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Identification of Elastic Constants of Glass Alloys

K. Vítek [1,*]

[1] *CTU in Prague, Faculty of Mechanical Engineering, Department of Mechanics, Biomechanics and Mechatronics, Technicka 4, 166 07 Prague 6, Czech Republic*

* *karel.vitek@fs.cvut.cz*

Abstract: An analytical structural limit state assessment needs a better knowledge of the material constants as one of their basic inputs. The methodology, used for determining the structure material elastic constants, is based on mechanical tests, being mostly tensile ones, applied on partially loaded specimens. There be can glass materials used as different alloy elements, therefore the glass elastic constants can vary considerably. However, using classic glass tensile specimens for tensile tests can be problematic, due to their production and implementation of tensile tests. Experimental methods for identifying the glass Young's modulus of elasticity and Poisson's ratio are based on a comparison of the displacement measurements applied on the glass beam, or curved rod, samples, combined with their displacements calculation.

Keywords: Young's Modulus; Poisson's Ratio; Glass; Elastic Constant.

1 Introduction

Glass can create a number of inorganic compounds. In chemical terms the conventional glass is a solid solution of various silicates of sodium, potassium, calcium, or lead or barium which are accompanied by other compounds, especially metal oxides. Selecting the components and their relative representation is possible to influence the properties of the glass in relatively wide limits, see Tab. 1, [2]. The variation of elastic constants of the glass is considerable. Therefore, it is desirable to have an operational method for identification of elastic constants - Young's modulus of elasticity and Poisson's ratio - for structures of glass materials. Strength of glass significantly depends on the surface properties, the dimensions of the sample and internal defects. The classic shapes of samples for the realization of tensile tests are quite complicated, which limits their production capabilities and possible behavior of glass samples during the experiment. The elastic constants of the glass must be verified on samples of a simple shape with a smooth surface to eliminate the negative properties of glass. The Etalon bar method [1] is based on two simple physical and numerical experiments. The elastic constants result from comparison displacements v and u (see Fig. 1) of a curved rod which is loaded by bending and torsion. The displacement v (for given force F) is a dominant function of Young's modulus E. Therefore, comparing the displacement measurement with its calculation it determines E. The Poisson's ratio ν is then determined by comparing the displacement measurement $u(E, \nu)$ with its calculation.

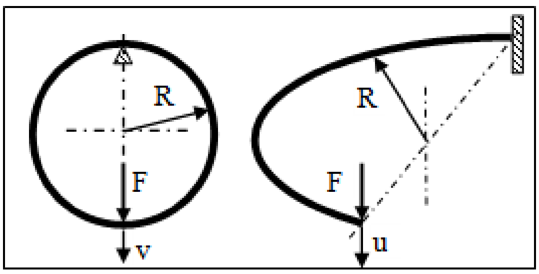


Fig. 1: Displacements v, u of curved rod.

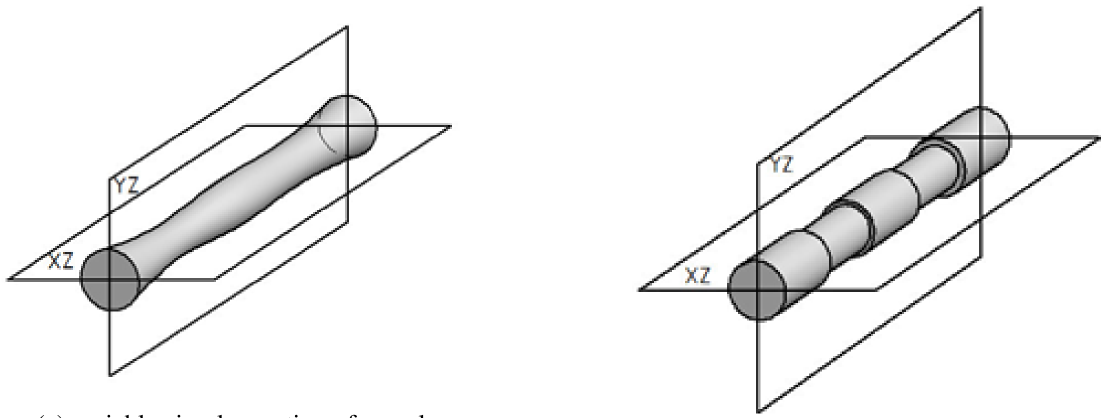
Tab. 1: Physical and mechanical properties of common glass.

