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Damage Localization Using Modal Analysis on Small Cement Specimens

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Abstract: The paper presents damage detection and localization on small cement specimens (40 × 40 × 160 mm) using modal analysis. Two levels and three different positions of induced cracks were localized using different methods based on modal characteristic changes. The sensitivities of these techniques for localization of the damage were investigated and mutually compared. The suitability of the techniques for damage localization was discussed in the conclusion.

Keywords: Damage Localization; Modal Analysis; Natural Frequency; Natural Mode; Health Monitoring.

1 Introduction

The experimental modal analysis is a well-known method for measuring the modal characteristics (natural frequencies, natural mode shapes and damping) of large structures [1]. This method has been also used for damage identification and localization on these structures [2]. The main aim of this experiment is to verify the possibility of usage of a modal analysis for damage localization also on small test specimens, where the right determination of material properties [3] and damages is also very important.

2 Specimens and Induced Damage

Nine specimens – three sets of three specimens – were used for the purpose of the test. They were made from the cement paste. The used cement was CEM I 42.5 R from the factory Radotín. The dimensions of the tested specimens were 40 × 40 × 160 mm. The water cement ratio was 0.4. All specimens were measured using modal analysis in the virgin state and then the damage (crack) was made by cutting the specimens in three different distances. One set of specimens was cut in the distance of 30 mm from the end, the second one in the distance of 50 mm and the third one in the distance 80 mm from the end of the specimen. At first all specimens were cut to the depth of 1/4 of the height of the specimen. Then the modal analysis was carried out on all specimens again. The next step was cutting of specimens to the depth of 1/2 of their height. Then the modal analysis was carried out on all specimens again.

3 Modal Analysis

The experimental modal analysis was carried out on all specimens three times, at first in the virgin – non-damaged state, then in state when specimens were cut to the 1/4 of their height and the last measurement was done in the state when they were cut to the 1/2 of their height.

The specimen was hanged to satisfy the free-free support conditions. The points of measurement were chosen on three perpendicular sides of the specimen (Fig. 1 and Fig. 2). Three reference acceleration transducers were placed to one corner of the specimen and each point of measurement was struck by the impact hammer (Fig. 1). Both signals, the excitation force and the reference acceleration, were recorded and transformed using Fast Fourier Transform (FFT) to the frequency domain, and the Frequency Response Functions (FRF) were evaluated from these signals using the vibration control station Brüel&Kjær Front-end 3560-B-120 and the program PULSE 13.4. The test was repeated five times for each point and resultant readings were averaged. From an averaged FRF, the modal characteristics were determined for each specimen and a damage level.

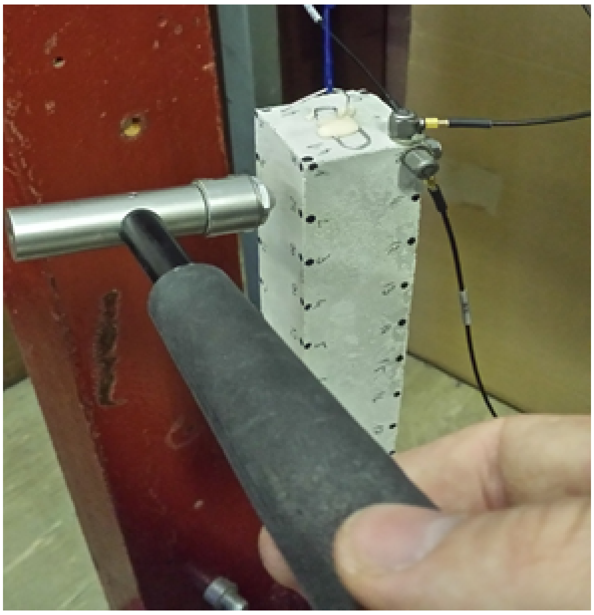


Fig. 1: Position of three reference acceleration transducers of type 4519 – 003 Brüel&Kjær and used impact hammer 8206 Brüel&Kjær.

Fig. 2: The net of measurement points on the specimen damaged to 1/2 of its height at the position 80 mm from the end.

