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Comparison of Theoretical Dynamic Analysis of the Footbridge with Experiment

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Abstract: This paper is aimed at dynamic analysis of the footbridge, which was excited by pedestrians and crowd of vandals. Mathematical models of pedestrians were considered as harmonic forces, which were moving over the structure with constant velocity. Second approach for description of loading, induced by pedestrians, was the Rimless wheel model, which takes into account kinematic aspects of trajectory of human body's centroid during walking and it is based on the motion of inverted pendulum. Models describing action of vandals were considered as force defined by periodic triangular pulses and viscoelastic SDOF model of human body, which was activated by kinematic excitation of the contact point between SDOF model and structure. The investigated structure statically acts as simply supported beam. It is slab-on-girder bridge with steel girders and concrete deck. The response was evaluated by using modal decomposition techniques with theoretically obtained mode shapes. Computed results were compared with experiment.

Keywords: Human-Induced Vibration; Rimless-Wheel Model; SDOF Models of Pedestrians.

1 Modal Analysis

A three-dimensional computational model was created in the software Dlubal/RFEM, for the purposes of calculating of the eigenvalues and the eigenvectors as the results of theoretical modal analysis. This software uses the Subspace Iteration Method with Sturm's control or Lanczos algorithm for calculation of the mode shapes and appropriate natural frequencies. Computed mode shapes have been subsequently normalized with respect to the mass matrix $\mathbf{M}$. % and thus $\mathbf{\Phi}^T\,\mathbf{M}\,\mathbf{\Phi} = \mathbf{E}$, where $\mathbf{\Phi}$ is the modal matrix and $\mathbf{E}$ is the unit matrix.

Experimentally obtained natural frequencies are summarized in Tab. 1. The logarithmic decrement, which is necessary for forced vibration analysis, was determined from the record of natural vibration. The structure was excited by force impulses, which were produced by gradually jumping people.

2 Forced Vibration Analysis

Due to the magnitude of the first natural frequency, forced vibration analysis had to be done, for a purpose of verification of comfort criteria of pedestrians during walking over the structure. The value of this frequency

Tab. 1: Computed and Measured natural frequencies.

frequency - mode	Computed [Hz]	Measured [Hz]	
%	$\frac{f_{\text{mea}}-f_{\text{theor}}}{f_{\text{mea}}}100\,[\%]$ $f_{(1)}$ - 1st bending mode	2.71	2.72
%	-5.26 $f_{(2)}$ - 1st torsional mode	4.59	5.06
%	4.25 $f_{(3)}$ - 2nd bending mode	8.81	9.31
%	0.20 $f_{(4)}$ - 2nd torsional mode	12.12	12.28
%	-4.17		

is in the range of resonance and therefore, significant magnitudes of acceleration could be achieved during excitation of the structure.

Synchronous walkers were simplified by the harmonic force. This approach, according to Bachmann see [2], was derived from the theory of Fourier series and it can be prescribed by relation

$$P\left(t\right) = mg\left(1 + \sum_{n=1}^{N} \alpha_n \sin\left(2n\pi f_p t - \phi_n\right)\right) \tag{1}$$

where mg is static weight of each walker, α_n is the dynamic loading factor of n-th harmonic defined e.g in [2], f_p is the pacing frequency and ϕ_n is the phase shift of n-th harmonic.

Runners have been expressed by the model defined in Japanese regulations. It is similar to the harmonic model according to Bachmann. The reason why the Japanese model was used instead of Bachmann's is, that Bachmann's model provided too high acceleration of structure in comparison with experiment. This was caused by higher values of coefficients α_n for runners. Mathematical model, used for description of runners, is defined as

$$P(t) = \alpha\, m_p\, g\, \cos\left(2\, \pi\, f_p\, t\right) \tag{2}$$

where α is correction coefficient, which depends on the number of footfall per second.

Both of these models were moving over the structure with constant velocity of motion v_p. Last model, which was used for description of moving pedestrian, is based on the assumption, that pedestrian's vertical trajectory of centroid can be simplified by motion of the inverted mathematical pendulum. This model is called the Rimless wheel model.

Effects of vandalism on the structural vibration were verified by using periodic triangular pulses and viscoelastic Kelvin-Voigt model of human body, see e.g. [1]. Periodic triangular pulses mean rhythmical gymnastic jumping or swinging on the spot. The passive SDOF models of human body were activated by kinematic excitation of contact point between SDOF model and the structure. Kinematic excitation of viscoelastic models was used for computing of the contact forces, which were subsequently used for excitation of the structure. The relation for kinematic excitation, the dashed line in Fig. 1, can be written in form

$$r(t) = A_r \left|\sin\left(\frac{\pi\, v_p}{d_p}\, t\right)\right| \tag{3}$$

where A_r is the amplitude of trajectory of COM (center of mass), v_p is the velocity of pedestrian, d_p is the length of step and t denotes time.