the property	unit	value range
compressive strength	MPa	700 - 1200
tensile strength	MPa	30 - 90
bending strength	MPa	40 - 190
Young's modulus	GPa	50 - 90
Poisson's ratio	-	0.14 - 0.32
coefficient of thermal expansion	K^{-1}	$6{\cdot}10^{-6}$ - $9{\cdot}10^{-6}$
density	Kgm^{-3}	2200 - 6000

2 Glass Test Specimens and Their Computational Models

A rod smooth specimen of circular cross section from of glass is relatively easily produced. To identify the Young's modulus of elasticity E of the glass sample calculation is sufficient to compare the computational deflection of the beam or curved rod v_c simple (using Castigliano's theorem) with the experimentally measured deflection $v \equiv v_m$ (see Fig. 1). According to Eq. 1, where U is elastic strain energy, F is force in direction of v_c or u_c, J_o is the principal second moment of cross section of area, J_T is polar second moment of cross section of area, M is internal bending moment in the beam or rod, T is internal torque in the rod, L is length of the beam or rod, ds is element of the length of beam or rod. In case that the glass produced samples have a variable circular section, it is possible to realize the segment model of a beam or rod, see Fig. 2. To identify the Poisson's ratio the procedure is similar using a sample in a shape of a curved rod loaded by bending and torsion, as shown in Fig. 1 - where $u \equiv u_m$. Poisson's ratio ν is determined comparing the measured displacement u_m with the calculated displacement u_c (using Castigliano's theorem) - according to Eq. 2. Segment solutions of displacement u_c of curved rod refines the identification of ν.

$$v_C = \frac{\partial U}{\partial F} = \frac{\partial}{\partial F}\int_L \frac{M(s)^2}{2E \cdot I(s)} \cdot ds = \frac{1}{E} \cdot \int_L \frac{M(s) \cdot \frac{\partial M(s)}{\partial F}}{I(s)} \cdot ds = v_m \Rightarrow E = \frac{\int_L \frac{M(s) \cdot \frac{\partial M(s)}{\partial F}}{I(s)} \cdot ds}{v_m} \tag{1}$$



(a) variable circular section of sample

(b) segment approximation of the sample

Fig. 2: Experimental beam shape to identify Young's modulus of elasticity E.

$$
\begin{aligned}
u_C &= \frac{\partial U}{\partial F} = \int\limits_L \frac{M(s)\cdot\frac{\partial M(s)}{\partial F}}{E\cdot I(s)}\cdot ds + \int\limits_L \frac{T(s)\cdot\frac{\partial T(s)}{\partial F}}{G\cdot J(s)}\cdot ds = \\
&= \int\limits_L \frac{M(s)\cdot\frac{\partial M(s)}{\partial F}}{E\cdot I(s)}\cdot ds + 2(1+\nu)\int\limits_L \frac{T(s)\cdot\frac{\partial T(s)}{\partial F}}{E\cdot J(s)}\cdot ds = u_m \Rightarrow \nu \\
\nu &= \frac{u_m - \int\limits_L \frac{M(s)\cdot\frac{\partial M(s)}{\partial F}}{E\cdot I(s)}\cdot ds - 2\cdot\int\limits_L \frac{T(s)\cdot\frac{\partial T(s)}{\partial F}}{E\cdot J(s)}\cdot ds}{2\cdot\int\limits_L \frac{T(s)\cdot\frac{\partial T(s)}{\partial F}}{E\cdot J(s)}\cdot ds} = \frac{u_m - \int\limits_L \frac{M(s)\cdot\frac{\partial M(s)}{\partial F}}{E\cdot I(s)}\cdot ds}{2\cdot\int\limits_L \frac{T(s)\cdot\frac{\partial T(s)}{\partial F}}{E\cdot J(s)}\cdot ds} - 1
\end{aligned}
\tag{2}
$$