4 Methods Used for Damage Localization

The damage detection and localization was carried out based on the comparison of natural frequencies and mode shapes using different methods. The basic method how to identify damage is the change of natural frequency between the virgin *XX* and damage *YY* states of the specimen.

$$\Delta f_{(j)} = \frac{f_{(j),obs,YY} - f_{(j),obs,XX}}{f_{(j),obs,XX}} 100 \tag{1}$$

where $f_{(j),obs,XX}$ is the j^{th} measured natural frequency of the specimen in the stage of investigation XX and $f_{(j),obs,YY}$ is the j^{th} measured natural frequency of the specimen in the stage of investigation YY. The next used method for damage identification is the Modal Assurance Criterion $MAC_{(j)}$.

$$MAC_{(j)} = \frac{|\{\mathbf{r}_{(j)}\}^T_{obs,XX}\{\mathbf{r}^*_{(j)}\}^T_{obs,YY}|^2}{\left(\{\mathbf{r}_{(j)}\}^T_{obs,XX}\{\mathbf{r}^*_{(j)}\}_{obs,XX}\right)\left(\{\mathbf{r}_{(j)}\}^T_{obs,YY}\{\mathbf{r}^*_{(j)}\}_{obs,YY}\right)} \tag{2}$$

where $\{\mathbf{r}_{(j)}\}_{obs,XX}$ – the natural mode shape vector of the specimen in the initial stage of investigation XX and $\{\mathbf{r}_{(j)}\}_{obs,YY}$ – the natural mode shape vector of the specimen in the stage of investigation *YY*. For damage localization we used also the Coordinate Modal Assurance Criterion ($COMAC_{(x)}$) [4].

$$COMAC_{(x)} = \frac{\left(\sum\limits_{j=1}^{n} \left|r_{(j),x,XX} r_{(j),x,YY}\right|\right)^2}{\left(\sum\limits_{j=1}^{n} \left|r_{(j),x,XX}\right|^2\right)\left(\sum\limits_{j=1}^{n} \left|r_{(j),x,YY}\right|^2\right)} \tag{3}$$

where $r_{(j),x,XX}$ is the ordinate of the j^{th} natural mode shape in the x^{th} measured point of the specimen in the initial stage of its investigation XX, $r_{(j),x,YY}$ is the ordinate of the j^{th} natural mode shape in the x^{th}

measured point of the specimen in the stage of its investigation YY and n is the number of the compared pairs of natural mode shapes.

The best results we obtained from the method Change of Mode Surface Curvature $CAMOSUC_{(j),x}$ [5]:

$$CAMOSUC_{(j),x} = \frac{r_{(j)XX,x+1} - 2r_{(j)XX,x} + r_{(j)XX,x-1}}{h^2} - \frac{r_{(j)YY,x+1} - 2r_{(j)YY,x} + r_{(j)YY,x-1}}{h^2} \quad (4)$$

where $r_{(j),XX,x}$ is the ordinate of the j^{th} natural mode shape in the x^{th} measured point of the specimen in the initial stage of the investigation *XX*, $r_{(j),YY,x}$ is the ordinate of the j^{th} natural mode shape in the x^{th} measured point of the specimen in the stage of the investigation YY and h is the distance of two investigated points in the direction, in which the change of the mode surface curvature is evaluated.

The last used method was the change of the diagonal members of the modal flexibility matrix $\Delta\delta_r$ [6].

$$\Delta\delta_r = \delta_{rr,YY} - \delta_{rr,XX} \quad (5)$$

where $\delta_{rr,XX}$ and $\delta_{rr,YY}$ are the diagonal members of the modal flexibility matrix $[\delta]$ in the stages XX and YY.

$$[\delta] = \left[\mathbf{R}_{(j)}\right] \left[\mathbf{1}/\omega^2_{(j)}\right] \left[\mathbf{R}_{(j)}\right]^T \quad (6)$$

where $[\mathbf{R}_{(j)}]$ is the modal matrix composed from n measured and mass-normalized natural mode shapes and $[\mathbf{1}/\omega^2{}_{(j)}]$ is the diagonal matrix built up from the inverse values of the square powers of the natural angular frequencies.

5 Damage Localization

We used five different methods for damage detection and localization on the small test specimens. For each specimen and each damage state, nine natural frequencies and mode shapes were evaluated – 3 bending modes in the plane XY, 3 bending modes in the plane XZ and 3 torsional modes. For damage evaluation only torsional modes and bending modes in the plane XZ were finally used because changes in the bending modes in the plane XY were very small.

