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# Theory vs. experiment

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**Abstract**. As generally accepted today, the model is a purposefully simplified concept of a studied phenomenon invented with the intention to predict – what would happen if … Accepted assumptions (simplifications) consequently specify the validity limits of the model and in this respect, the model is neither true nor false. The model, regardless of being simple or complicated, is good, if it is approved by an appropriate experiment. When we are modelling particular phenomena of Mother Nature, the question of truth becomes thus irrelevant since the models we are designing, checking and using, either work or do not work to our satisfaction. So it is obvious that we rather strive for robust models with precisely specified limits of validity and not for philosophically defined categories of truth and falsehood. From it follows that it is the validity of models, theories and laws that is of primary importance.

## Introduction

The author ponders about things that necessarily come into engineering mind when the results obtained by theoretical, numerical and experimental approaches in solid continuum mechanics are correlated and compared with a pious wish to ascertain which of them are ‘truer’ or closer to 'reality'. This invokes many questions. How is the truth related to consistency and validity of theoretical, numerical and experimental models we are inventing and employing? What is the role of thresholds in physics, engineering, computation and in an experiment? The doubts stemming from uneasy answers to above pertinent questions are complemented by discussing examples from theoretical, numerical and experimental results obtained by solving dynamical problems in solid continuum mechanics.

## Model vs. experiment

To get rid of doubts we often claim that it is the experiment, which ultimately confirms the model in question. But experiments, as well as the subsequent numerical treatment of models describing the nature, have their observational thresholds. And sometimes, the computational threshold of computational analysis is narrower than those of an experiment. From this point of view, a particular experiment is a model of nature as well.

Another mental hindrance we might have, in our incessant quest for truth, is the lack of precise definitions of certain mechanical quantities. Definitions of conceptually defined quantities as force, stress, energy, etc. are rather intuitive and often circular.

**Examples and hints**

Practical examples from the field of stress wave propagation tasks will be presented with the intention to show remedies required to avoid ‘well-conceived’ ideas, errors and blunders.

**Example_1. Wavefront singularities and threshold uncertainty**

It is a good practice to have a solid theoretical background at hand. When the stress wave propagation tasks, induced by shocks and impact, are treated, one should be aware of the fact that there are two fundamental stress wave types – namely the longitudinal and transversal ones. This fact was explained by Christiaan Huygens, (1629 – 1695). The wavefronts of longitudinal (also called P, primary, irrotational, dilatational, extensional) and transversal (S, shear, rotational, equivolumetrical) are schematically depicted in Fig. 1 for an elastic halfspace loaded by a point force whose time distribution is prescribed by the Heaviside function. Also, the appearance of the wavefront of so the called Schmidt’s wave, introduced by the surface points, which were already hit by the longitudinal wave, become sources of the slower – i.e. transversal – waves, is depicted.

In this respect, it should be emphasized that a frequently used term ‘the sound velocity in solids’ is wrong and unjust. Actually, we have distinct speeds for 1D, plane stress, plane strain, and 3D cases.

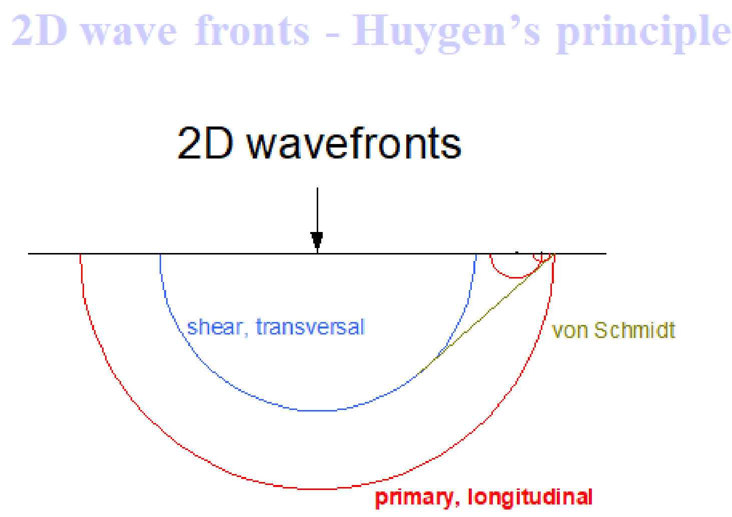


Fig 1. Wave fronts in elastic half space due to the sudden application of a point force

These phenomena are easily ‘proved’ by finite element computations as shown in Fig. 2. The upper part of the figure shows the velocity distribution of in an elastic solid due to a point loading, whose time distribution is given by the rectangular pulse. The depicted vector directions justify the accepted terminology – longitudinal and transversal waves. Also, the appearance of so-called Rayleigh waves, characterized by elliptic patterns of particles in the vicinity of the body’s surface, is shown. See [1].

The lower figure, also a result of finite element computations, clearly shows the appearance of Schmidt’s waves and the effect of reflections on the opposite boundary.

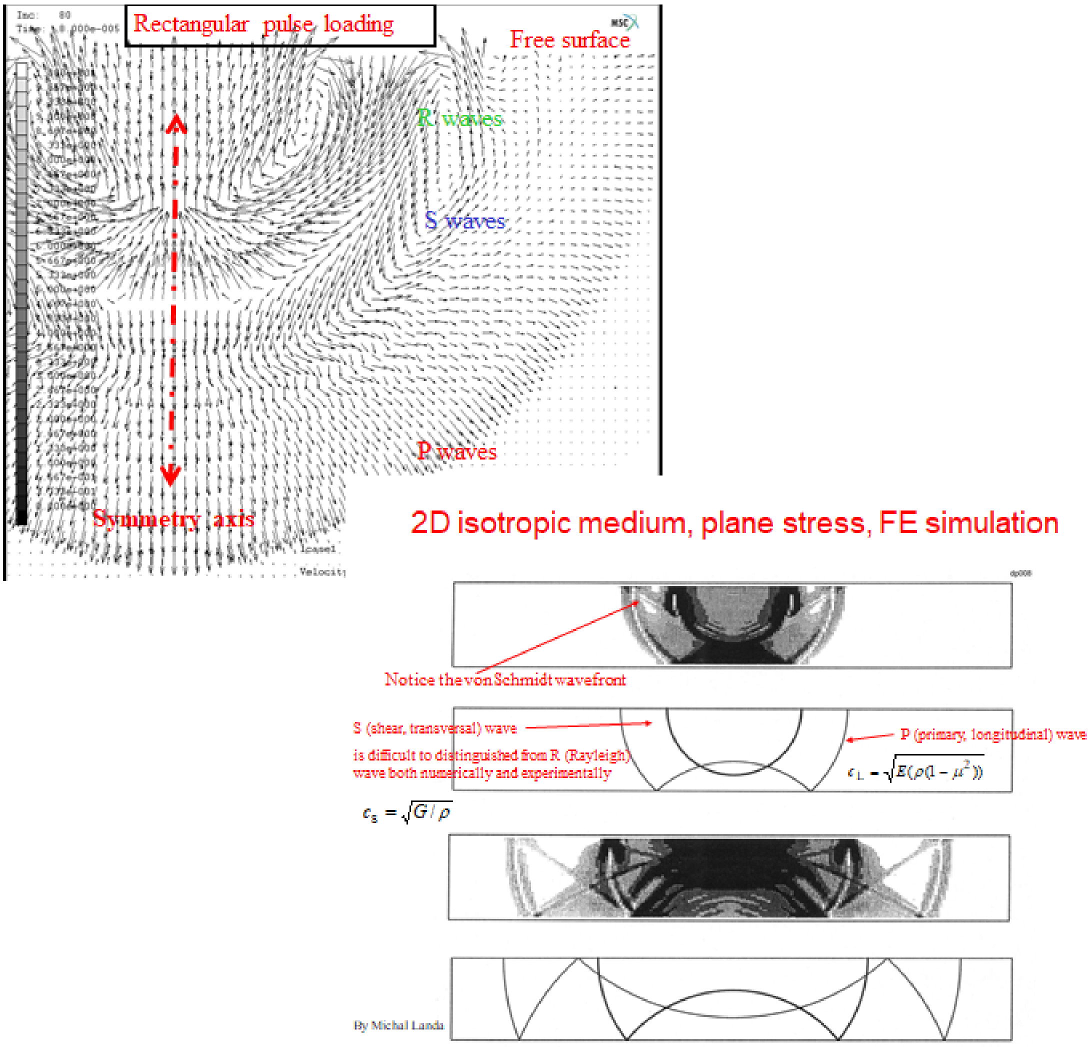


Fig. 2. Computed wavefronts

**Lamb's problem**

The task is schematically illustrated in Fig. 3. The original Lamb's analysis employs Fourier superposition of harmonic waves for the transient normal loading on a half-space. Later Cagniard presented a numerical method for the inversion of the Fourier's transform, and Brepta [2] and others were able to carry out the inversion analytically even for cases with more complicated boundaries. The analytical solution the Lamb's problem on a free boundary is relatively simple and is available in a closed form giving space-time distributions of radial and axial displacements. It is attributed to Pekeris, whose analytical formulae, presented in [3], are easy to evaluate. Computed results, depicted in space-time coordinates in Fig. 4, show two significant singularities as well as arrivals of longitudinal, shear and Rayleigh waves.

