# FEM Calculation of Gear Mesh on Single-stage Axle Gearbox for Trains

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**Abstract:** This article describes the methodology and calculations of gear mesh optimization by FEM for single-stage axle gearbox. Results of the calculations are used for examination of changes in the gear mesh of the tooth flank under load, particularly in term of improved uniform load distribution through a width of the tooth. The aim of the methodology is to design optimized tooth profile with application modifications of tooth flank on the basis of analysis of calculation model by FEM. The article compares several variants of modifications according to distribution of contact pressure.

**Keywords:** FEM; gear mesh; modifications; contact pressure; axle gearbox; gear; optimization; train; shaft; deviation; single-stage.

### **1** Introduction

This article describes the problem of position and shape changes of gears in the gear-mesh under load and methodology, which handles compensation for these negative phenomena. Application of the finite element method (FEM) for the calculation of gearing with respect to real shapes of individual components of the transmission system can accelerate the gear mesh optimization process [1].

The results of the calculations are used for examination of changes in the gear mesh of tooth flank under load, particularly in term of improved uniform load distribution through a width of tooth. The aim of methodology is to design optimized tooth profile with application of transverse, longitudinal and angular modifications on the basis of analysis calculation of model by FEM.

Basic technical data about single-stage axle gearbox (Fig. 1): gear ratio 3,857; nominal power 340kW; max. input torque 3528Nm; max. input shaft speed 4200min<sup>-1</sup>; operating temperature  $-30^{\circ}/+45^{\circ}$  [2].



Fig. 1: Single-stage axle gearbox with coupling and wheel-set output shaft.

Fig. 2: Relations affecting the gear mesh.

### 2 The Reasons of the Inequality of Gear Mesh under Load

The final gear mesh conditions are influenced by several factors. The most important factors are deviations from the ideal shape of gears, clearance, assembly variations and transmission system parts deformation. The final position of gear mesh is heavily affected by loaded geometry of gearbox, shafts, bearings and gears. These components have fixed relations among each other and are shown in the Fig. 2.

Based on calculations by FEM for individual elements according Fig. 2, these conclusions can be made. An essential element that influences the design process is the shaft. Another important part is the gearbox, which optimized construction, must have sufficient rigidity for the position of the bearings. The part which affects the calculation on the lower level is bearing. Separate problem is the stiffness of own teeth.

### **3 FEM Model and Calculation of Gearbox and Shafts**

After loading of CAD model into FEM connection between upper and lower part of the gearbox by the contact was made. Both parts of gearbox are bolted together. Transfer reaction forces from the shafts are applied at the center point of bearing and the transfer of the gearbox is simulated as type "Coupling" - Fig. 3.

The result of the calculation (Fig. 4) is the spatial layout of points which characterizes the entry position of the points of the shaft ( $f_{xU\_deform\_i}$ ,  $f_{zU\_deform\_i}$ ).



Fig. 3: FEM model of gearbox with simulation of bolts in dividing plane.



Fig. 4: Deformation of gearbox in scale.

The output shaft has locked rotation around own axis Y. On the input shaft is applied load torque 1960Nm. The calculation is performed in two steps. At first, are activated tensile forces into the bolts between the output gear and the hub, and then is applied the load torque on the input shaft.

#### 4 Summation of all Parameters Affecting the Gear Mesh

The result is a displacement and rotation of gears. Calculation starts by displacement of points in the shaft fits. Deviations of the support and the shaft in directions X and Z are calculated according to formulas (1) for direction X and same formulas with index Z (changed X to Z) for direction Z.

$$f_{xL_i} = f_{xU_prod_i} + f_{xU_deform_i} + \left(f_{L_clearance_i} + f_{L_deform_i}\right) \cdot \cos \varepsilon_i \tag{1}$$

where:

 $f_{xU\_prod\_i}$  – production deviation from the ideal position of support in the dir. X [mm];  $f_{xU\_deform\_i}$  – deviation result from the deformation of gearbox in the dir. X [mm];  $f_{L\_clearance\_i}$  – radial clearance in loaded bearing [mm];  $f_{L\_deform\_i}$  – radial deviation resulted from the deformation of the bearing "i" [mm];  $\epsilon$  – angle of the direction of load bearing in the plane XY [°].



In the Fig. 5 are shown representations of the new shaft positions and in Tab 1 are shown resulting values.

Fig. 5: Representation of total displacements and rotation of gears in the plane XZ and plane ZY.

