Phase-shifting Method and its Comparison with Procedure traditionally used in Photoelasticity

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Abstract: The main goal of this paper is to perform the comparison of phase-shifting method with procedure, which was used in photoelasticity before advent of computers. In order to make this old procedure friendlier for use, its code was written in MATLAB software. The phase-shifting method is a procedure from digital photoelasticity. It provides isochromatic and isoclinic parameters in the full field of the specimen. In this contribution the phase shifting is carried out by three different loads acting on the specimen. The code was also written in MATLAB. Thin disc, annular discs and beam were used as models. The results are compared in certain positions on the models.

Keywords: photoelasticity; phase-shifting method; isoclinic; isochromatic; Cauchy stress tensor.

1 Introduction

Photoelasticity is an experimental method, which enables to evaluate stress state in transparent materials. Photoelasticity used polarized light and phenomenon of certain transparent materials called birefringence [1]. It is possible to solve static, dynamic tasks and problems associated with residual stresses.

This contribution is focusing on problematics of gathering data from photoelasticity measurement. There exists two types of fringe patterns, which is possible to observe. Isoclinic and isochromatic fringe patterns is representing directions of principal stresses and amount of difference of principal stresses, respectively. In work [2] was presented a procedure, which allow to obtain Cauchy stress tensor in the full field of specimen trough shear difference method. The isoclinic and isochromatic fringes were rendering manually by user and then approximated by Bézier curves. The values of isochromatic and isoclinic parameters in points between two curves were determined by the weighted arithmetic mean. This work quite well, when the density of curves is high.

In order to determine the influence of the density of interferometric fringes on results, there were used technique called phase-shifting method [3]. This procedure can provide data in the full field of the specimen. The necessity of determination of the isoclinic and isochromatic fringes manually is eliminated. There can arise some errors caused by isoclinic isochromatic interaction.

2 Phase-shifting Method

There exists many modification of this procedure. Asundi and Liu algorithm [4] was chosen for this contribution. A circular polariscope was used for this measurement (Fig. 1b). For this type of testing rig it is possible, after involving Jones' calculus, to express the light intensity as:

$$I = \mathrm{I}_0 + \mathrm{a}^2[\cos^2\beta \sin^2(\delta/2) \sin^2(2\varphi - 2\theta) + \sin^2(\beta) \cos^2(\delta/2) + \\ +0{,}5 \sin(2\beta) \sin(\delta) \sin(2\varphi - 2\theta) + \sin^2(\delta/2) \cos^2(2\theta - 2\varphi) \sin^2(\beta - 2\varphi)], \quad (1)$$

where I_0 is the background light and a is amplitude of the light vector.

The intensity of light is a function of orientations of the optic members and retardation δ introduced by the model. To calculate the isochromatic parameter δ (principal stresses difference) and isoclinic parameter θ (orientation of principal stresses) there must be considered polariscope configuration according to Tab. 1.

Tab. 1: Intensity equations for circular polariscope arrangements ($\theta = \varphi$).

configuration	φ [deg]	β [deg]	intensity equation	
1	45	90	$I_1 = \mathrm{I_b} + \mathrm{I_m} \cos\delta$	bright field
2	45	0	$I_2 = \mathrm{I_b} - \mathrm{I_m} \cos\delta$	dark field
3	90	135	$I_3 = \mathrm{I_b} - \mathrm{I_m} \sin\delta \sin 2\theta$	where
4	45	45	$I_4 = \mathrm{I_b} + \mathrm{I_m} \sin\delta \cos 2\theta$	
5	90	45	$I_5 = \mathrm{I_b} + \mathrm{I_m} \sin\delta \sin 2\theta$	$\mathrm{I_b} = \mathrm{I_0} + \mathrm{a}^2/2$
6	45	135	$I_6 = \mathrm{I_b} - \mathrm{I_m} \sin\delta \cos 2\theta$	$\mathrm{I_m} = \mathrm{a}^2/2$