The existence of singularities in our models (and the solid continuum is a model) always indicates certain unjustified assumptions and/or simplifications accepted at the beginning of the modelling process. In this case, the singularities occur due to the application of the point force, which, strictly speaking, is a forbidden phenomenon in continuum mechanics.

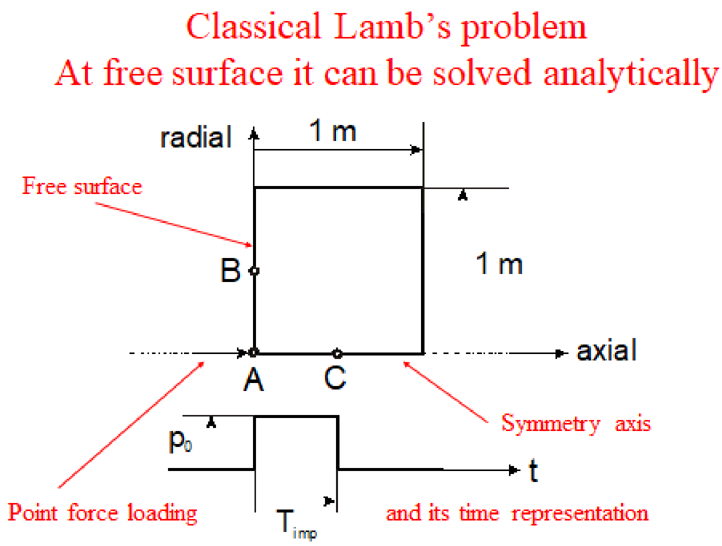


Fig. 3. Lamb's problem

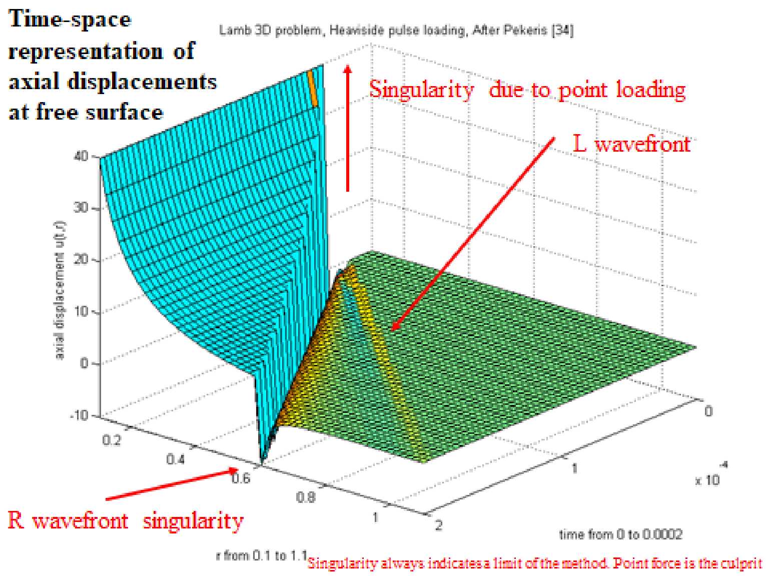


Fig. 4. Lamb's problem – analytical solution

Seemingly unproblematic model of elastic continuum has embedded singularities in it. For example a point force, a frequently used tool in engineering analysis, is a forbidden entity in continuum mechanics since it leads to a singularity response – this is manifested by the fact that the displacement under the application of a point force tends towards infinity. To a certain extent this property is retained when the continuum is treated by means of a FE model. Actually, it is smeared out by the existence of shape functions but with diminishing meshsize it is well observable by the increase of displacement under the application of a point (nodal) force. The FE mesh made of 'null-sized' elements would provide the infinite displacement under the application of a nodal force as the continuum model. So making a finer and finer mesh we are representing better and better those continuum properties that are mathematically correct but physically unattainable. This is a sort of paradigm we are used to live with. Singularity in displacement response to a point loading, Rayleigh waves and crack analysis are well-known examples both in continuum and its FE representation. See [4].

**Example 2. – Dispersion**

In finite element computations of stress wave propagation tasks, the dispersion phenomena significantly influence the accuracy and reliability of obtained results. That's why we focus our attention on the subject of dispersion in detail.

Dispersion is a medium property characterized by the fact that the velocity of the propagating harmonic wave depends on its frequency. Consequently, the wave packages, being composed of the whole spectrum of frequencies, are distorted during their passage through the medium.

A seemingly academic theoretical subject of spatial and temporal dispersion in discretized continuum solid mechanics still has significant practical consequences for current users of commercial finite element packages.

Studying the subject of dispersion allows understanding and minimizing the unpleasant side effects typical for the finite element computations of non-stationary dynamics tasks of solid computational mechanics. Also, the 'correct' determination of mesh size and the time step can be safely determined. It is shown that the Fourier analysis is an excellent tool for checking the obtained results and for the safe estimation of their validity.

In solid continuum mechanics, we distinguish *material*, *geometrical*, *spatial* and *temporal* types of dispersion. The *material dispersion* is induced by the damping properties of the medium through which the wave propagates. The *geometrical dispersion* occurs due to changing the geometrical shapes of waveguides through which the waves propagate, The *spatial dispersion* is typical for the numerical approaches based on discretization in space, in which originally continuum structure of material is replaced by small finite parts – elements, and finally, the *temporal dispersion* is typical for the numerical solution of transient tasks – the output quantities, instead of being described by continuous functions of time, are evaluated in discrete time intervals only.

In solid continuum mechanics, there is a group of the wave propagation tasks which are purely dispersionless. As examples, the propagation of stress waves in unbound 1D, 2D, and 3D spatial geometries could be mentioned. See [1], [5]. The theoretical cases described and solved there can be efficiently exploited as useful etalons for judging the seriousness and amount of dispersion errors in discrete media.

On the other hand, the FE model of continuum is generally of dispersive nature, see [7], [8]. To show and explain dispersion phenomena in detail, let's study the time response of a thin rod modelled by 1D constant strain elements, while the behaviour of idealized dispersionless solid continuum, represented by the wave equation, can serve as a suitable etalon.

Studies concerning the finite element dispersion analysis of 2D and 3D elements are numerous – see [7], [8], [9]. The conclusions, shown for the 1D wave equation and for its 1D finite element model, are quantitatively valid for 2D and 3D elements as well.

It should be emphasized that in 1D continuum, there is only a single speed of wave propagation, i.e. $c_0 = \sqrt{E/\rho}$ .

However, due to the existence of normal and shear stresses in 2D and 3D continua, there are two kinds of stress waves that could propagate in an unbound medium. See [1], [5]. There are the *primary* (P, *longitudinal*) and *transversal* (S, *shear*) waves – their speeds are different. The speed formulas and values (in m/s), for a typical steel material ($E = 2.1\times10^{11}$ Pa, $\rho = 7600$ kg/m$^3$, $\mu = 0.3$), are as follows.

$$c_0 = \sqrt{E/\rho} = 5189 \qquad \dots \quad \text{1D wave, slender bar,}$$

$$c_1 = \sqrt{(2G+\lambda)/\rho} = 6020, \qquad \dots \quad \text{P wave for plane strain and for 3D,}$$

where $\lambda = \dfrac{\mu E}{(1+\mu)(1-2\mu)}$, $G = \dfrac{E}{2(1+\mu)}$.

$$c_2 = \sqrt{G/\rho} = 3218 \qquad \dots \quad \text{S wave, shear, both for 2D and 3D,}$$

$$c_3 = \sqrt{E/(\rho(1-\mu^2))} = 5439 \quad \dots \quad \text{P wave for plane stress.}$$

In Fig. 5 the speeds of wave propagation for 1D constant strain elements with consistent and diagonal mass matrices (L1C and L1D) are compared with those of 2D constant strain elements for an equilateral triangle with consistent and diagonal mass matrices. The right-hand part of the figure depicts the propagation speeds the longitudinal (upper group of curves) and transversal (lower group of curves) waves as functions of the dimensionless wavenumber. As we have seen before, the elements with consistent mass matrix (plotted by dashed lines) overestimate the 'correct' speed of propagation (horizontal lines) while the speed of propagation for elements with the diagonal mass matrix is systematically underestimated. In the discretized 2D continuum the speed of propagation is furthermore influenced by the direction of the wave propagation, indicated by the $\delta$ parameter in the figure. This means that a discretized 2D medium exhibits the anisotropy behaviour, not existing in an ideal 2D model of solid continuum. See [10].

