

## Determination of group velocities of Lamb waves in unidirectional carbon-epoxy plate

Zuzana Lašová<sup>1,a</sup>, Robert Zemčík<sup>1,b</sup>

<sup>1</sup>Faculty of Applied Sciences, University of West Bohemia in Pilsen, Czech Republic

<sup>a</sup>zlasova@kme.zcu.cz, <sup>b</sup>zemcik@kme.zcu.cz

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**Abstract.** The group velocity of zero-order symmetric Lamb wave mode ( $S_0$ ) in unidirectional composite plate is strongly dependent on the direction of propagation. The group velocities in chosen directions were determined experimentally using piezoelectric patches. The collected data were processed by wavelet transform to find a time-of-arrival of wave packages relevant to the  $S_0$  mode. Group velocities in more directions of propagation were calculated using finite element analysis. The numerical results show good agreement with measured velocities.

### Introduction

Lamb waves are a kind of elastic waves propagating in plates and hollow cylinders. They have attracted attention of many researchers due to their potential use in detection of structural flaws known as structural health monitoring. This is especially useful in case of layered composite structures suffering by internal hidden flaws such as delamination [3, 4]. Structural health monitoring is based on extraction of information from time signals collected by a network of sensors applied on the structure. In each time record, forward and reflected wave packages need to be identified precisely to identify those reflected by the damage.

The group velocity can be understood as a speed of transfer of energy carried by wave packages. It is generally defined by

$$c_g = \frac{\partial \omega}{\partial k}, \quad (1)$$

where  $\omega$  is angular frequency and  $k$  is wavenumber. With the knowledge of group velocities it is possible to distinguish the wave packets associated with unique wave modes. There is theoretically infinite number of Lamb wave modes, but they can be divided to two families: symmetric ( $S_n$ ) and anti-symmetric ( $A_n$ ) modes in respect to the middle-plane. In low frequency range only two fundamental modes,  $S_0$  and  $A_0$ , exist.

The group velocity of symmetric zero mode ( $S_0$ ) in transversally isotropic composite plate is strongly dependent on angle of wave propagation in respect to direction of fibers. Rhee et al. [2] used ultrasonic pulser-receiver to find a group velocity curve in respect to angle of propagation in unidirectional composite plate. Jeong [1] used a lead break as a source of acoustic emission and two piezoelectric sensors to determine group velocities in quasi-isotropic laminate plate.

In this work, three piezoelectric transducers were used for experimental determination of group velocities in directions of propagation  $0^\circ$ ,  $25^\circ$ ,  $45^\circ$  and  $90^\circ$  in respect to the fiber direction. It is not always possible to localize time of arrival of Lamb wave modes in temporal

domain, therefore methods of time-frequency analysis have to be applied. For the processing of recorded time signals wavelet transform (WT) is preferred by most of authors [4, 5].

### Wavelet transform

The time signals obtained by measurement or by finite element analysis were processed by wavelet transform. This method is advantageous mainly because the time signals are non-stationary and have character of harmonic waves. Continuous wavelet transform (CWT) of arbitrary function  $f(t)$  is defined as [1]

$$CWT_f(\tau, s) = \int_{-\infty}^{\infty} f(t)\psi_{\tau,s}^* dt. \quad (2)$$

The CWT formally resembles Fourier Transform, but the difference is in the basis function  $\psi_{\tau,s}$  which is derived from mother wavelet function  $\psi$  by scale  $s$  and time shift  $\tau$ :

$$\psi_{\tau,s} = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right). \quad (3)$$

Variable scale and time shift provide better resolution both in frequency and time, which is crucial in determination of time of arrival of wave packets. There are some options for the choice of mother wavelet function, in this work it was a complex Morlet wavelet:

$$\psi = e^{i2\pi ft} e^{-\frac{t^2}{2s}}. \quad (4)$$

### Measurement of group velocities

The group velocity was measured in unidirectional carbon-epoxy plate having dimensions  $770 \times 770 \times 1.8$  mm hanged on a frame by elastic strings and secured in vertical position by a clip (Fig. 1).

