Application of Enhanced Frequency Response Function Technique in Increasing the Accuracy of Modal Parameters

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Abstract: The paper deals with the use of a technique known as Enhanced Frequency Response Function (EFRF), which allows separate the different modes of a structure. The results of the separation are responses of isolated systems with one degree of freedom by means of which modal parameters of analyzed structure can be improved. The paper explains the basic principle of EFRF technique and also presents its application in experimental modal analysis.

Keywords: EFRF; Experimental Modal Analysis.

1 Introduction

The estimation of modal parameters is the most important part of the experimental modal analysis. Currently, there are many well processed techniques and algorithms that perform the estimation in the frequency or time domain. Many of them are implemented in commercial software that is delivered with precision measuring instruments. Very effective and relatively simple tool for the estimation of modal parameters is Complex Mode Indicator Function (CMIF). This method is often applied to multiple reference measurements when it is necessary to process a large amount of measurement data. CMIF is based on singular value decomposition of a complex matrix of frequency response functions [1, 2, 4]. The decomposition of FRF matrix is being performed respectively at each spectral line and leads to matrix of singular values, matrix of left singular vectors and matrix of right singular vectors. The left singular vectors represent normalized mode shapes, right singular vectors correspond to modal participation factors. These vectors are the basis to obtain the enhanced response functions. Based on these functions it is possible to determine the values of complex poles and modal scaling factors.

2 Enhanced Frequency Response Function

Enhanced Frequency Response Function can be defined as a weighted average of all of the measured frequency response functions. The singular vectors obtained from SVD are used as discrete weighting functions. Basically, they create discrete modal filters which transform the data from the physical space to the modal space. This allows to isolate the individual modes of the structure and to create as many single degree of freedom systems as there are dominant modes in the given frequency range. When the transformation is complete, then for any s-th mode the EFRF can be calculated as follows [4–6]:

$$\mathrm{EFRF}(\omega)_s = \{U\}_s^H \; [H(\omega)] \; \{V\}_s, \tag{1}$$

where $\{U\}$ is the left singular vector, $\{V\}$ is the right singular vector. The both vectors are unitary and can be determined from the following equation

$$[H(\omega_n)] = [U(\omega_n)] \; [\Sigma(\omega_n)] \; [V(\omega_n)]^H, \tag{2}$$

where $[H]$ is complex Frequency Response Function matrix, $[U]$ is the left singular matrix, $[V]$ is the right singular matrix, $[\Sigma]$ is the diagonal singular value matrix.

The CMIF function is graphically represented as the plot of singular values on a log magnitude as a function of frequency [6]. Therefore, CMIF is defined as

$$CMIF(\omega) = \text{diag}\left([\Sigma(\omega)]\right) \tag{3}$$

Matrix $[H]$ can be formally expressed as

$$[H(\omega)] = \sum_{r=1}^{2N} \{\varphi\}_r \frac{Q_r}{j\omega - \lambda_r} \{L\}_r^H \tag{4}$$

where $\{\varphi\}_r$ is the r-th mode shape, $\{L\}_r$ is the r-th modal participation factor, Q_r is the scaling factor for the r-th mode. By substituting (4) to (1)

$$\text{eFRF}(\omega)_s = \{U\}_s^H \left[\sum_{r=1}^{2N} \{\varphi\}_r \frac{Q_r}{j\omega - \lambda_r} \{L\}_r^H \right] \{V\}_s = \sum_{r=1}^{2N} \{U\}_s^H \{\varphi\}_r \frac{Q_r}{j\omega - \lambda_r} \{L\}_r^H \{V\}_s \tag{5}$$

The effect of this operation is to attenuate the contribution to the summation of all modes except mode s, thus enhancing this mode. The degree of enhancement is dependent on the inner product of the left singular vector and modal vectors. If the modal vectors are mutually orthogonal, then the EFRF will be completely decoupled. Otherwise, if the modal vectors are not orthogonal with respect to $\{U_s\}$, then those modes have some contribution to the EFRF for mode s. When r equals s, the vectors are parallel and the principal mode is not attenuated (Fig. 1). Since the vectors $\{U\}$ and $\{V\}$are normalized to unitary vectors, the EFRF function is actually defined as the response function of one decoupled single mode. Therefore for the r-th mode

$$\text{EFRF}(\omega)_r \equiv \frac{Q_r}{j\omega - \lambda_r} \tag{6}$$

Eq. (5) shows that EFRF is only related with the modal scaling factor, natural frequency and damping of the given mode. In addition, modal scale factor corresponds to the residue, therefore by successively analyzing the individual Enhanced FRFs there is possible to ensure that all mode shapes are correctly normalized.