Indication of	Total Displacement [mm]		Total Rotation [ ° ]	
midpoint of gears	$d_{xL}$	$d_{zL}$	d <sub>z</sub> – Plane XY	$d_x$ – Plane ZY
O <sub>12</sub>	0.02622381	-0.04098320	0.00655374	0.01529100
O <sub>34</sub>	0.02077178	-0.03239230	0.00939368	0.01688300

Tab. 1: Resulting values of new midpoint position of gears.

# **5** FEM Model and Calculation of Gear Mesh

The solution of the contact problem is created in the gearing (3D model - gear and pinion). The contact bond among tooth flank is added to the model and meshed. The gears are connected with the center point of rotation with help of interaction type "Coupling". The subsequent rolling of the gear (pinion) and the implementation of all moments are applied through these points. Simulation of gear-mesh is performed quasi-statically (Fig. 6).



Fig. 6: FEM model of gearing (gear mesh).

# 6 The Variants of Modification Used for FEM Calculation

In the Tab. 2 are shown descriptions of modifications (seven variants) used for the FEM calculation of gear mesh and average and maximum values of contact pressure (FEM).

From the program KISSsoft is maximum contact stress (pressure) for pinion  $\sigma_{H_p} = 1029 \text{Nmm}^{-2}$  and for gear  $\sigma_{H_g} = 1025 \text{Nmm}^{-2}$  (according DIN 3990). In the Tab. 2 are shown average and maximum values of contact pressure. Contact pressures on gears for seven variants of FEM calculations are shown in Fig. 7.

Description of modifications (variants)	Max. contact stress [Nmm <sup>-2</sup> ]	Average contact stress [Nmm <sup>-2</sup> ]
The ideal gear mesh (without modification) $\rightarrow$ <b>Ideal</b>	1038	731
Displacement and rotation (without modification) $\rightarrow$ <b>Def</b>	858	760
Transverse modification ( $c_{aa} = 0.015 \text{ mm}$ ) $\rightarrow$ <b>Trans_0.015</b>	993	808
Transverse (0,015mm) + longitudinal (0,009mm = $c_b \rightarrow pinion$ ) modification $\rightarrow$ <b>Trans_Long</b>	865	801
Longitudinal modifications ( $\Delta\beta \rightarrow 0,015 \text{ mm} = c_b$ ) $\rightarrow$ Long_0.015	1136	725
Longitudinal modifications ( $\Delta\beta \rightarrow 0,009 \text{ mm} = c_b$ ) $\rightarrow$ Long _0.009	959	694
Longitudinal modifications ( $\Delta\beta \rightarrow 0,007 \text{ mm} = c_b$ ) $\rightarrow$ Long _0.007	899	694

Tab. 2: Description of modifications used for the FEM calculation and average and maximum values of contact pressure for each variant.

## 7 Conclusion

Results from the application of the methodology - the calculations of the modifications with help of the finite element method confirmed the applicability in the design of longitudinal, transverse and angular modifications. Already the first step where are calculated deformation of the gearbox and shafts, provided important results for the design of angular modification. By summing the calculated deviations of deformations, manufactured deviations from the ideal shape of the bearing and clearances is calculated final position from which can be easily calculated the angle modification.

In the Fig. 7 is shown (Hertz) contact pressure on tooth flank without modification and deformation of shaft, gearbox (ideal); with respect of deformation of shaft and gearbox (def); and for five variants of modifications (with respect of deformation of shaft, gearbox, etc.).

The best solution was achieved with usage of longitudinal modification  $c_{bb} = 7 \ \mu m$  (Long\_0.007). FEM results with 7  $\mu m$  modification was compared in program KISSsoft and they corresponded.



Fig. 7: Comparison of the average contact pressures on the pinion tooth flank along tooth width.

# References

- K. Petr, P. Žák, V. Dynybyl, J. Kanaval, FEM Optimization of Gear Flanks Shape Modifications. In Proceedings of the 3rd International conference Power Transmissions '09. Thessaloniki: Sofia Publications, 2009, vol. 1, p. 93-100. ISBN 978-960-243-662-2.
- [2] K. Petr, V. Dynybyl, Methods of testing gearboxes of rail vehicles, In Applied Mechanics and Materials. Uetikon-Zurich: Trans Tech Publications, 2014, p. 277-282. ISSN 1660-9336. ISBN 978-3-03785-977-3.