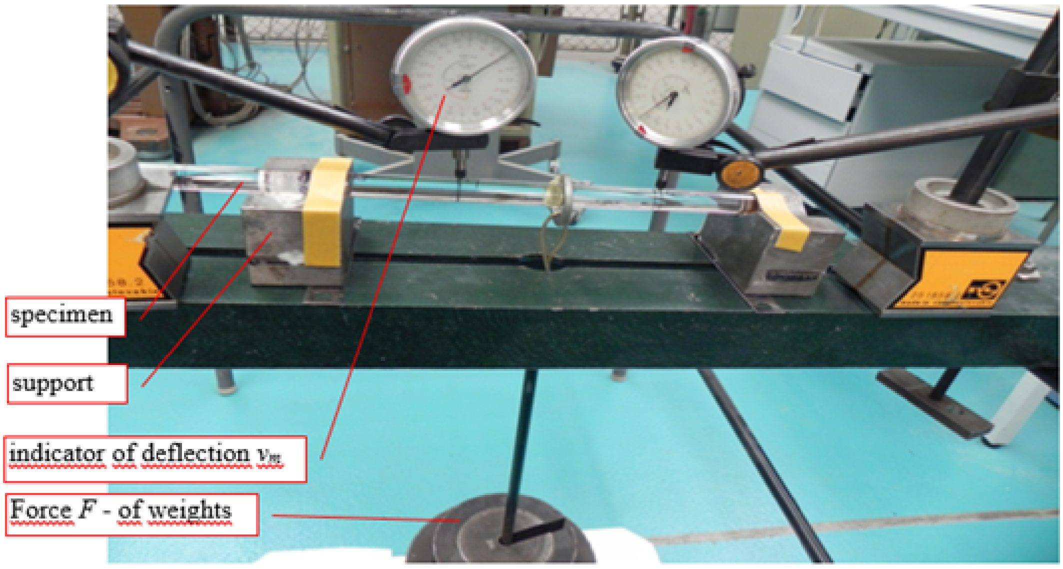


Fig. 3: Measuring the deflection of a glass beam specimen.

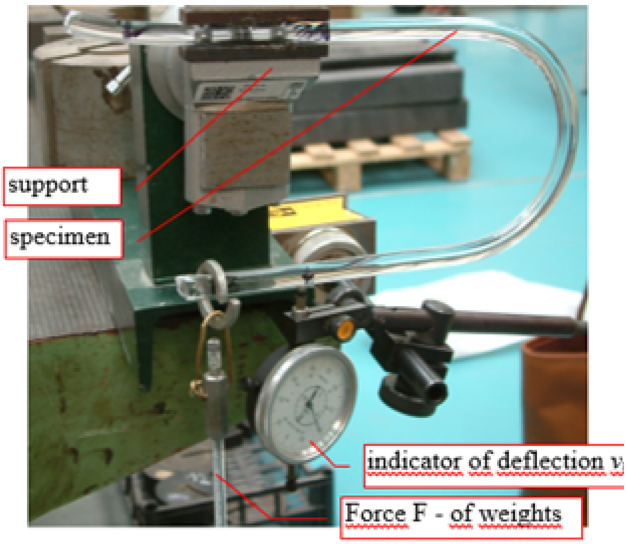


Fig. 4: Deflection of a glass curve rod specimen.

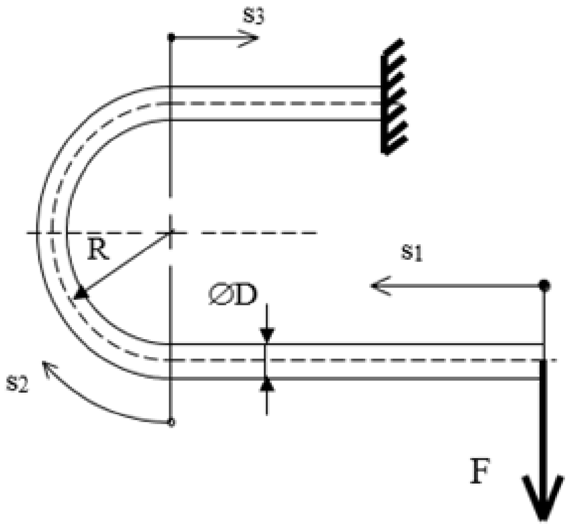


Fig. 5: Model of curved rod specimen.