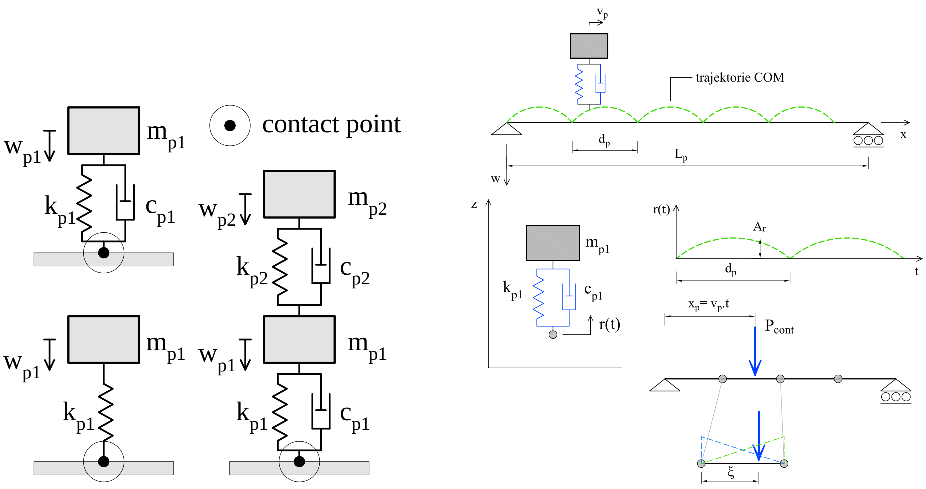

Fig. 1: Biodynamic/Viscoelastic models of human body.

The experiment was consequently focus on the assessment of the serviceability of the structure. The structure was loaded by the synchronous walkers and joggers and group of vandals. The response of the load-bearing structure was measured by four accelerometers Brüel & Kjær type 8344 and by the vibration control station Brüel & Kjær Front-end 3560-B-120. The accelerometers were placed on the bridge deck at the center and quarter points of footbridge span to determined influence of higher natural modes of vibration. One of these sensors was situated to the other side of the bridge deck due to participation of torsional mode shapes on vibration of the structure. Experimental data were taken from [3].

3 Procedure of Computation

Due to the fact, that the forces were moving across the structure, it was necessary to calculate equivalent nodal loading with respect to the fact, that in general moment t could force $P(t)$ acted between four general FE nodes. Therefore, the algorithm for recalculation of harmonic force into FE nodes was written in MATLAB software. The nodal forces were computed by using linear interpolation functions for two variables. One of them can be defined as

$$N^C(\xi,\eta) = \left(1 - \frac{\xi}{L_\xi}\right)\frac{\eta}{L_\eta} \tag{4}$$

where ξ and η are unknown, L_ξ is the length corresponding to the ξ variable and L_η is length in η direction.

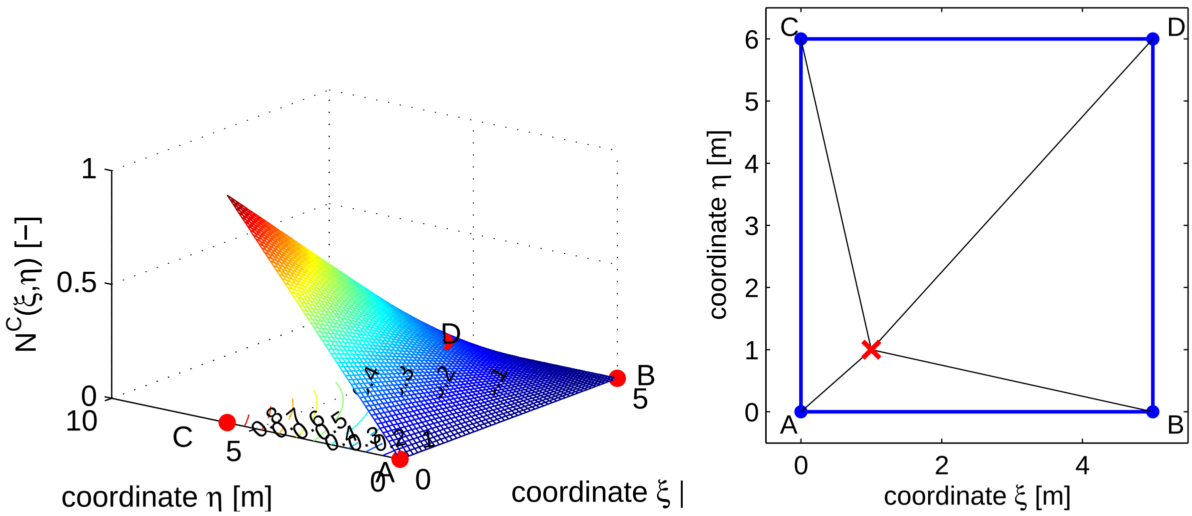

Fig. 2: Graphical interpretation of interpolation function for FE node.