3 Analytical Model

The EFRF algorithm was tested on a mechanical system with 4 degrees of freedom (Fig. 1). This system was described by the following matrix equation of motion

$$[M]\{\ddot{x}\} + [B]\{\dot{x}\} + [K]\{x\} = \{0\} \tag{7}$$

where $[M]$, $[B]$, $[K]$ are mass, damping and stiffness matrices, respectively; and $\{x\}$, $\{\dot{x}\}$, $\{\ddot{x}\}$ are displacement, velocity, and acceleration vectors, respectively.

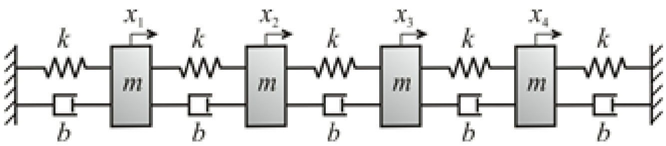

Fig. 1: The model of 4-DOF mechanical system.

A free vibration analysis of the system has been carried out to get discrete time responses, which were subsequently transformed to the frequency domain and arranged into a matrix of frequency responses. Theoretically, this matrix represented a FRF matrix that is a result of multi-reference measurement of modal parameters. In this case, matrix of frequency responses was 4×4, which corresponds to 4 input DOFs and 4 output DOFs. Before applying the EFRF algorithm, it was necessary to obtain the singular vectors $\{U\}$ and $\{V\}$. For this purpose, the Complex Mode Indicator Function had to be computed. In Fig. 2, plot of the diagonal elements of singular value matrix $[\Sigma]$ solved from the matrix of frequency responses at each spectral line is shown. The peaks detected on the CMIF plot indicate the existence of modes, and the corresponding frequencies of these peaks give the damped natural frequencies for each mode [3]. The number of modes detected in CMIF determines the minimum number of degrees-of-freedom of the analytical system and the order of the system equation used in the algorithm.

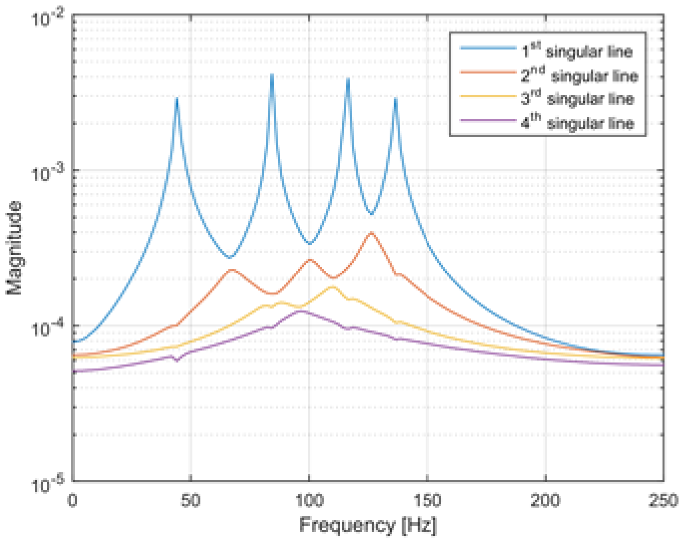

Fig. 2: Plot of Complex Mode Indicator Function of analytical model.

The Enhanced Frequency Response Functions of the systems were subsequently calculated according to the Eq. (1) at each spectral line. The isolated responses of the individual modes are shown in Fig. 3. Each of them is represented by a complex-valued vector. If the modes are correctly decoupled, a simple single degree of freedom parameter estimation algorithm can be used to estimate the modal parameters of the enhanced mode.

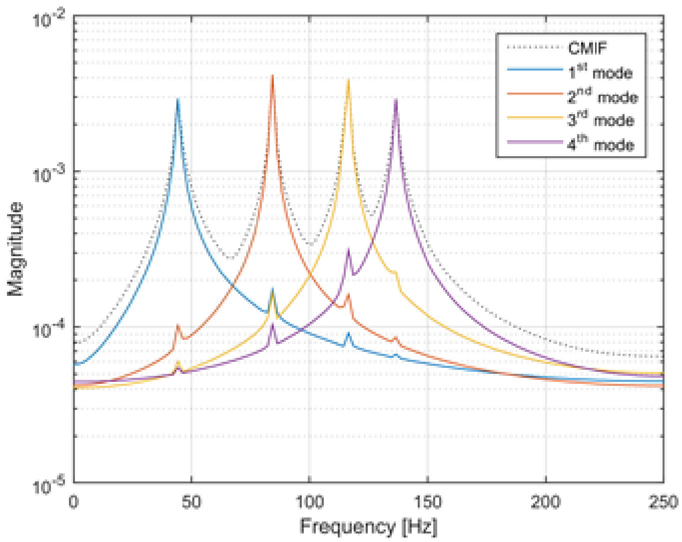

Fig. 3: Enhanced Frequency Response Functions of analytical model.