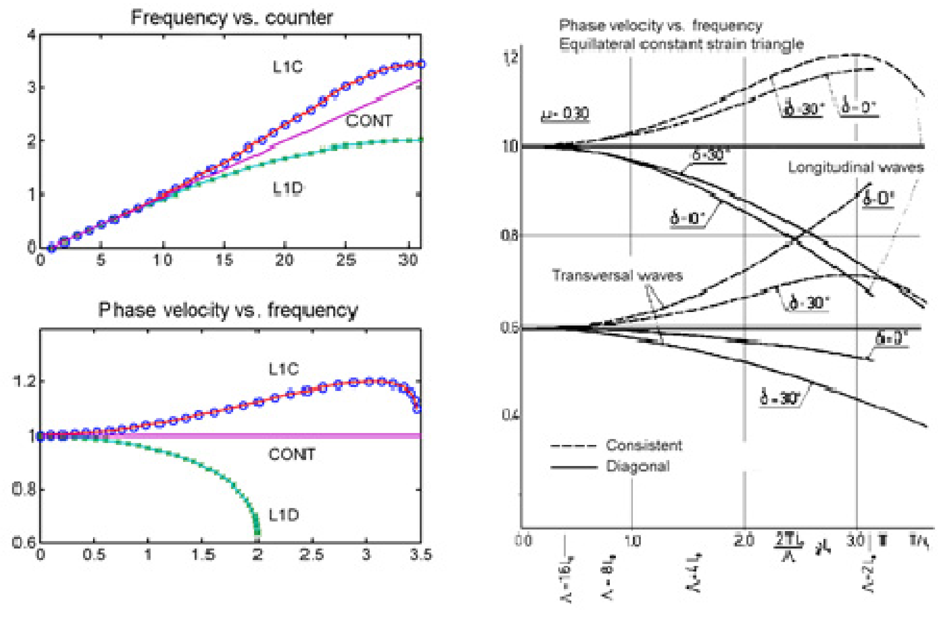


Fig. 5. Dispersion properties for constant strain elements in 1D and 2D

On the $x$-axis there are indicated values of another parameter, namely $\Lambda = k \times l_0$, where $k$ is the number of elements that could be 'squeezed' into the considered wavelength. One sees that if there are 16 elements into a wavelength, then the dispersion error is negligible – for only 2 elements, the dispersion error is considerable. On should emphasize that the dispersion errors could be minimized, but not completely eliminated.

Generally, the results of modelling should be independent of the method being used. This is not always so as shown in Fig. 6, where the same task – propagation of a halfsine pulse – is treated using consistent and mass matrix formulations respectively, resulting in different positions of the dispersion side effects, i.e. the ‘false’ vibrations appearing – either in front of or behind the pulse.

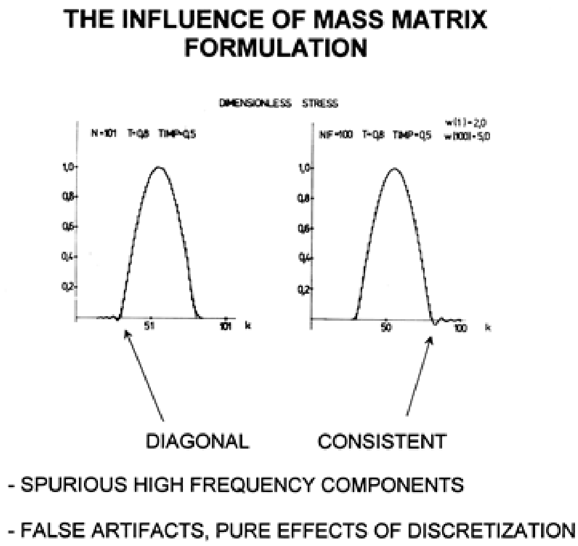


Fig. 6. Consequences of mass matrix formulation

The dispersion analysis of higher order 1D elements (i.e. quadratic and cubic), see [7] revealed that using these elements in 1D, we get the a chance to increase the cut-off limit of the highest available frequency but at the expanse of higher computing costs and of the appearance of embedded band pass filters. The dimensionless phase velocities as functions as dimensionless frequencies for 40 linear (L1), 20 quadratic (L2) and 13 cubic (L3) finite elements are depicted in Fig. 7. Approximately, the same number of degrees of freedom was used. One can observe that for the higher order elements, the first part of dispersion curves are ‘improved’, the dispersion curves, however, consist of distinct branches that are separated by certain frequency regions (called the cut-off filters) through which the particular frequencies cannot propagate. For the L2 element, the second part of the dispersion function is called the *acoustic branch*. For the L3 element, there is also the third part which is traditionally called the *optical branch*. The attributes C and D appearing by the L1, L2 and L3 identifiers signify the consistent and diagonal mass matrices formulations. Generally, using the higher order elements we get a longer spectrum with ‘acceptable’ dispersive properties, not differing much from that of the ideal continuum. But at the cost.

The effects of dispersion induced anisotropy in 2D modelling of stress wave propagation are shown in Fig. 8 for a body assembled of square elements with bilinear shape functions. You might notice it is direction dependent. Different meshes, characterized by the ratio of the mesh size ($q$) to the wave length ($\Lambda$), are considered.

The polar dispersion diagrams for bilinear and serendipity biquadratic finite elements with the consistent mass matrix and for four different dimensionless wavelengths of propagation wave are shown in Fig. 9.