The damage influences especially the natural frequencies (Fig. 3). The decrease of frequencies caused by damage was up to 47 %. The next used method for damage identification was $MAC_{(j)}$. The damage also changed the natural modes (Tab. 1), for some modes the $MAC_{(j)}$ decreased to 0.4. So the damage can be identified using these two methods even on small specimens.

Then we started with damage localization. We used three different methods ($COMAC_{(x)}$, $CAMOSUC_{(j),x}$ and $\Delta\delta_r$.). The best results we obtained from the $CAMOSUC_{(j),x}$ (Fig. 4).

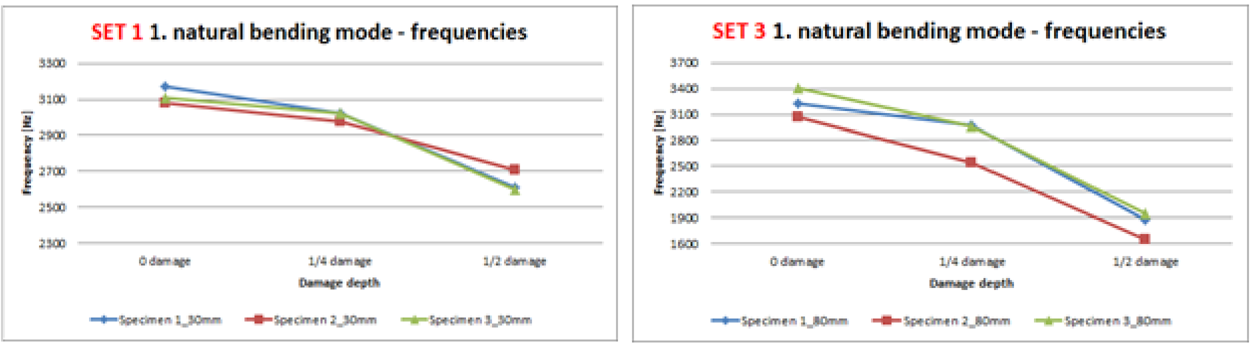
Tab. 1: Comparison of the undamaged and damaged specimen using MAC - damage in the distance 30 mm.

Natural mode No.:	1	2	3	4	5	6
MAC - 0 × 1/4 damage	0.98	0.98	0.97	0.89	0.68	0.82
MAC - 0 × 1/2 damage	0.95	0.93	0.40	0.75	0.51	0.61