From intensity equations in Tab. 1 the isoclinic and the isochromatic parameter can be expressed as:

$$\theta = (1/2)\,\mathrm{tg}^{-1}[(I_5 - I_3)/(I_4 - I_6)], \tag{2}$$

$$\delta = \mathrm{tg}^{-1}\{[(I_5 - I_3 + I_4 - I_6)/(\sin(2\theta) + \cos(2\theta))]/(I_1 - I_2)\}. \tag{3}$$

Due to tg^{-1} function and $(1/2)$ multiplier in formula (2), the isoclinic parameter is determined in range $\langle -\pi/4; \pi/4 \rangle$ and there is no information whether the angle refer to first or second principal stress. From equation (3) it can be seen, that an ambiguity of θ leads to improper determination of isochromatic parameter. It is possible to implement the three-load method to resolve this behavior and determine the photoelastic parameters in correct way.

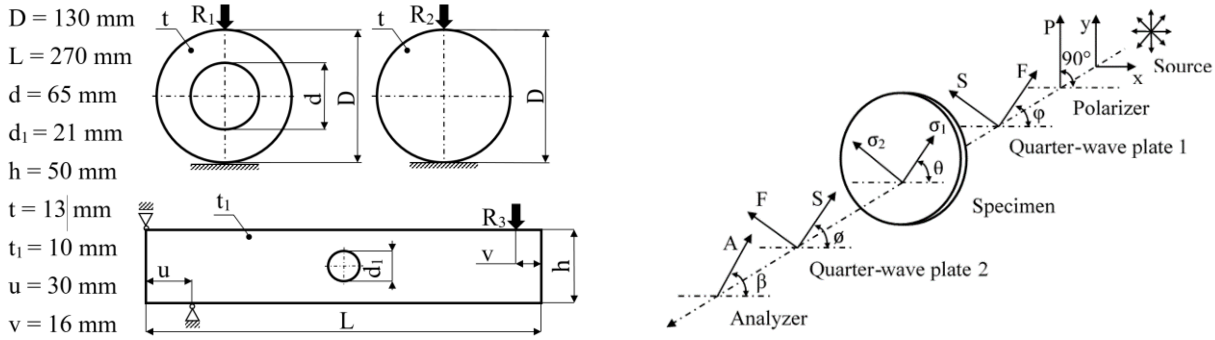
2.1 The Three-Load Method

The model in testing rig is subjected to three different loads. For practical measurement it is useful if the difference between actual and subsequent load is constant. For loads is valid relation $\mathrm{F}_1 > \mathrm{F}_2 > \mathrm{F}_3$, therefore for retardation $\delta_1 > \delta_2 > \delta_3$. In regions where the retardation has been mathematically wrong determined, the relation becomes $\delta_1 < \delta_2 < \delta_3$. In any point in the stressed specimen one can find out six possible relations between retardations [4].

The main purpose of this procedure is to determine the principal stresses orientation always related to one of the major stress. If the isoclinic parameter have changed the isochromatic parameter must be recomputed according to formula (3).

3 Experiment and Results

Polariscope with sodium light was used for visualize stress state in three specimens (see Fig. 1a). Two quarter-wave plates were inserted into the polariscope. A schematic of polariscope can be seen on Fig. 1b.



a) three investigated specimens　　b) orientation of optic members

Fig. 1: Models and scheme of polariscope for circular polarization.

The six intensity images for annular thin disc, disc and beam model are shown on Fig. 2. The images correspond to the maximum applied load. The eighteen images were captured by Canon EOS 60D for each specimen. The two discs were subjected to the vertical compressive force and the beam to the bending force.

