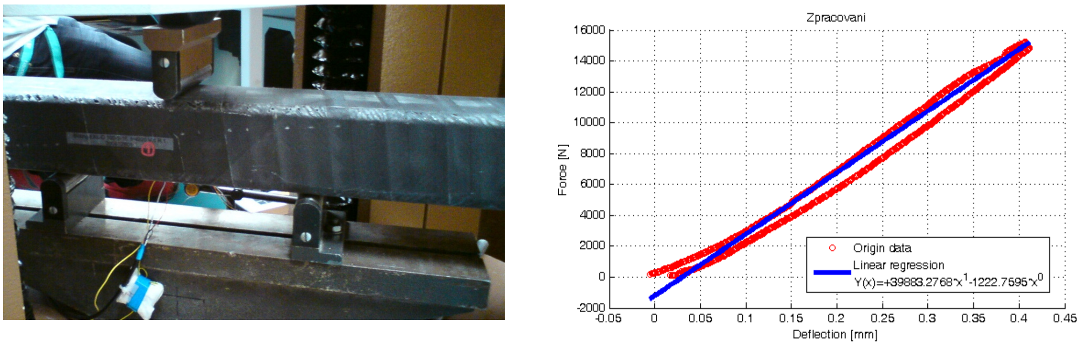

Fig. 6: Standard bending test (600mm between supports)

The three point bending test in the modified configuration was motivated by the need to measure deflection through the whole beam length, including local deformations. The assumption was that the top surface deformation will be the similar as the deformation of the bottom surface or the deformation in the beam middle plane. This assumption could be fulfilled just if the local beam compliance below the loading force actuator and around the supports are negligible. More detailed measurement was done, where the beam deformation was captured using a line of displacement sensors at the beam top surface, see Fig. 7.

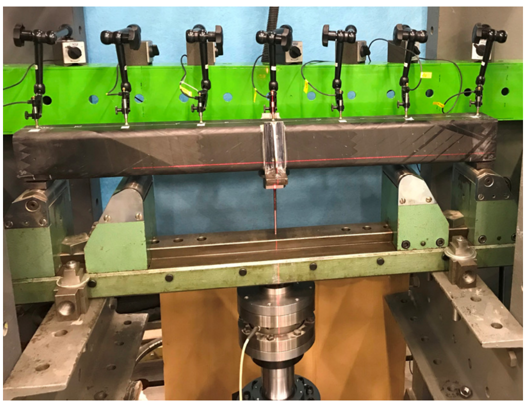

Fig. 7: Modified bending test with top surface deflection measurement

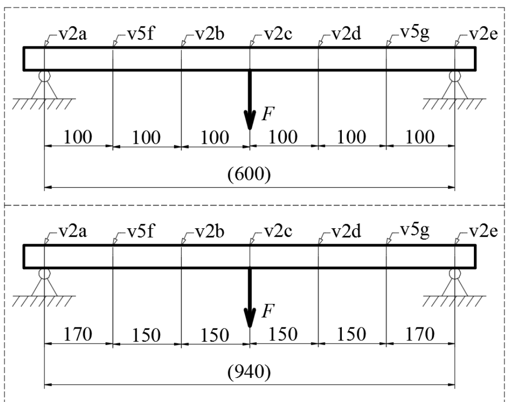

Fig. 8: Test setups

Due to the local beam compliance in supports area, the results of the first modified test were not useable. The effect was studied more deeply by a final element model of the experiment. For the second modification, the coupon support areas were modified, using bonded aluminium inserts, see Fig. 9.

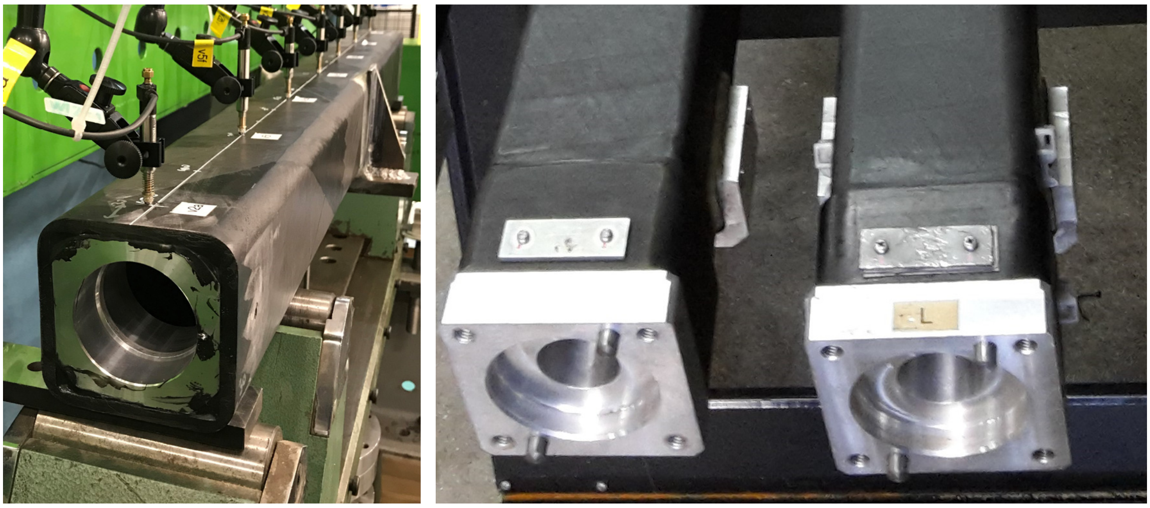

Fig. 9: Detail of aluminium reinforcement – a) test coupon, b) real application

Experimental study was done 3-times for each setup and the captured deformations are shown in Fig. 10. The deformation in supports area was greatly reduced for the second modified test. The assumption is, the captured deformation in supports area for second modified test is deformation of load and measure frame.