6 Conclusion

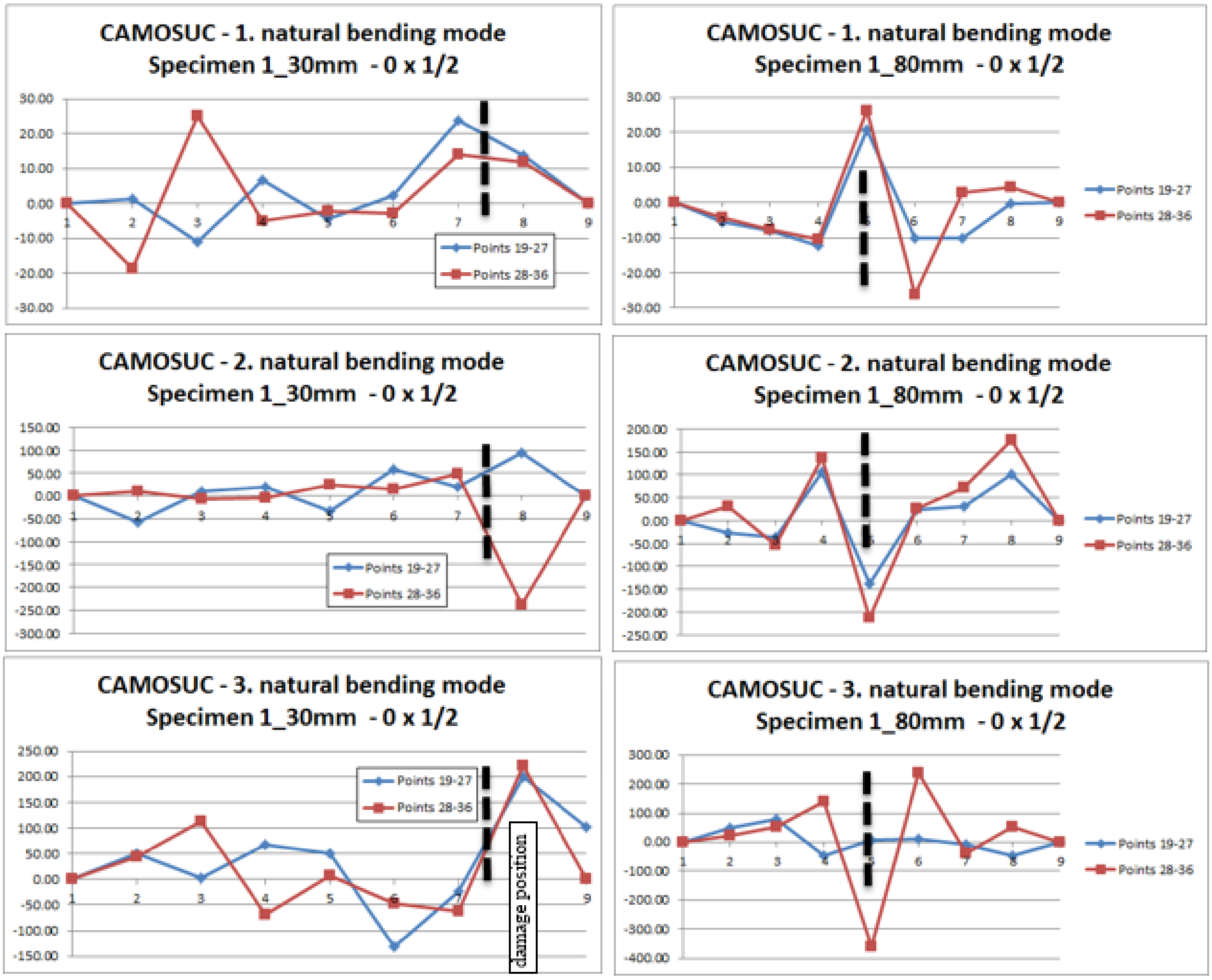
The paper presents use of modal analysis for damage detection and localization for small test specimens. The presented methods can be very well used for damage identification because changes in natural frequencies and changes in MAC are very sensitive to the damage. Some frequencies decreased even to 47 % (Fig. 3) and MAC (Fig. 4) decreased to the value 0.4.

The localization of the crack using modal analysis was not as accurate as we expected. Only one method $CAMOSUC_{(j),x}$ gave us results which corresponded with the position of the crack (Fig. 4) but only for the cracks with the depth 1/2 of the specimen thickness. Therefore the method can be used only for approximate localization of the deep crack, for shallow cracks it did not give good results.



(a) specimens with damage in the distance 30 mm (b) specimens with damage in the distance 80 mm

Fig. 3: The changes of the natural frequencies corresponding to the 1st bending mode shapes.



(a) specimen with damage in the distance 30 mm (b) specimen with damage in the distance 80 mm

Fig. 4: The changes of the first three natural bending mode shapes.

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References

[1] J. Benčát, D. Papán, Dynamic modeling and testing of a cable-stayed pedestrian bridge, in proc.: Proceedings of the 11th International Conference on Civil, Structural and Environmental Engineering Computing, Civil-Comp 2007, St. Julians; Malta, 9p.

[2] S.W. Doebling et al., Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review, Los Alamos, Los Alamos National Laboratory, 1996.

[3] P. Padevět, T. Otcovská, O. Zobal, Variation of material properties of cement pastes with various types of fly ash during maturation, WSEAS Transactions on Applied and Theoretical Mechanics 9 (2014) 88-96.

[4] N. Maia, J. Silva, Theoretical and Experimental Modal Analysis. Textbook, Research Studies Press Ltd. Taunton, England. 1997.

[5] M. Pirner, S. Urushadze, Aplikace dynamické odezvy na identifikaci stavebních konstrukcí a jejich poškození. ÚTAM AVČR, Praha 2001.

[6] T. Plachý, Dynamic Study of a Reinforced Concrete Beam Damaged by Cracks, (in Czech) PhD Thesis, FCE CTU in Prague, Praha 2003.

[7] L. Melzerová, M. Šejnoha, P. Klapálek, Bending tests and simulations of GLT beams, Applied Mechanics and Materials 611 (2014) 54-59.

[8] M. Polák, T. Plachý, J. Máca, Experimental modal analysis of the footbridge across Vltava River in Prague damaged by flood in June 2013, Applied Mechanics and Materials 617 (2014) 3-6.