4 Experimental Model

Measurement object was a steel rod of circular cross section with 22 mm in diameter. Its length was 0.8 mm. The object was put on a very soft pad (Fig. 4a). Geometrical model of the measured structure was represented by a single straight line. There were defined 18 input DOFs at which the structure was excited and six output DOFs at which responses were measured by uniaxial accelerometers. The excitation was done by impact hammer. The position of each DOF is evident from Fig. 4b. The measurement was performed in frequency range up to 3200 Hz.

Measured frequency response functions were exported to Matlab, where they were processed the same way as the data obtained from analytical model. In this case, the FRF matrix was 18 $\times$ 6. CMIF plot for analyzed

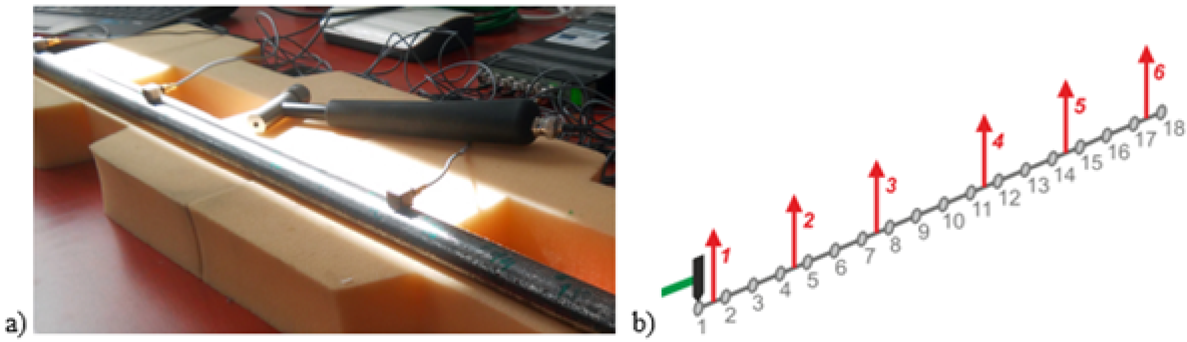

Fig. 4: a) Measurement object with applied accelerometers, b) Geometrical model of the structure.

Tab. 1: Natural damped frequencies of the analyzed structure.

Mode	0.	1.	2.	3.	4.	5.	6.
Frequency [Hz]	36	160	432	838	1374	2050	2826

structure is shown in Fig. 5. As can be seen, the structure has 7 modes in the given frequency range. The first one is rigid, the others are flexible. Every mode shape of vibration is given by the corresponding singular vector $\{U\}$ that was determined from singular value decomposition of the FRF matrix. The mode shapes of the individual flexible modes are shown in Fig. 6. Values of natural damped frequencies are noted in Tab. 1.

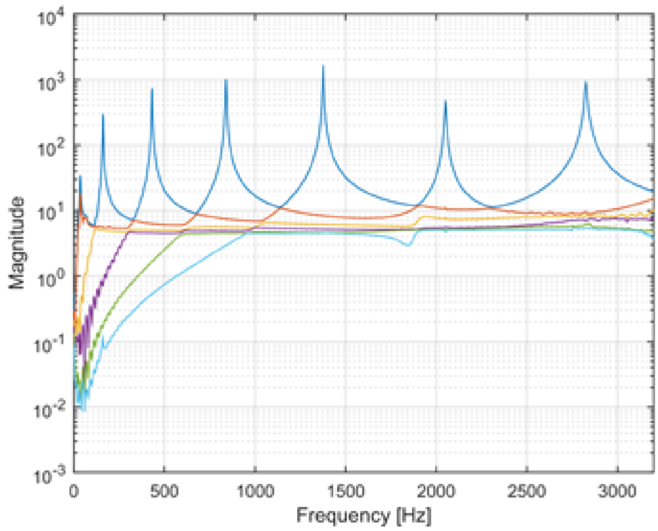

Fig. 5: CMIF plot of the analyzed structure.