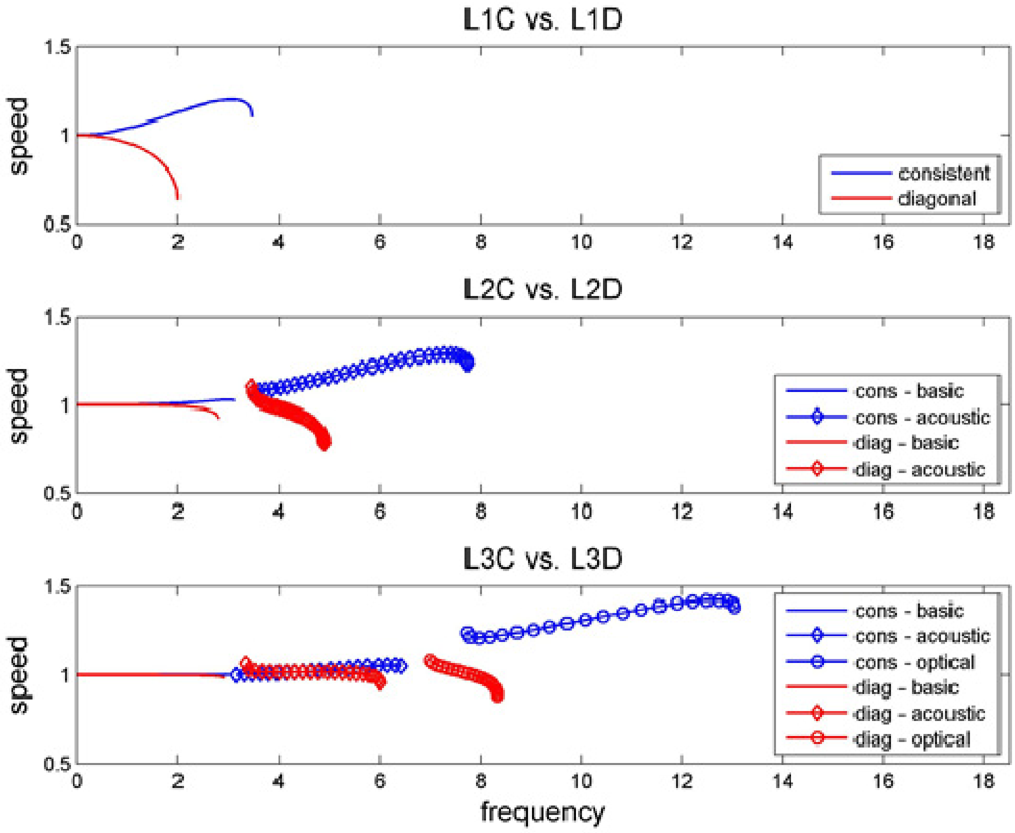


Fig. 7. The frequency spectra of higher order 1D elements

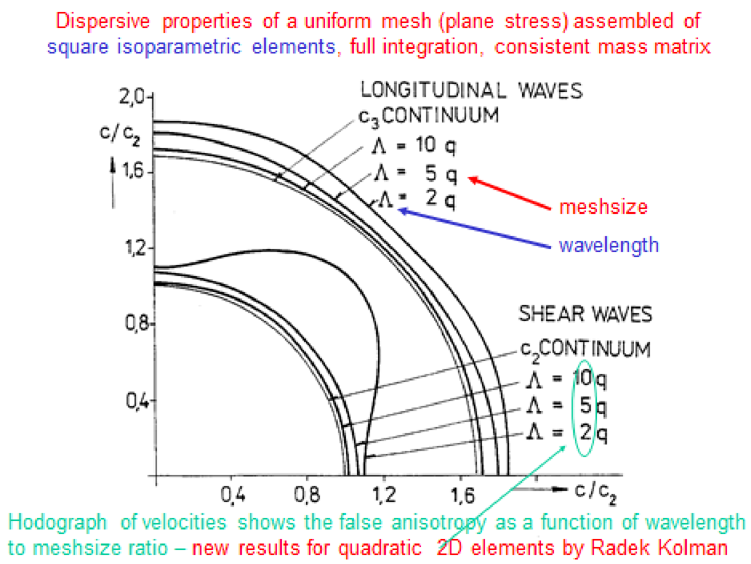


Fig. 8. Bilinear square element

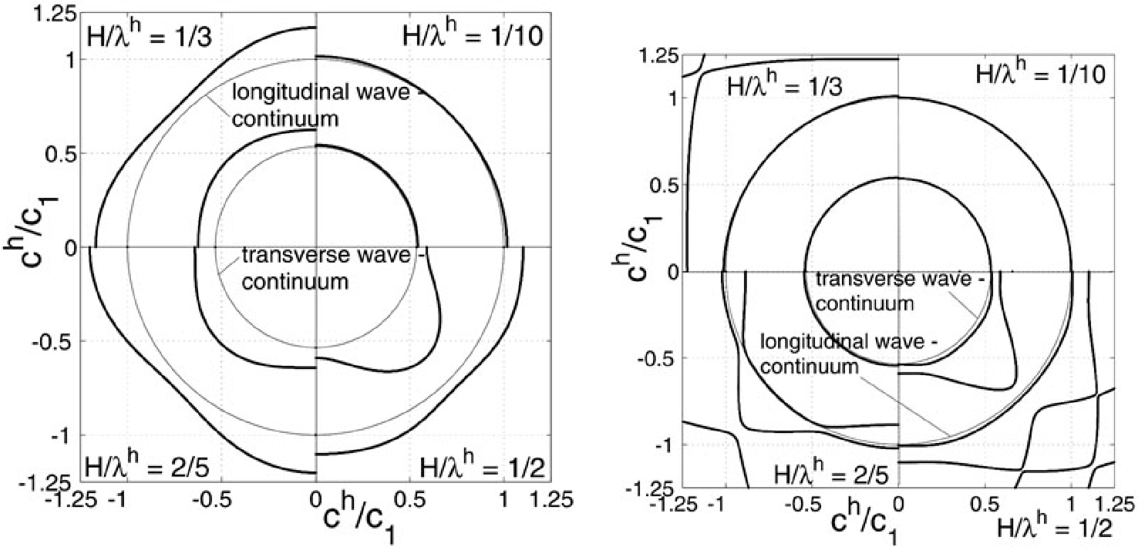


Fig. 9. Quadratic square elements

Polar dispersion diagrams for a plane square 4-node bilinear (on the left) and 8-node serendipity biquadratic (on the right). Finite elements with the consistent mass matrix for four different dimensionless wavelengths are presented. See [11].

**Example 3. – Computational and experimental thresholds**

Limits of all kind of analyses (analytical, numerical and/or experimental) depend on the observational resolution – threshold, which is typically a minimum value of a signal that can be detected by the system.

Validity limits are checked by comparison with analytical methods, comparison with experiment, self-assessment, and statistical tools and by FFT analysis.

In this paragraph, the investigation of the stress wave propagation in an impact loaded shell by Double Pulse Holographic Interferometry is compared with a corresponding finite element analysis. The task is schematically depicted in Fig. 10. The shell was loaded by an exploding wire.

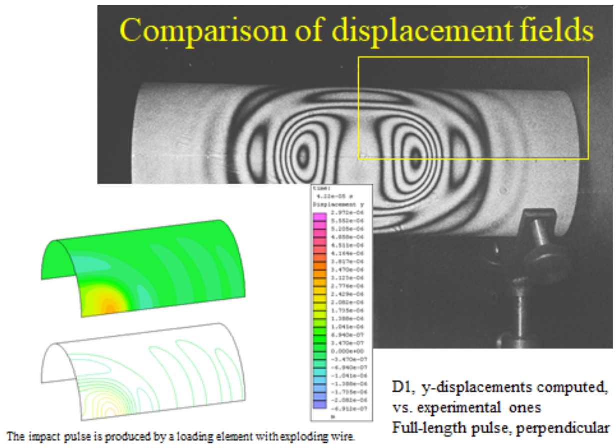


Fig. 10. Impact loaded shell

Velocities, as functions of time, were experimentally recorded and compared to those obtained by finite element computations with two different mesh densities. A rather unsatisfactory agreement or both approaches is presented in Fig. 11.

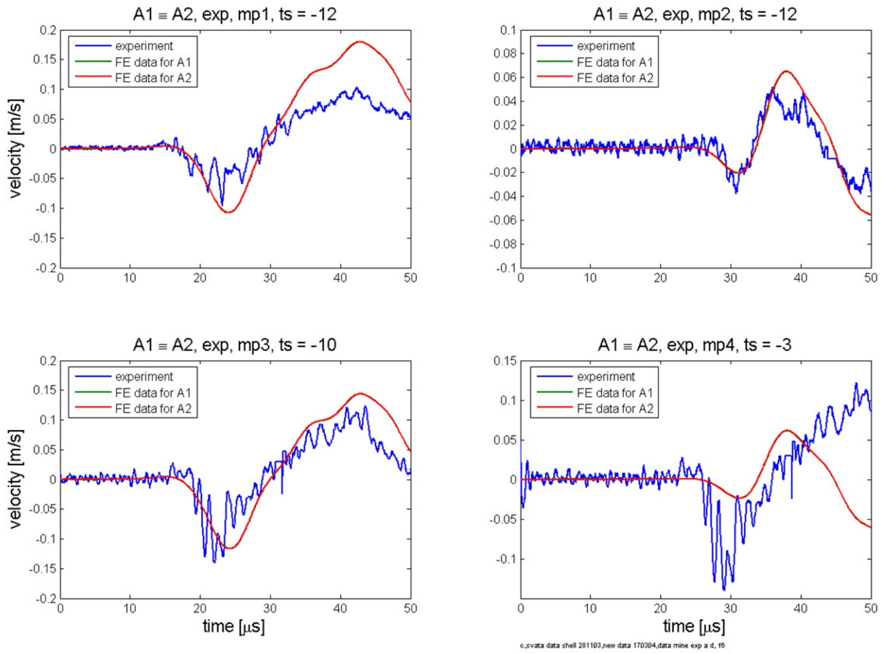


Fig. 11. Experiment vs. two FE meshes at four locations

*Finite element limits*

The FE shell results for the coarse (A1) and fine (A2) meshes are not distinguishable in the scale of the figure. Since there is no visible difference of results with mesh refinement one can conclude that time step and mesh size are set correctly. From this fact, however, we cannot deduce that the results are 'physically correct'. They are 'correct' within the validity range applicable to this particular FE model – the shell element a priori assumes what is happening within the shell thickness and does not take into account the actual wave processes occurring there.

*Experimental limits – the first part*

Applying the *Matlab improfile function* to experimental data allows determining both the absolute and relative thresholds of the measurement used. The procedure is outlined in Fig. 12.