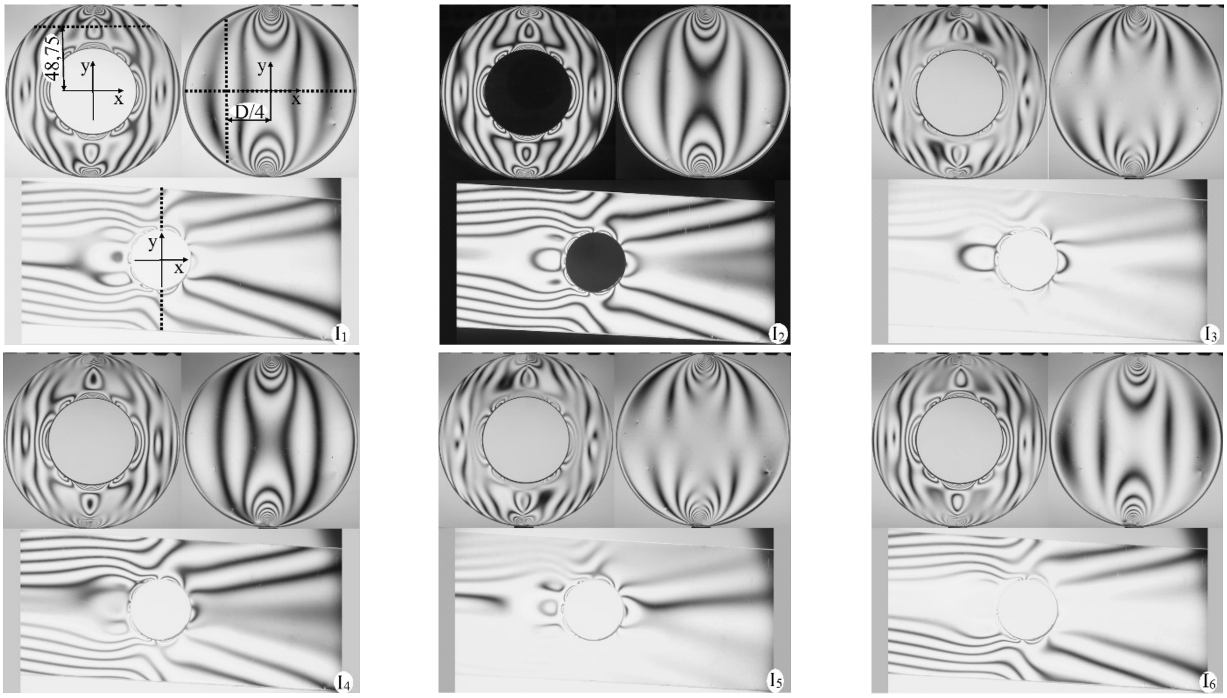
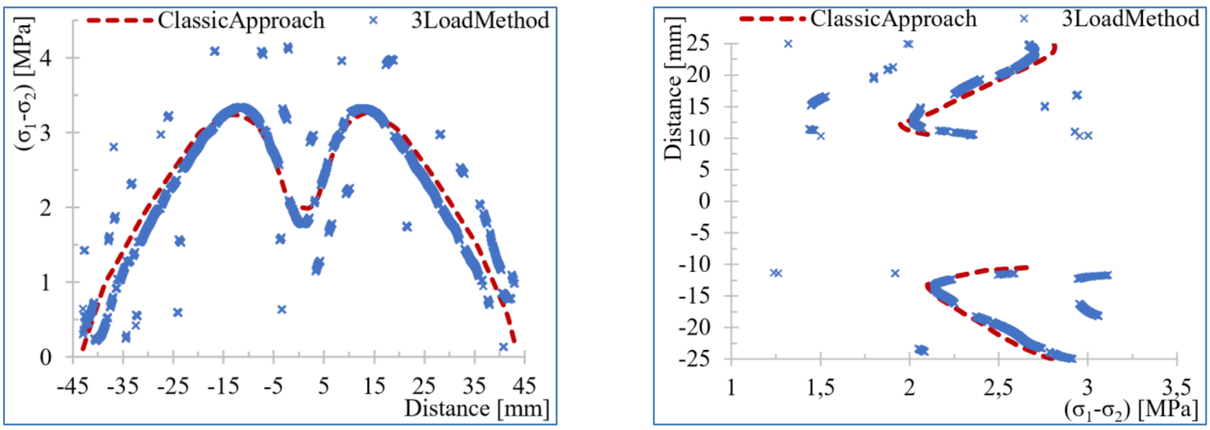


Fig. 2: Fringe patterns corresponding to polariscope arrangement mentioned in Tab. 1.