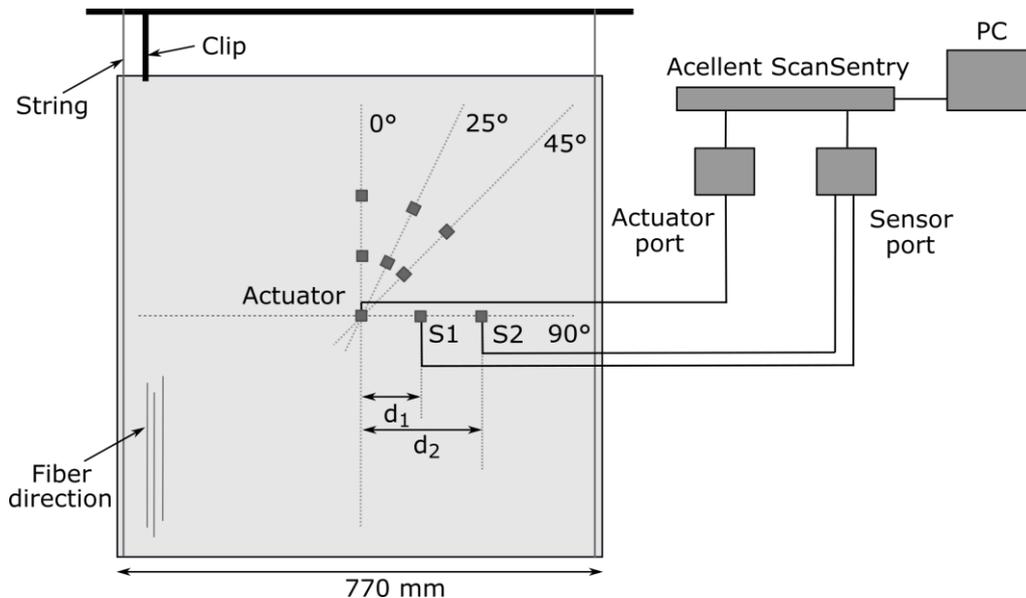


Fig.1 Measurement set-up

The transducers were piezoelectric patches DuraAct P-876.SP1. These transducers consist of piezoceramic layer ( $10 \times 10 \times 0.5$  mm) and a protective plastic foil ( $15 \times 15 \times 1$  mm). The patches

were attached to the surface of the plate by double-sided tape. The actuator was placed in the center of the plate and the pair of sensors was sequentially collocated in directions  $0^\circ$  (direction of fibers),  $25^\circ$ ,  $45^\circ$  and  $90^\circ$  in constant distances  $d_1 = 100$  and  $d_2 = 200$  mm from the actuator.

The elastic waves in the plate were excited and measured by Acellent ScanSentry, which is special system for the structural health monitoring. This device serves both as pulse generator (with built-in amplifier) and one-channel data acquisition system. The exciting pulse was five-cycle sine wave burst with central frequency of 200 kHz modulated by Hann window (Fig. 2).

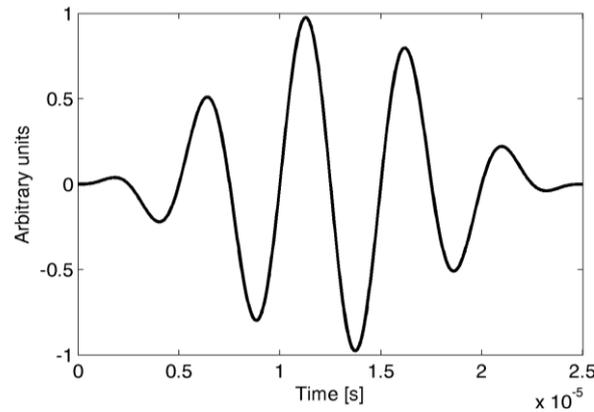


Fig. 2 Five-cycle sine burst with frequency 200 kHz

The time signals from pairs of sensors (see Fig. 3 a) and b)) were processed by wavelet transform. The time history of frequency component 200 kHz (Fig. 3 c)) was obtained as a relevant row of the scalogram (matrix of real parts of WT coefficients). The peaks in this graphs correspond to the passages of wave packages.  $S_0$  has the highest group velocity, therefore it can be identified as the first significant peak [5].

As we know the mutual distance of the sensors, the group velocity was determined from the difference of times of arrival of corresponding wave packages. Calculated group velocities of  $S_0$  mode in investigated directions of propagation are presented in Table 1.

Tab. 1 Experimentally determined group velocities for frequency 200 kHz.