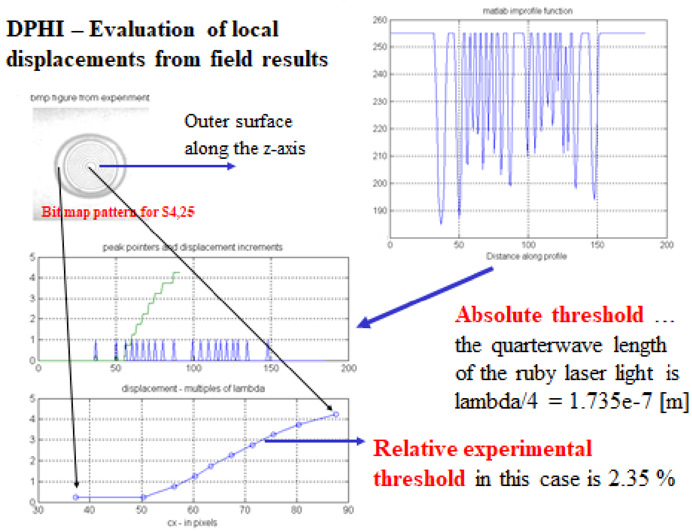


Fig. 12. Experimental threshold analysis

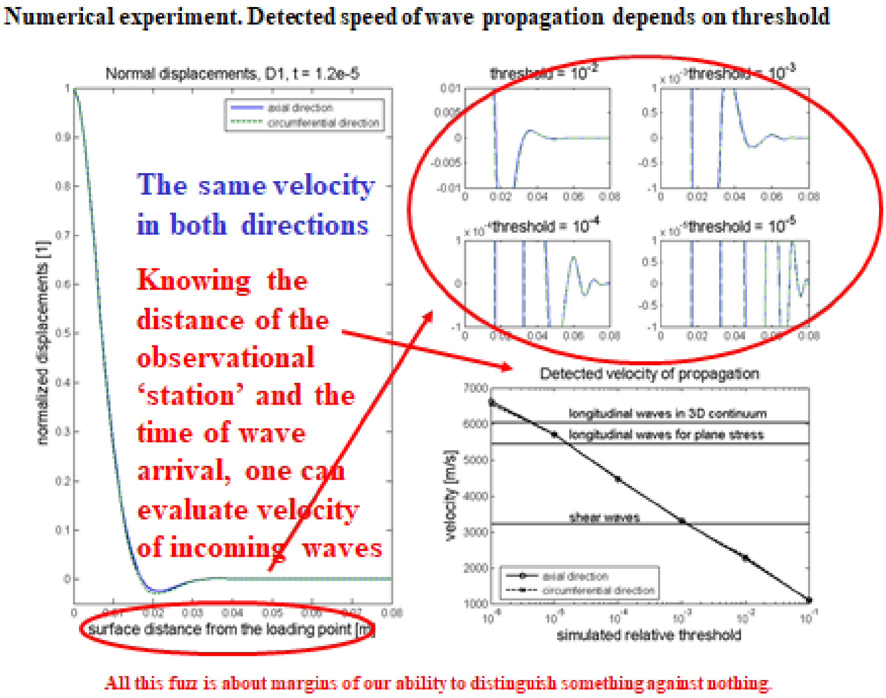


Fig. 13. Numerically simulated threshold. The registered speed of propagation depends on the threshold.

The threshold issue is important when the speed of propagation is to be determined by experimental means. Observing the first measurable response at a certain time in a given distance from the loading point, one can estimate the propagation velocity. This way the estimated value depends on the threshold value of an apparatus being used for the measurement of a particular physical quantity and is usually constant at the experiment site.

The observational threshold is usually constant for the considered experimental setup being used for the measurement of a particular physical quantity.

The influence of the threshold value on the measured results could be achieved by a numerical thought experiment.

What would be a common sense approach? Sitting at a certain observational node, whose distance from the loaded node is known, one would estimate the speed by measuring the time needed for the arrival of the 'measurable' or 'detectable' signal. And the measurable signal is such a quantity that is – in absolute value – greater than a 'reasonable' observational threshold. And what is a proper value of it is a good question.

Imagine a standard finite element double-precision computation giving at a certain time the spatial distribution of displacements at a node on the surface of a body. Assume that the distance of our observational node from the loading node is known. Now, let's set a 'reasonable' value of the threshold and apply a sort of numerical filter on obtained displacements, which erases all the data whose absolute values are less than the mentioned value of the threshold. This way, for a given threshold value, we get a certain arrival time and from the known distance we obtain the propagation speed. Working with displacements normalized to their maximum values allows us to consider the threshold values as the relative ones.

Varying the simulated threshold value in the range from $10^{-6}$ to $10^{-1}$ we will get a set of different velocities of propagation. As a function of the simulated threshold they are plotted in Fig. 13. Material constants for the standard steel were used. The horizontal lines represent the theoretical speeds for longitudinal waves in 3D continuum, for longitudinal plane stress waves in 2D continuum as well as for the shear waves. Obviously, the shear wave speeds are identical both for 3D and 2D cases.

It is known that the longitudinal waves carry substantially less amount of energy than these of the shear and Rayleigh waves and that the surface response, measured in displacements or strains, is of substantially less magnitude for the former case.

One can conclude that for a correct capturing of the longitudinal velocity value, the relative precision of at least of the order of $10^{-6}$ is required. This is a tough request. The relative threshold of the order of $10^{-3}$ is more common in experimental practice. However, in an experiment with the relative precision of the order of $10^{-3}$, one would not detect the arrival of longitudinal waves and might wrongly conclude that the first arriving waves are of the shear nature and would wrongly estimate the velocity of propagation of the order of 3000 m/s.

All this fuzz is about margins of our ability to distinguish something against nothing. This is, however, crucial for any meaningful human activity.

Physical and technical aptness and reliability of the model depends not only on a threshold value but – and mainly – on application of the model within the limits for which it was constructed, i.e. proper satisfaction of initial and boundary conditions, material assumptions, mesh properties, etc.

*Experimental limits – the second part*

Trying to explain the differences in velocity distributions, presented in Fig. 11, we focused our attention to the data related to the experimental loading process. As mentioned before, the experimental loading was secured by exploding wires. Actually, four experiments were carried out to get velocity distributions at the above-mentioned four locations. The time distribution of four randomly taken pulses is depicted, together with an ‘average pulse’ that was used as an input for the numerical computation, in the first row of Fig. 14.

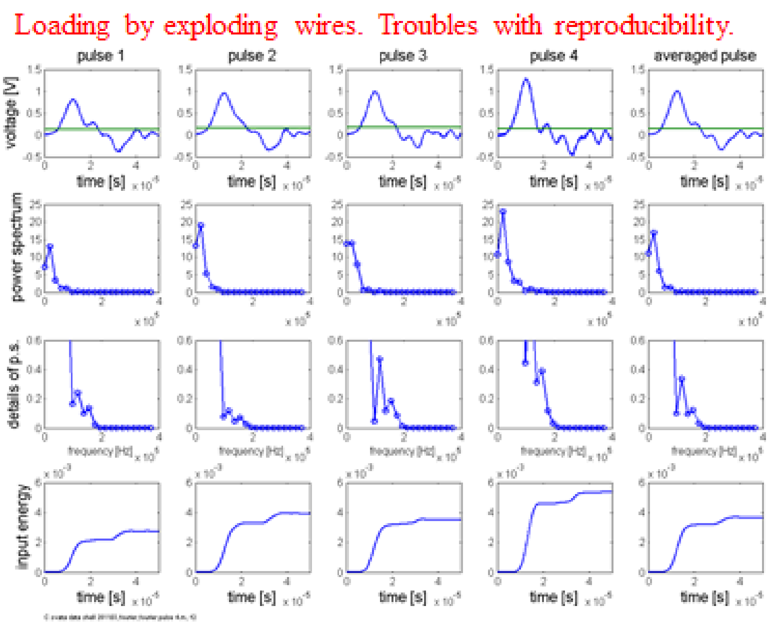


Fig. 14. Troubles with reproducibility

Shown discrepancies in time distributions of kinematic quantities, computed for individual pulses, indicate that differences in the loading distributions play a more important role than originally believed.

In two subsequent rows of Fig. 14 one can observe corresponding Fourier's power spectra of loading pulses in full and detailed views. The spectra were computed a posteriori by FFT. First twenty discrete FFT frequencies are plotted only. Having, however, a proper hardware or using a real time simulation package, the FFT analysis is readily available at the experimental site as well, and could be obtained immediately after the experiment (meaning the explosion of the wire) has been carried out.

The fourth line of subfigures presents the sum of potential and kinetic energies supplied by the particular pulse to the loaded body. This quantity – a result of FE computations – is not readily available to the experimentalist.

