

Numerical Investigation of 3-D Strain Constraint in Lab Test Specimens

ŠEDEK Jakub^{1,a}

¹VZLÚ – Aerospace Research and Test Establishment, Czech Republic

^asedek@vzlu.cz

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Abstract. The strain constraint investigation using finite element method is presented. Three-dimensional (3-D) models of standard test specimens (M(T), C(T) and SE(T)) are analysed from the point of view of out-of-plane constraint. The constraint factor α was defined according to Newman Jr. J. C. New relationship of constraint in dependence on the ratio of plastic zone size versus thickness is proposed. The global and local behaviors of constraint factors are presented. The local constraint is adopted via equivalent thickness in dependence on the through-thickness position on the crack front.

Introduction

Crack growth in metal materials is significantly influenced by stress state in the vicinity of crack tip. Stress state changes due to finite dimensions of cracked element according to the distance from free surface, where plane-stress state is dominant. In contrast the plane-strain can prevail when moving inside the element in dependence on the distance from free surface. The stress state is being considered according to parameters which characterize strain constraint, i.e. strain restriction. The rate of constraint is generally determined based on the stress and strain fields.

Strain constraint can be assessed by analytical methods of fracture mechanics, but often it is necessary to utilize numerical methods. Flat specimens are generally analyzed by simplification to two-dimensional problem with assumption of plain-strain or plain-stress. However more complex view on the problem is suitable to analyze using three-dimensional (3-D) approach. In many cases the symmetry can be applied and the cost of problem will be simplified a lot; e.g. middle crack tension specimen (M(T)) analyzed by application of symmetry in three orthogonal planes. The results of the analysis are 3-D stress and strain fields, which serves determination of strain constraint.

Analysis of Strain Constraint Behavior in Test Specimens

Analysis basis. In the plane of test specimen the crack from certain length is influenced by the finite specimen width. The effect is called in-plane constraint. On the contrary the specimen thickness influences the stress in the plane perpendicular to the plane of flat specimen and the effect is called out-of-plane constraint. In this work the latter case the out-of-plane constraint is analyzed.

The 3-D finite element elasto-plastic analysis of test specimens is carried out according to pioneer work of Newmann and Bigelow [1]. The elastic-perfectly plastic material is used in order to apply the results on modified Dugdale model of crack employed for crack growth prediction. 3-D finite element models of lab specimens, namely middle crack tension M(T),

compact tension C(T) and side edge crack tension SE(T) specimens were created and analyzed in ABAQUS SW (for example of C(T) finite element model see Fig. 1). One-quarter models were created to reduce computational cost. 57 000 elements of C3D8R type with reduced integration scheme were used. The smallest element size in region of yield material was 0.03 mm.