Relatively easily manufacturable glass samples, see Fig. 2a, c, can be, in some cases, modeled as rods having constant circular cross-sections. The deflection v_m results, measured on the glass beam (Fig. 2a) samples by using the three-point bending test, are presented in Fig. 3. And the deflection results v_m, measured on the cantilevered glass test samples, shaped as curved rods, Fig. 2c, when loaded in bending, are presented in Fig. 4. The curved rod computational model, see Fig. 5, consists of the three areas s_1, s_2, s_3. When we use a set of weights representing a set of loading forces in these experiments, then we use only of the regression line linear term of the deflection measurements v_m, which we compare with deflection v_c obtained using Eq. 2.

Analogously, also the bending moment $M(s)$ of the sample in the calculation of deflection will be a function of the same loading force F, which is used in the calculation of deflection v_m measured.

The curved cantilever beam (Fig. 6), loaded with a combination of bending moment and torque, which is used to identify the Poisson's ratio ν, is tested analogously. In the computational model (Fig. 7), the integration area of the circular curve rod, starts at the loading force F action point, which has a coordinate s_1 and ends at position s_2, where the curved rod is clamped.

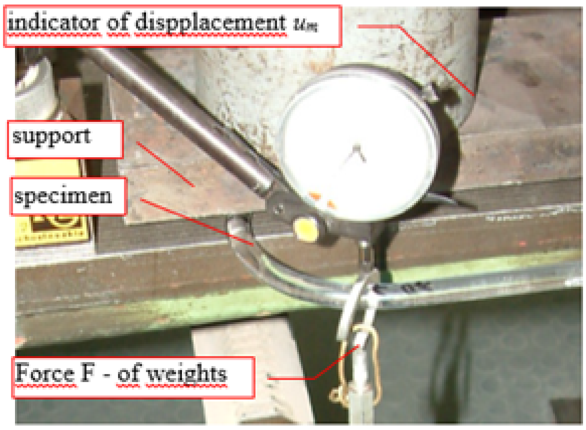


Fig. 6: Displacement of a glass curve rod specimen.

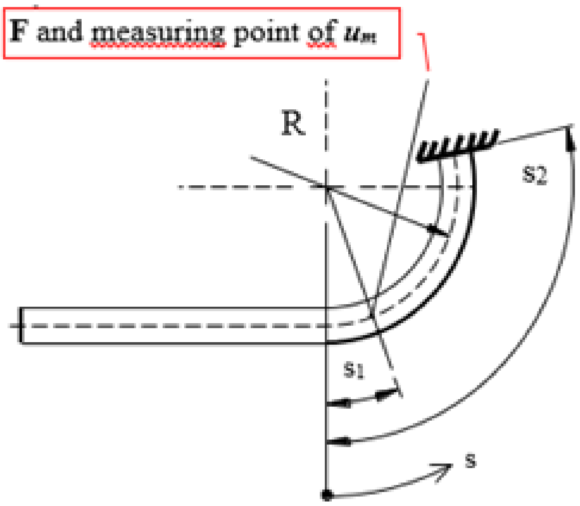


Fig. 7: Model of curved rod specimen.

These analytical models can further be refined by respecting the specimen dimensional changes when calculating the displacements. The rod is divided into a suitable number n of computational areas in which we can measure the rod dimensions, according to Fig. 2b, which will be used in the calculation model. The Young's modulus of elasticity E identification can be solved by analogy with Eq. 3 and the identification of Poisson's ratio can be solved similarly to Eq. 4.

$$E = \frac{1}{v_m} \sum_{i=1}^{n} \frac{1}{I_i} \int\limits_{L_i} M_i(s) \cdot \frac{\partial M_i(s)}{\partial F} \cdot ds \tag{3}$$

$$\nu = \frac{u_m - \frac{1}{E} \sum\limits_{i=1}^{n} \frac{1}{I_i} \int\limits_{L_i} M_i(s) \cdot \frac{\partial M_i(s)}{\partial F} \cdot ds}{\frac{2}{E} \sum\limits_{i=1}^{n} \frac{1}{J_i} \int\limits_{L_i} T_i(s) \cdot \frac{\partial T_i(s)}{\partial F} \cdot ds \cdot} - 1 \tag{4}$$

References

[1] Vítek, K.: Identification of Elastic Constants by Bar Etalon, Proceedings of the 40th Annual Conference on Experimental Stress Analysis, Czech Technical University in Prague, Prague, pp. 261–266.

[2] Retrieved from: http://geologie.vsb.cz/loziska/suroviny/sklo.html