Angle of propagation [ $^\circ$ ]	0	25	45	90
Group velocity [ $\text{ms}^{-1}$ ]	8791	4600	2860	2272

### Finite element model of the plate

The finite element model was a one half of the plate of same thickness 1.8 mm as measured plate and dimensions  $770 \times 375$  mm with directions of fibers parallel to  $x$ -axis. The reason for using this model was that the group velocity in the longitudinal direction is expected to be up to 4 times higher than the group velocity in transversal direction. Otherwise the wave packets in longitudinal direction reflected from the edges may reach the measured nodes before transversal wave packets.

The model was created using linear hexahedral elements with 2 elements per thickness. Material parameters were partly measured and taken from literature. Density  $\rho$  was determined experimentally. Tensile moduli  $E_{11}$ ,  $E_{22}$  and Poisson's ratio  $\nu_{12}$  were measured by tensile tests of specimens  $0^\circ$  and  $90^\circ$  cut from the plate. The shear moduli  $G_{12}$  and  $G_{23}$  were taken from [5]. All material data are presented in Tab. 2. The propagation of Lamb waves was modelled in Abaqus v.6-14 explicit solver.

It is not possible to include piezoelectric behavior in the explicit solver, therefore Lamb waves were excited mechanically. The time-variable displacements in-plane  $u_x$  and  $u_y$  identical with the actuating pulse in measurement (five-cycle sine burst with frequency 200 kHz) were applied to the nodes in the central location of the actuator.

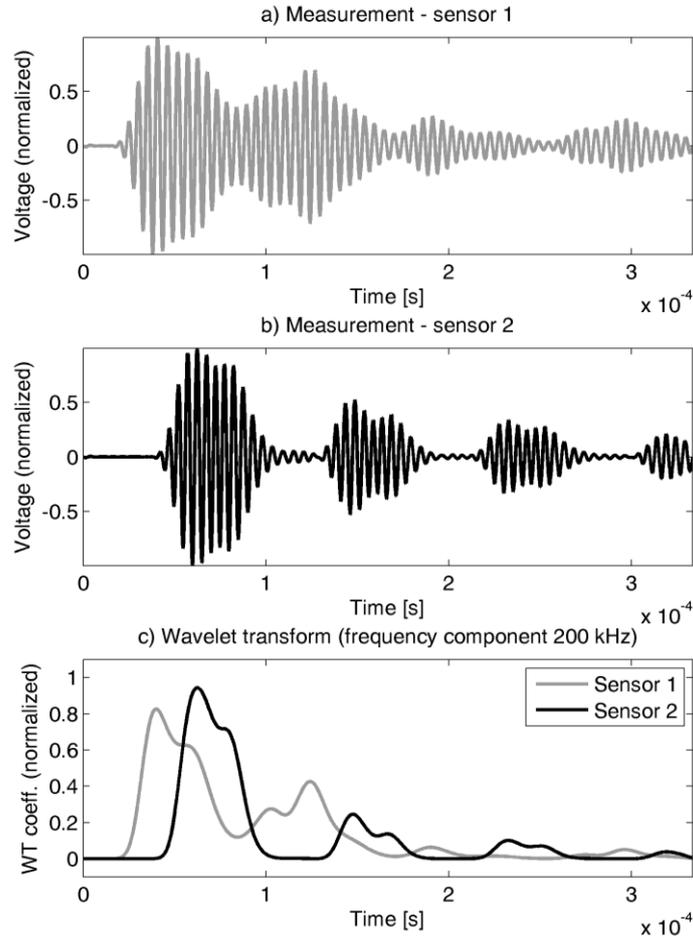


Fig. 3 Time records from a pair of sensors (a, b) and time history of frequency component 200 kHz (c).

Tab. 2 Material parameters of composite plate.

$\rho$ [kg m <sup>-3</sup> ]	$E_{11}$ [GPa]	$E_{22}$ [GPa]	$\nu_{12}$	$G_{12}$ [GPa]	$G_{23}$ [GPa]
1560	122.0	8.0	0.308	5.97	3.75

The time histories of out-of-plane displacement  $u_z$  were collected from pairs of nodes in directions of propagations in range  $[0, 90]^\circ$  with step  $5^\circ$ . The nodes closest to required angles and distances  $d_1 = 10$  mm and  $d_2 = 20$  mm from actuator were used as sensors.