Subsequently, the differential equation of motion in modal coordinates can be written in form

$$\ddot{q}_j(t) + 2\,\xi_j\,\omega_j\,\dot{q}_j(t) + \omega_j^2\,q_j(t) = \boldsymbol{\phi}_j^T\,\mathbf{p}(t) \tag{5}$$

where ξ_j is the damping ratio of j-th mode shape, ω_j is the natural circular frequency of j-th mode shape $\boldsymbol{\phi}_j$, $\ddot{q}_j(t), \dot{q}_j(t), q_j(t)$ are generalized coordinates in modal domain and their time derivatives. The damping matrix $\mathbf{C}$ was assembled according to the Rayleigh damping model.

The Eq. (5) was solved also in MATLAB by using Newmark β integration method with zero initial conditions, which means that the structure was considered as inactive at the beginning of the numerical analysis. Some of the obtained results are visualized in Fig. 3.

4 Conclusion

Currently, loading of structures induced by pedestrians is mostly simplified by the DLF model of harmonic force, which is placed in the most efficient point of the structure, where the appropriate mode shape has maximal ordinate. Therefore, some of alternative approaches to modeling of human induced vibration have been presented in this paper. One of the new models is the Rimless wheel model of pedestrian, which is based on the movement of the mathematical inverted pendulum. The other model, appropriate for modeling of pedestrians or vandals, is SDOF viscoelastic Kelvin-Voigt's model.

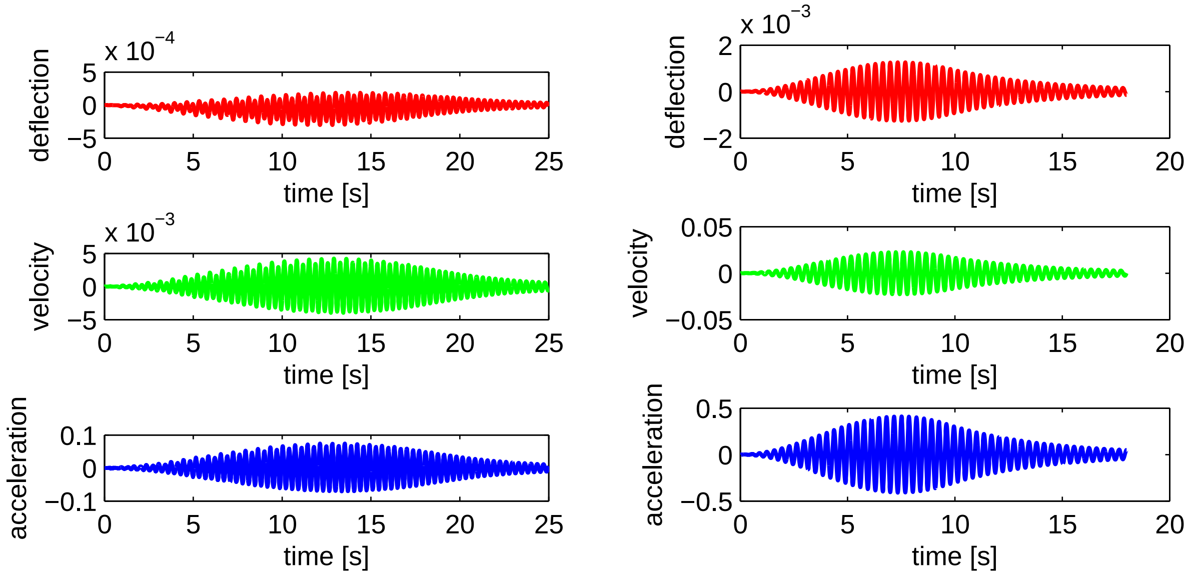

Fig. 3: Obtained results of time behaviour of middle-node, two synchronous walkers $f_p = \frac{f(1)}{2}$ (*left*) and two synchronous runners $f_p = f(1)$ (*right*).

The comparison of theoretically and experimentally obtained data showed, that Bachmann's model provided too high values of acceleration for the case of runners than the Japanese model. This was caused by higher values of coefficients α_n for runners defined in [2]. On the other hand, data obtained by using Bachmann's model were in better agreement with experiment than data obtained by Japanese model for the case of synchronous walkers. Rimless wheel model of two synchronous runners provided sufficient results compared to the experimental data. The worst agreement between theory and experiment was in the case of excitation of the structure by crowd of people, which mean that the walkers, runners or vandals were not perfectly synchronized between themselves and the structure. Time behaviour of mid-point is visualized in Fig. 3 for loading produced by 2 synchronous walkers and 2 synchronous runners.

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