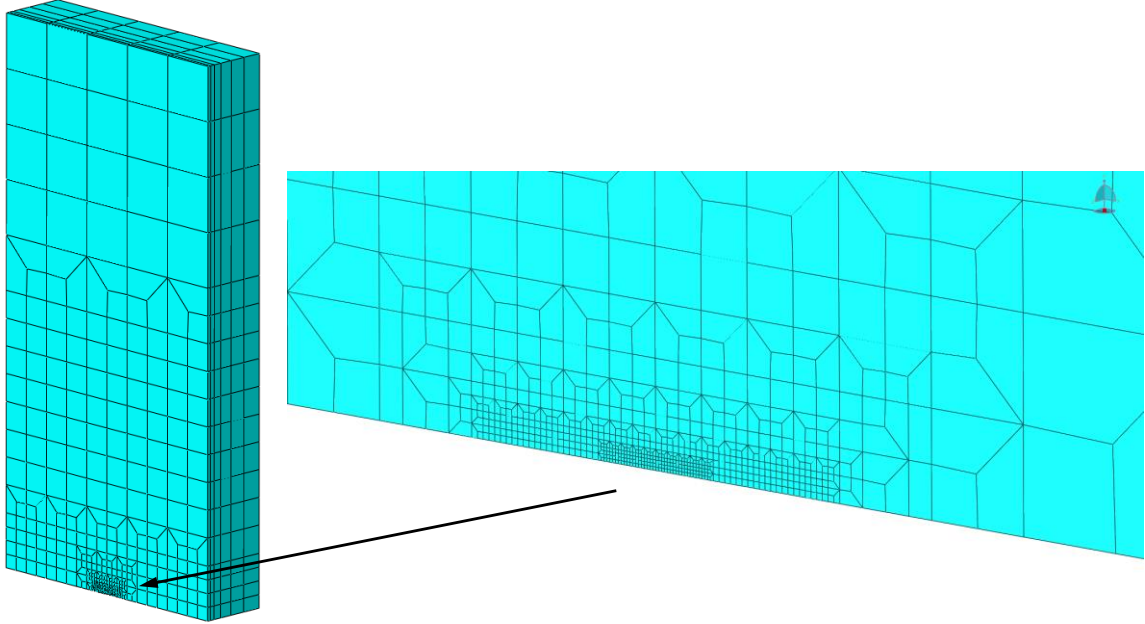


Fig. 1: One-quarter finite element model of M(T) lab test specimen.

Constraint factor α according to Newman [1] was determined through whole thickness and in several through-thickness layers individually. The layers were gradually thinner close to free surface. The definition of global constraint factor α_g is as follows:

$$\alpha_g = \frac{1}{A} \iint \alpha da, \quad (1)$$

where A denotes normal cross-section area through yielded material in the uncracked ligament and α is local constraint factor defined as the ratio of normal stress σ_{yy} to the flow stress σ_0 along the crack front; Eq. 2.

$$\alpha = \frac{\sigma_{yy}}{\sigma_0} \quad (2)$$

Flow stress is defined as average of strength and yield stress; Eq. 3.

$$\sigma_0 = \frac{\sigma_{Rm} + \sigma_{yield}}{2} \quad (3)$$

When applying finite element analysis, the Eq.1 can be expressed as Eq. 4.

$$\alpha_g = \frac{1}{A_T} \sum_{i=1}^N \left(\frac{\sigma_{yy}}{\sigma_0} \right)_i A_i \quad (4)$$

A_i is the area of yielded element i in the plane $y = 0$, $\left(\frac{\sigma_{yy}}{\sigma_0} \right)_i$ is normalized normal stress in the element i and A_T is the area of all yielded N elements in the crack plane.

Guo [2] presented the idea of equivalent thickness B_{eq} in order to describe the constraint change through the thickness and expressed Newman's data in dependence of the rate of specimen thickness B and plastic zone size. The equivalent thickness B_{eq} is expressed in dependence on the through-thickness location according to Eq. 5, where z denotes the distance from the center and B is specimen thickness.

$$B_{eq} = 1 - (2z/B)^2 \quad (5)$$

Analysis Results. In present work, the constraint factor α was determined for several types of lab test specimens (M(T), C(T) and SE(T)) using finite element method. Constraint factor α was depicted in relation to the logarithm of the rate of specimen thickness B and plastic zone size derived from FE analysis with elastic material ($B_{eq}/r_p^{FEM lin}$); see Fig 2.