Results were processed same way as experimental data. The calculated group velocities are presented and compared with the experimental results in Fig. 4. It is apparent the approximated dependency curve approaches an ellipse, which corresponds with the group velocity wave curve in unidirectional composite determined by Rhee et al. [2]. The variations of group velocities in angles  $70^\circ$  and  $75^\circ$  are caused by an interference of  $S_0$  mode with shear wave mode, as can be seen in Fig. 5.

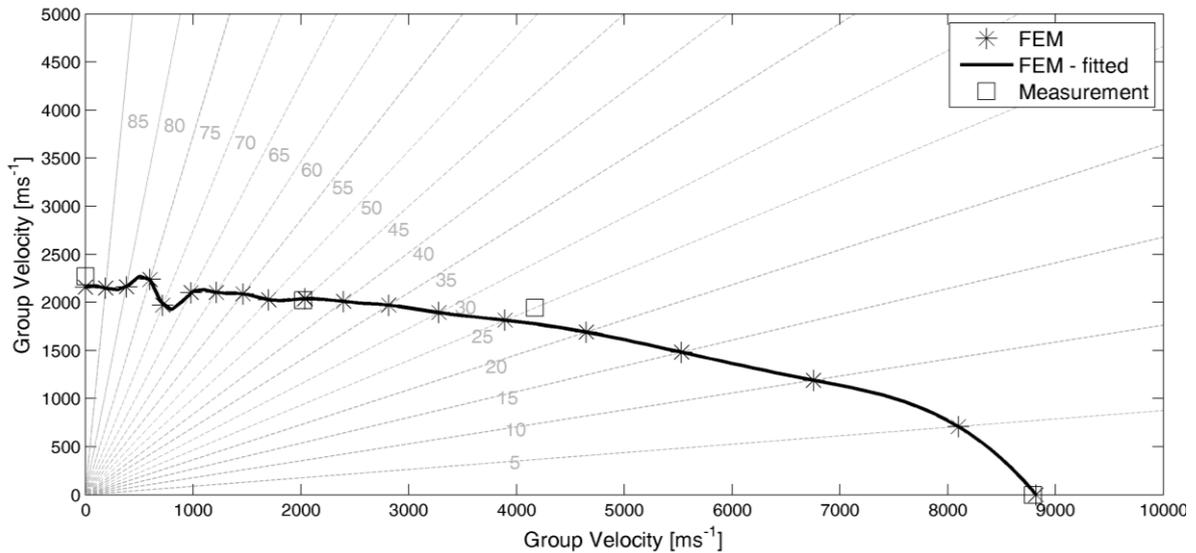


Fig. 4 Angular dependency of group velocity of  $S_0$  mode (frequency 200 kHz).

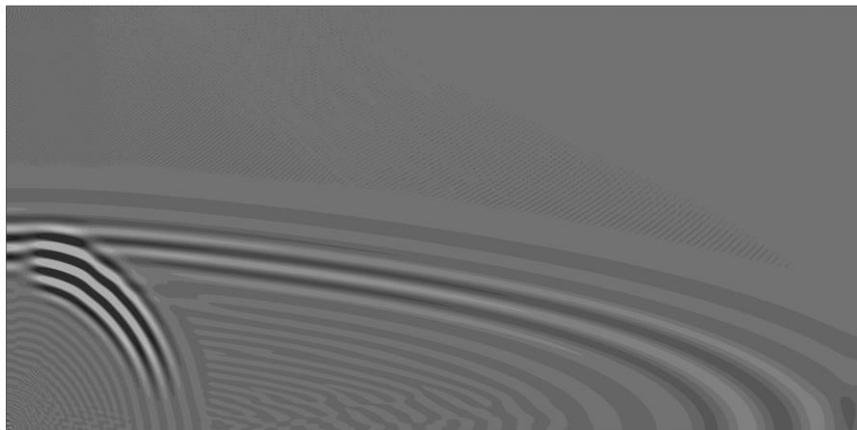


Fig. 5 Out-of-plane displacement of the plate in time  $t = 180$  ms from numerical analysis.

## Conclusions

Dependency of group velocity on direction of propagation of Lamb waves in unidirectional carbon-epoxy plate was determined using finite element model. The values in directions  $0^\circ$ ,  $25^\circ$ ,  $45^\circ$  and  $90^\circ$  were compared to the experimental results. The difference between numerical and measured velocities does not exceed 7%. However, group velocities in more directions have to be measured to verify the numerical model.

## Acknowledgements

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