There are, however, other quantities – available to the experimentalist – that might help. Realizing that the first term of the Fourier's transform analysis is $c_0 = \int_{-\infty}^{\infty} P(t)\, e^{\mathrm{i}0t}\, \mathrm{d}t = \int_{t_{\mathrm{initial}}}^{t_{\mathrm{final}}} P(t)\, \mathrm{d}t$, reveals that its physical meaning is the mechanical pulse. Denoting the discretized time evolution of reference pulse $\{\mathcal{P}\} = \{\mathcal{P}_1\ \mathcal{P}_2\ \ldots \mathcal{P}_n\}^{\mathrm{T}}$ we can express its mean value $\mathcal{P}_{\mathrm{mean}} = \frac{1}{n}\sum_{i=1}^{n} \mathcal{P}_i$ and the approximate the area under the pulse $\int_{t_{\mathrm{initial}}}^{t_{\mathrm{final}}} P(t)\, \mathrm{d}t$ by $\mathcal{P}_{\mathrm{mean}}\, \Delta T$. Which means the same.

For given pulses, this correlation is shown in the first column of Fig. 15. Without going to details one can show that there is another useful correlation, namely, between the Euclidian norm of the pulse quantities and the square root of input energy. For details see [4].

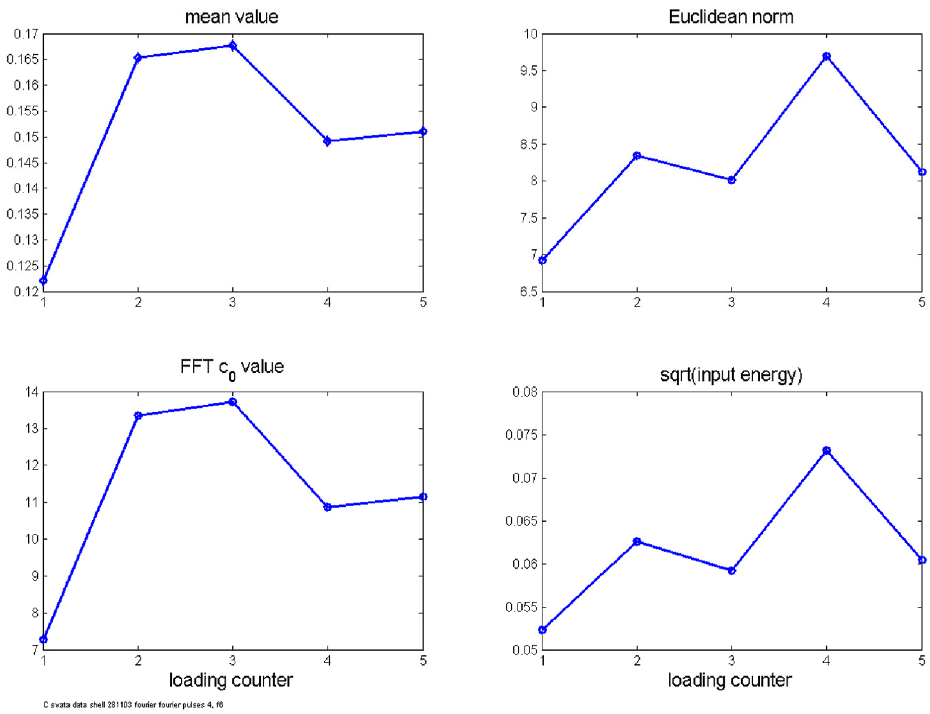


Fig. 15. A hint for analysis of experimental data – statistics and FFT

**Example – 4. In FE analysis, the speed of stress wave propagation depends on the integration operator**

In order to assess the reliability and precision of time integrating methods, a stress wave propagation task was computed using Newmark (denoted NM in the text) and central difference methods (CD). Comparison of axial strains at a certain location of the considered body, obtained by both methods, is presented in Fig. 23. The same time integration step (1e-7 [s]) was used in both cases. For the NM method the consistent mass matrix was employed, while the diagonal mass matrix was used for the CD method.

The left-hand subplot presents the strains in the whole computed time range showing the excellent agreement of results due to both approaches. The agreement documents that the accepted space and time discretizations are suitable for this geometry and loading – the differences are minimized. Two couples of horizontal lines indicate the areas that are shown in detail the in right-hand side subplots.

The difference between NM and CD solutions can be numerically assessed by means of a cantered correlation coefficient which can be geometrically interpreted as an angle between NM and CD time vectors by means of the cosine of an angle between two vectors.

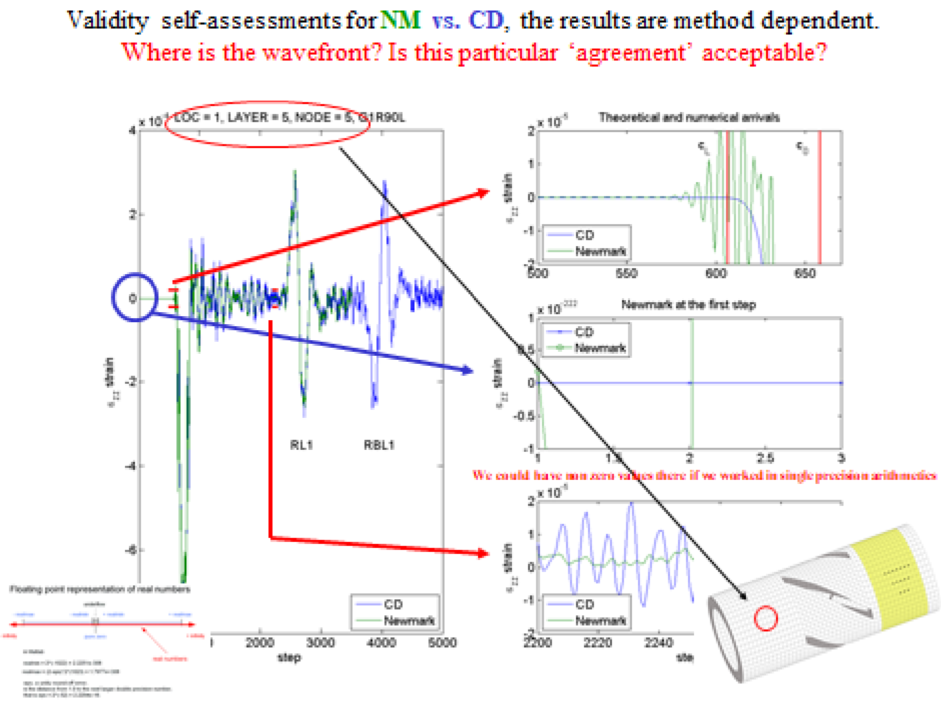


Fig. 16. Different speed of propagation for NM and CD operators

In the upper right-hand subplot of Fig. 23 the positions of theoretical arrivals of 3D longitudinal and 1D waves are indicated by red vertical lines. The actual axial strain distributions computed by the NM and CD methods are shown as well. From the analysis of dispersion properties of finite elements and that of time integration methods it is known that the computed speed of wave propagation for the CD approach with diagonal mass matrix underestimates the actual speed, while the NM approach with consistent mass matrix overestimate the actual speed. The presented results nicely show this.

What is less known is the fact, that the speed of propagating waves with NM-consistent modelling is actually 'infinitely' large.

To find the response of a body in time due to the impact loading requires solving the system of differential equations in which the inversion of stiffness or mass matrices (depending on the mass matrix formulation and on the time integration method being employed) is needed. [13]. This way, the sparse structure of relevant matrices might vanish, and a sort of unlimited range of influence (both in space and time) could prevail. If we worked with infinite number of significant digits we would get a nonzero response over the whole domain for any time greater than zero. Working, however, with standard precision arithmetic we will get no ‘measurable’ response in domains where the computed values are, in absolute value, less than the computational threshold – the smallest positive floating point number. Besides, the limited range of influence is secured by the fact that a substantial part of the energy is always carried by low frequency components. See [12].

Results of modelling should be independent of the method of solution, but in practice the agreement is both the method and the threshold dependent.

The shown infinite speed of propagation in FE analysis is an artefact of a particular method of treating differential equations in time by Newmark method. Even if this fact is fundamentally wrong, it does not devaluate the solution itself because its manifestation is well below the ‘reasonable’ threshold of observation.

From the engineering point of view the presented agreement is acceptable. Two methods are giving similar results, which a necessary but not sufficient condition for our belief that we are on the right track. From the point of view of numerical analysis theory it is important to be aware of these artefacts

**Limits of finite element analysis**

For fast transient problems as shock and impact the high frequency components of solutions are of utmost importance. In continuum, there is no upper limit of the frequency range of the response. In this respect continuum is able to deal with infinitely high frequencies. This is, however, a sort of singularity deeply embedded in the continuum model.

As soon as we apply any of discrete methods for the approximate treatment of transient tasks in continuum mechanics, the value of upper cut-off frequency is to be known in order to ‘safely’ describe the frequencies of interest.