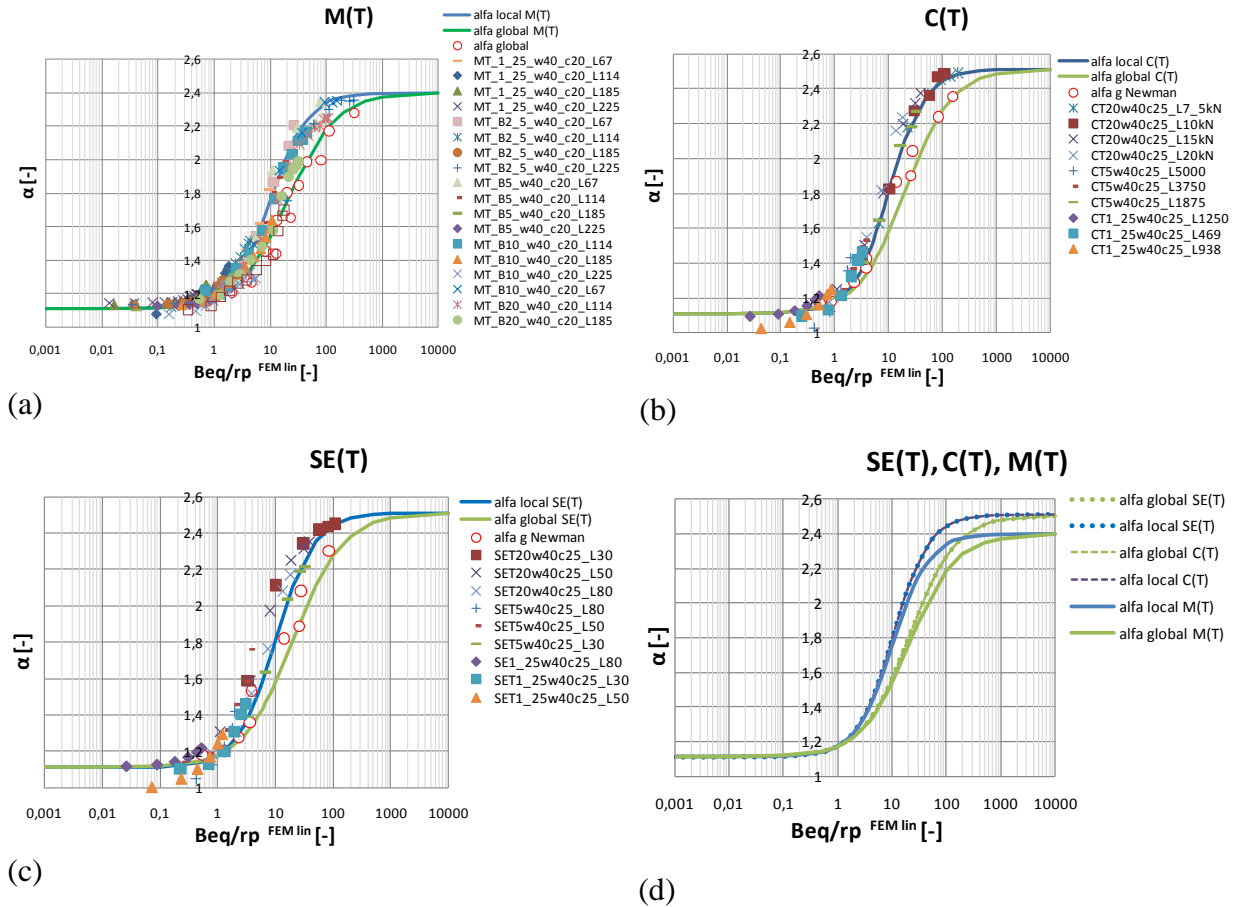


Fig. 2: Constraint factor α in dependence on the rate of specimen thickness B and plastic zone size according to FE elastic analysis $r_p^{FEM lin}$; (a) M(T); (b) C(T); (c) SE(T); (d) Comparison of derived α curves.

Based on the results of FE analyses the new relationship of constraint factor α was derived. The global factor α_{global} for whole specimen and local factor α_{local} for equivalent thickness are expressed via Eq. 6.

$$\alpha = 1,11 + \frac{\frac{1}{1-2\nu} - C}{1 + 20 \left(\frac{r_p}{B} \right)^\beta} \quad (6)$$

For α_{global} : $\beta = 1$ and B denotes specimen thickness and

for α_{local} : $\beta = 1,3$ and B is replaced with equivalent thickness B_{eq} according to Eq. 5.

Parameter C is expressed by relation $C = a + b\nu^\gamma$, as a function of Poisson's ratio ν . The relation incorporates the change of maximum value according to specimen type with coefficients a , b and γ dependent on specimen type.

Maximum values of constraint factor α in dependence on Poisson's ratio ν are shown in Fig. 2.

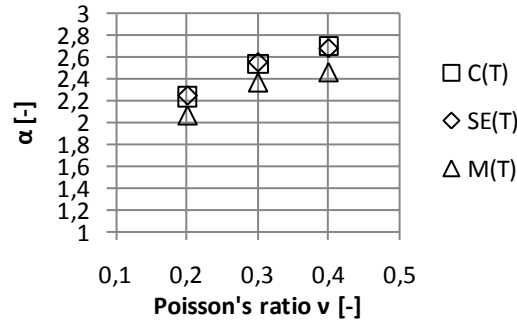


Fig. 2: Constraint factor α under plain-strain conditions in dependence on Poisson's ratio ν ; elasto-plastic FE analysis

Conclusions

The constraint factor α according to Newman is analyzed in this work. New relationship in the form of sigmoid curve in dependence on the logarithm of the rate of specimen thickness B and plastic zone size is presented. The maximum value of constraint factor α in plane-strain conditions differs a little according to specimen type, but the minimum value of 1,11 in plain-stress conditions for all analysed specimen types (M(T), C(T) and SE(T)) is identical.

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References

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