In the first three images the dashed lines denote positions where the results have been compared and they are in some meaning in symmetric positions on the models. For thin disc the vertical line lies in distance 1/4 of diameter from left. The discs are made from optically sensitive material CT 200 (material fringe value 17.4 N·mm^{-1}) and beam is made from polycarbonate (material fringe value 7 N·mm^{-1}).

3.1 Results Comparison

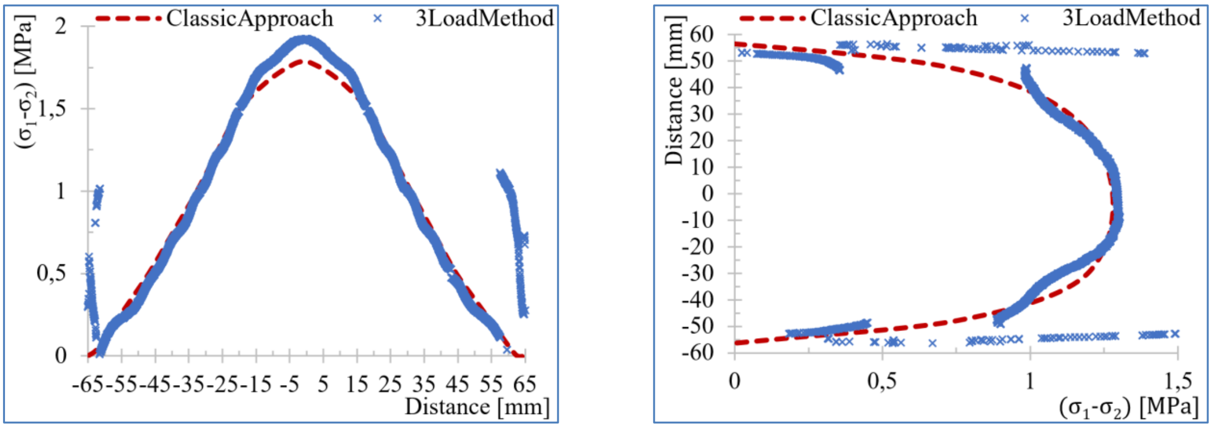
Results are compared on Fig. 3 and Fig. 4. No curve-fitting method was used for smoothing the data. Results on the boundary of the models can be affected by residual stresses and light scattering on the model's edge.



a) principal stress difference along horizontal line for thin annular disc

b) principal stress difference along vertical line for thin beam subjected to bending force

Fig. 3: Comparison of phase-shifting (Asundi's) algorithm and traditional approach for thin annular disc and thin beam.



a) principal stress difference along horizontal line   b) principal stress difference along vertical line

Fig. 4: Comparison of phase-shifting (Asundi's) algorithm and traditional approach for thin disc.

The Results on Fig. 3 and Fig. 4 correspond to force R_1=1240.9 [N] for thin annular disc, R_2=1678.9 [N] for thin disc and R_3=106.5 [N] for thin beam. It is apparent, that there must be implemented some post processing technique to obtain more accurate results especially on the boundary of the models. Refinement data could be used as input data for stress separation technique and then components of stress tensor can be determined.

4 Conclusion

It is apparent that there exists some differences in results obtained by two approaches. The traditional approach, which is fully computer aided, is suitable if there exists many isochromatic fringes in the field. On the other hand the phase-shifting method is more comfortable. It has clear physical and mathematical explanation, however the results in region with high density of isochromatic fringes are not good due to isochromatic-isoclinic interaction. This drawback can be eliminated by proper combination of results obtained for the three different loads. It can be proved, that with some assumption four intensity images from Tab. 1 are sufficient for calculation of photoelastic parameters.

Photoelasticity is exceptional experimental method due to its capability to directly visualise stress state in transparent models. Reflection photoelasticity is used for determination the strain distribution on surface of non-transparent parts. Solve stress condition caused by dynamic loads and residual stresses is also possible.

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