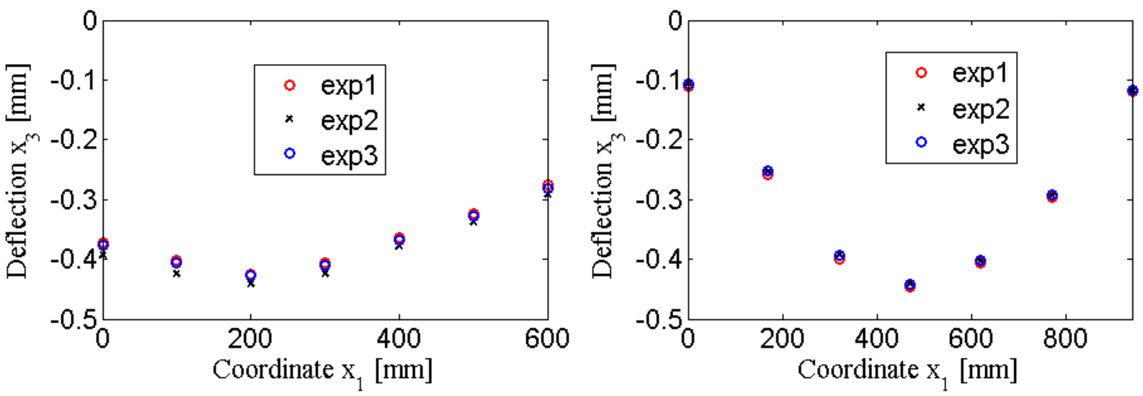

Fig. 10 Raw experimental data for 15kN – a) without end reinforcement, b) with end reinforcement

Results

The comparison of the beam deformation on the top/bottom/middle plane is shown in Fig. 11, both for the modified test without and with ending reinforcements; these data were taken from the finite element analysis, which was performed in ANSYS software.

The effect of local compliance is clearly visible. However, the comparison of the FEA and experimental results demonstrate that the compliance of the real experiment was higher than of the model. During the standard testing, the local deformations were about 40-50% of the total deformation. For the second modified testing, the local deformation was reduced to 5%.

The experimental results were verified by finite element simulations of the tests in the both configurations. The results of the modified test provides reliable for the verification of the developing analytic tools for the stiffness prediction of rigid composite beams, which was the original goal of the testing, see Table 2.

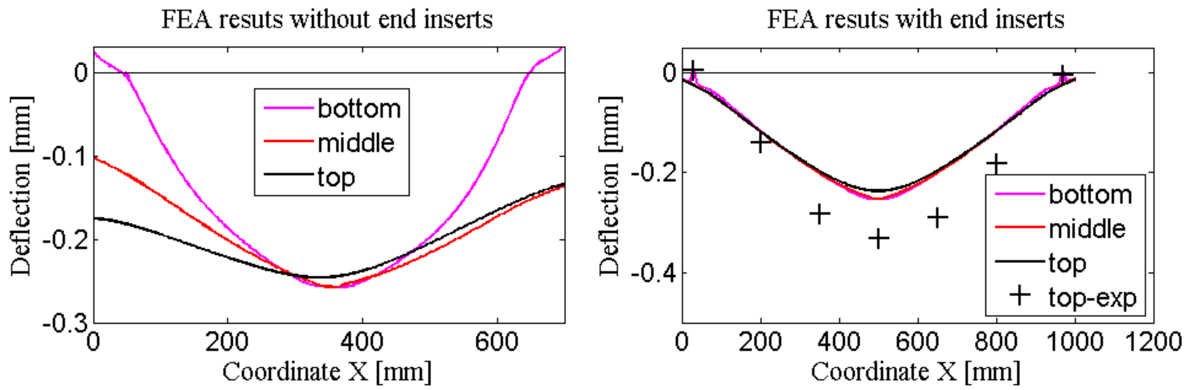

Fig. 11: Bending test FEA results for 15kN – a) without end inserts, b) with end inserts

Direct approach to revise shear stiffness was verified by a modified experiment. This evaluation of shear stiffness is suitable for hybrid carbon profile analytical tool. Deviation -12% in deformation is really good accordance of analytical model with experiment.

Table 2: Comparison of analytical methods with experiment (load 15kN)

Method	Support span [mm]	Deflection [mm]	Compliance [mm/kN]	Δ [%]
Original experiment	600	0.38	2.53e-02	0
Modified experiment	600	0.08	5.33e-03	-79
FEA modified exp. midline	600	0.14	9.33e-03	-63
Bernoulli	600	0.06	4.00e-03	-84
Direct shear model	600	0.10	6.67e-03	-74
Modified experiment v2	940	0.33	2.20e-02	0
FEA modified exp. v2	940	0.25	1.67e-02	-24
Bernoulli	940	0.22	1.47e-02	-33
Direct shear model	940	0.29	1.93e-02	-12

Conclusions

Direct approach to revise shear stiffness has proved as a suitable method for evaluation of shear stiffness of hybrid carbon beam profile. Deviation -12% from modified experiment is evidence of this method usability. The biggest advantage of direct approach to revise shear stiffness is not necessary evaluation of shear revise coefficient, which isn't simple to evaluate for hybrid carbon profiles and coefficient for isotropic material haven't fulfilled derivation condition.

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