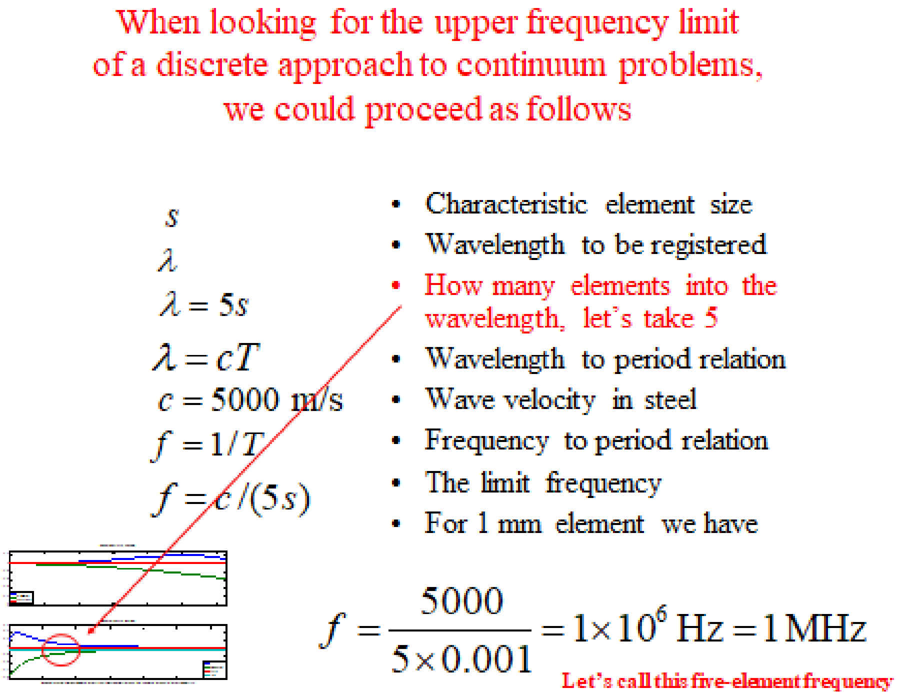


Fig. 17. Element size vs. frequency

A procedure leading to approximate correlation between the element size and the maximum frequency – the model assembled of elements of that size could safely transmit – is outlined in Fig. 17.

A rule of thump, which is easy to remember, is that a 1mm element is ‘good’ for 1 Mhz frequency.

**Example – 5. Impact induced stress wave energy flux.** Validation of FE and experimental approaches.

The temporal and spatial distribution of the stress wave energy flux in an axially impacted cylindrical tube, whose middle part contains four spiral slots, is studied experimentally and numerically. The high-speed recording of transient surface strains was used in the experiment, based on 1D theory, while the 3D finite element treatment was employed for the numerical analysis. The aim of the paper is to ascertain how reliable is the energy assessment based on transient recordings of surface strains and on subsequent 1D wave theory reasoning. The presented study is to determine how much of the impact energy, which is predominantly of axial (longitudinal) nature, is transferred into torsional (or shear) energy mode as well as into other energy modes not seen by the experiment. The task is sketched in Fig. 18.

Considering the linear elastic continuum and the validity of Hook's law in the form $\sigma_{ij} = C_{ijkl}\, \varepsilon_{kl}$, the internal stress power and the time rate of kinetic energy are as follows $W_{\mathrm{int}} = \dot{E}_{\mathrm{s}} = \dfrac{\mathrm{d}}{\mathrm{d}t}\int_V \tfrac{1}{2}\, C_{ijkl}\, \varepsilon_{kl}\, \varepsilon_{ij}\, \mathrm{d}V$ and $W_{\mathrm{kin}} = \dot{E}_{\mathrm{k}} = \dfrac{\mathrm{d}}{\mathrm{d}t}\int_V \tfrac{1}{2}\, \rho\, \dot{u}_i\, \dot{u}_i\, \mathrm{d}V$. The details, describing how these genetic formulae were evaluated in the FE analysis and in the experiment, are in [12].

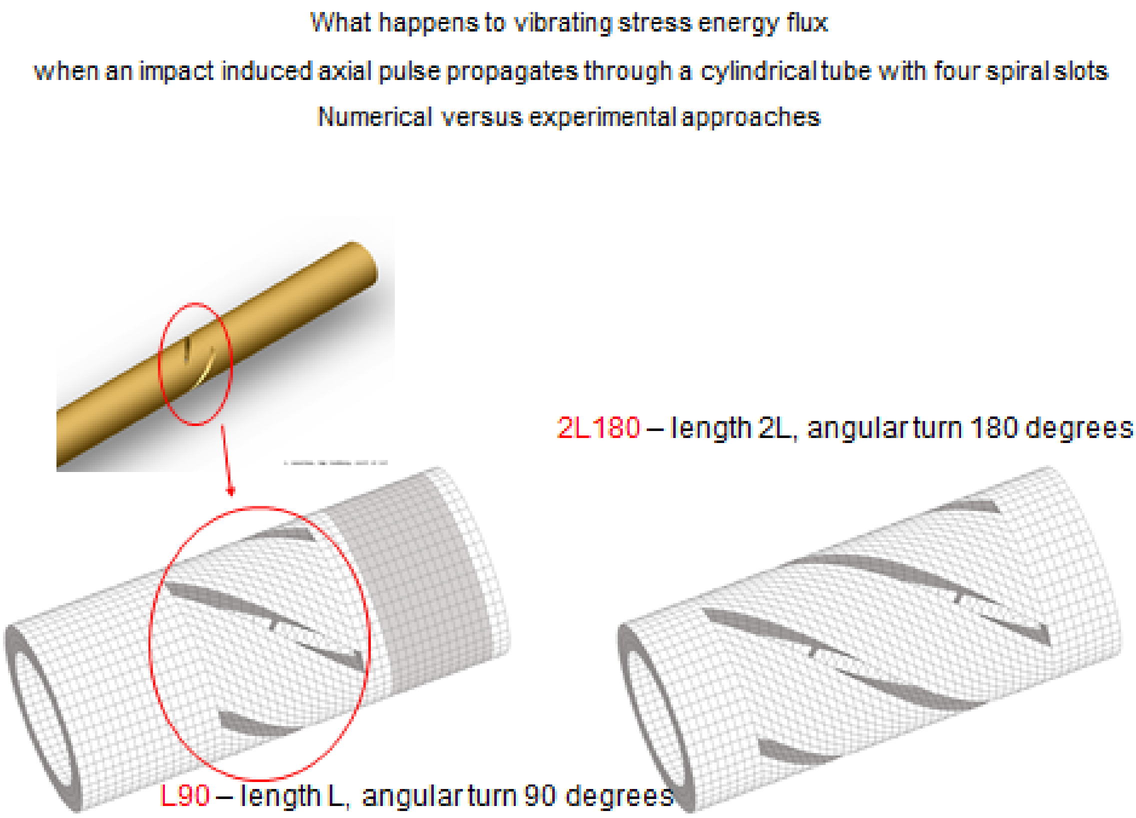


Fig. 18. FE approach

*In FE analysis* the energy flux computation can be more naturally assessed by analyzing the time history of energy in the whole body and in its parts.

*In experimental analysis* it is convenient to evaluate the amount of energy in the body as the cumulative energy flux through a cross sectional area, observed within a specified time interval.

The strain gauge rosette orientation and the gauge locations on the surface of the tube are indicated in Fig. 19. To avoid parasite bending signals the same strain gauges were positioned at the opposite parts of the surface. The 'experimental' tube is longer then the 'finite element' one, but the signals, reflected from the supported end of the tube, comes to the measurement locations long after the signal recording has been finished. A standard Wheatstone bridge set-up with and a fast transient recorder were employed.

The details of FE computations with emphasis on the proper choice of element sizes with respect to frequency contents to be transferred and of determination of integration methods and used time steps and an of experimental setup are in [12].

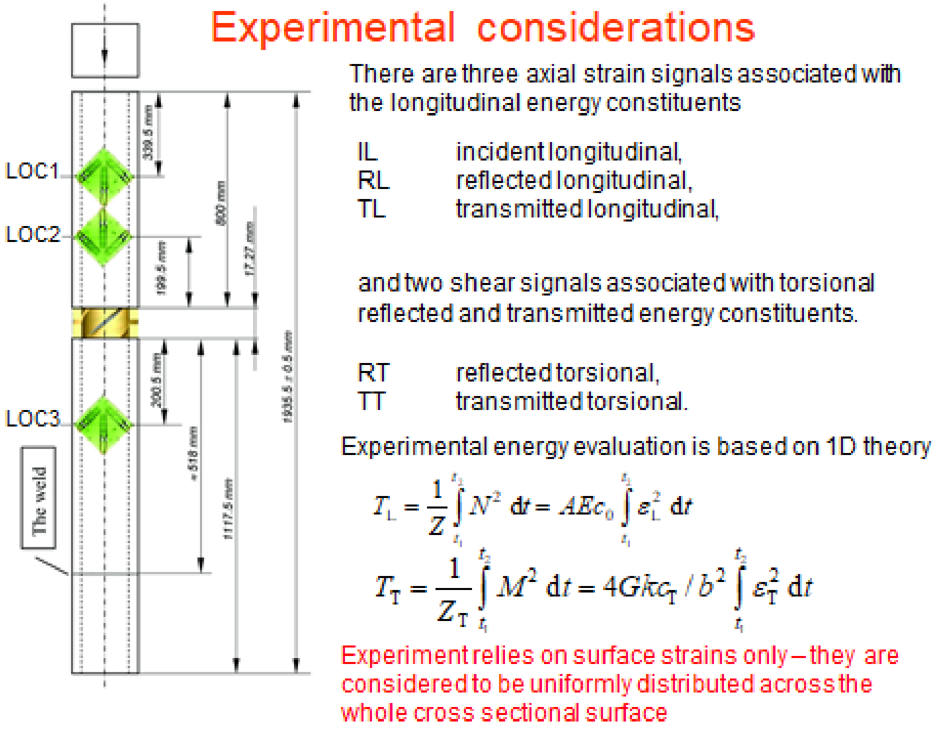


Fig. 19. Experimental approach

Comparison of axial strains obtained by an experiment and by the FE element analysis is in Fig. 20.

Is it a bad or a good agreement? Is experiment or FEA closer to reality? Where is the truth? Experimental people often say to FE analysts: All your high frequency components are just a numerical noise.

The cut-off frequency of experimental setup had the value of 100 kHz. The Nyquist frequency for the FE analysis, based on the integration timestep – which is a sort of experimental sampling interval – is 0.5 MHz.

There are many questions. Among them: What are the experimental limits in this particular case? Can all high-frequency components be attributed to numerical noise?

Possible answers: The 3D finite element analysis is compared with the experiment based on the 1D theory. Only axial and shear strains are measured – radial ones are not. The values of measured surface quantities (displacements, strains, velocities) are attributed to the whole cross-sectional area. 1D wave theory is used. Smaller frequency sampling rate is employed.

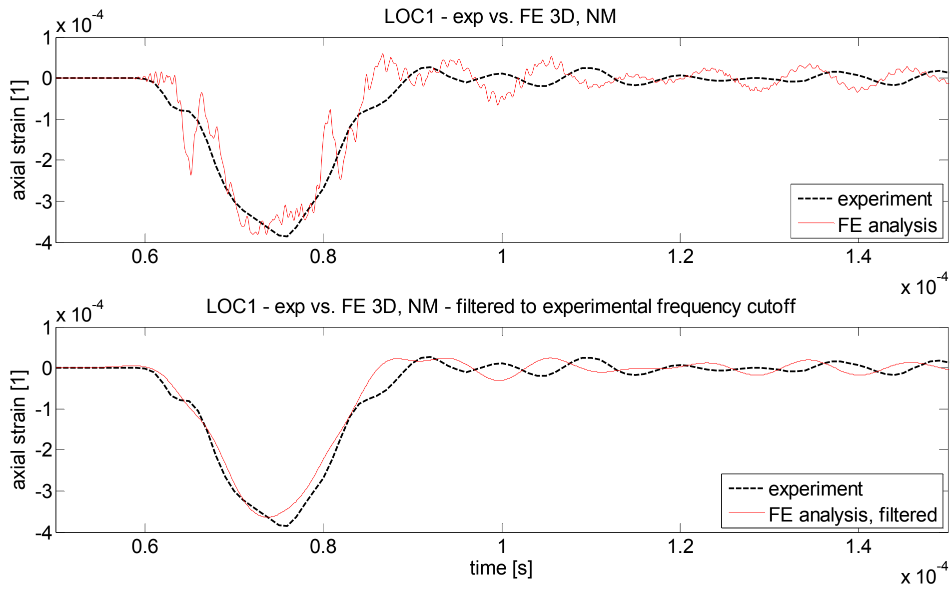


Fig. 20. Comparison of axial strains. Raw and filtered data.

The experiment, as conceived in this case, could not 'catch' the 'actual' frequency components higher than its upper frequency limit.

In this case the upper frequency limit of FE analysis is substantially higher, so the FE spectrum is longer.

A question arises what is the range of computed frequencies which are 'correct', especially in view of the fact that the FE transfer spectrum is method dependent. Notice the differences for NM and CD treatment.

Since an experiment with a finer time and frequency resolution is not available, the FE analysis should help itself to answer the question. Self-assessment by mesh- and timestep refinement could help. Remember the Richardson method, known from quadrature analysis, where the subsequent halving the integration increment is used for the quadrature error estimation.

Having small differences between two alternative approaches does not automatically imply that the results are correct. It only means that for a given loading and the employed time and space discretizations, there is almost no 'measurable' difference between results obtained by two types of approaches. One has to realize that the existence of close solutions, stemming from alternative approaches, is only a necessary, but not a sufficient, condition of 'correctness'. And what is 'correct', in the sense of correct modeling the Mother Nature, is difficult to define.

**Limits of continuum, FE analysis and experiment**

These limits are schematically depicted in Fig. 22, which is a sort of extension of Fig. 17, where element sizes vs. frequency are depicted. This time, the limit of continuum is limited from above by the austenite steel grain size, and far far above by the atom size. Maximum frequency that we are able to capture by the first class transient recorders is of the order of 1GHz. A range of a practical frequency range of FE analysis is depicted as well. This information might be complemented by a piece of wisdom attributed to S.C. Hunter who claims "To neglect corpuscular structure of matter the specimen should be at least $10^4$ times larger than the interatomic distance.

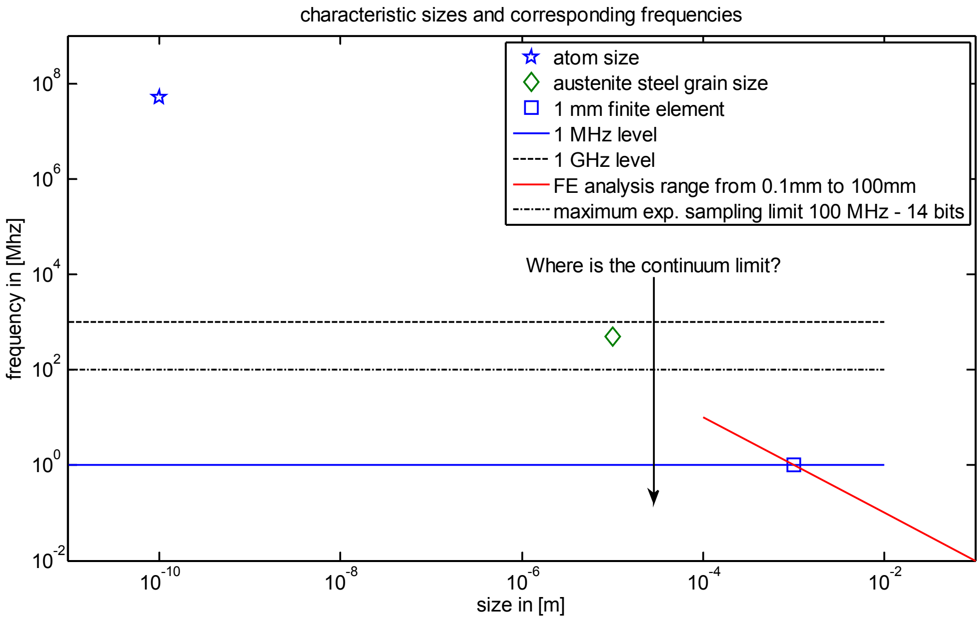


Fig. 22. Limits

**Conclusions**

Mechanical theories, principles, laws and models, used in engineering practice, cannot be proclaimed true or false. They are either right (working to our satisfaction) or wrong. Regardless of being simple or complicated, they are 'right', if approved by an appropriate experiment (i.e. conceived in agreement with accepted assumptions of the theory). History reveals that wrong theories might appear, but not being confirmed by experiments, are quickly discarded as ether or flogiston. Theories are right only within the limits of their applicability. We cannot claim that a theory being proved by an experiment is right. The only thing we can safely state is that such a theory is not proved wrong.

Generally, a singularity appearing in a model always means a serious warning concerning the range of validity of that model. Usually, a more general model – having a wider scope of validity – is invented removing that singularity. Very often there is no need to discard the older and simpler model, since it might perfectly work in the validity range for which it was conceived. The role of doubts on our way to understanding the nature is far from negative.

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