As the example, the Enhanced Frequency Response Function of the 2^{nd} mode is shown in Fig. 7a. The accuracy and quality of a modal weighting depends on a degree of multi-reference analysis. The more reference DOFs are available the better the modes are decoupled to each other. The EFRF shown in Fig. 7b was calculated from the FRF matrix of size 18×3.

5 Conclusion

The algorithm of Enhanced Frequency Response Function was explained and practically applied in experimental modal analysis. The EFRF technique allows separate the different modes of a structure. The results are responses of isolated systems with one degree of freedom by means of which modal parameters can be more accurately estimated. The singular vectors solved from SVD of FRF matrix are used as modal filters. The quality of filtering, as presented in the paper, depends on the number reference degrees of freedom. There is also possible to determine system poles of a measured object. The UMPA estimation procedure could be used for this purpose.

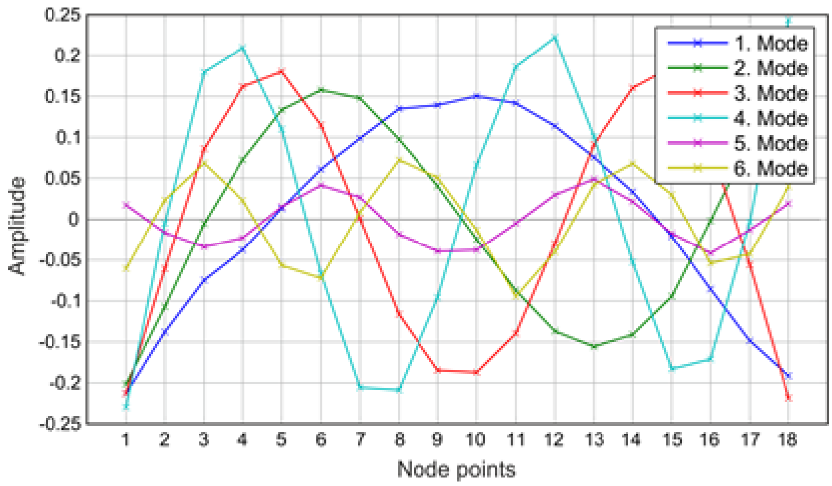

Fig. 6: Mode shapes of the analyzed structure.

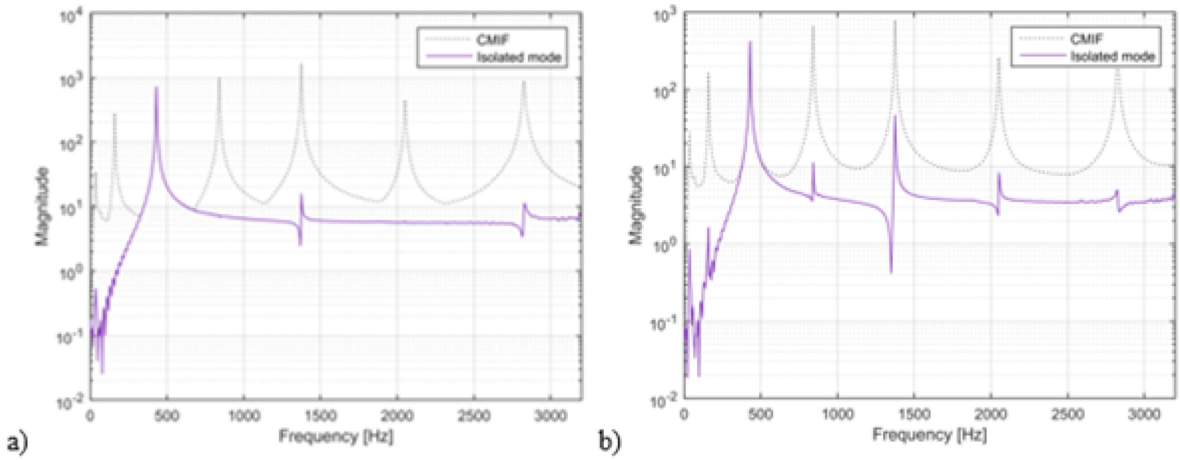

Fig. 7: EFRF of the 2nd mode for a) FRF matrix of size 18 $\times$ 6, b) FRF matrix of size 18 $\times$ 3.

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References

[1] C. M. Harris, G. P. Allan: Harris’ shock and vibration handbook, Fifth edition. McGraw-Hill, 2002.

[2] D. J. Ewins: Modal Testing - Theory, practice and application. 2nd edition, Wiley, 2000.

[3] F. Trebuňa, F. Šimčák, R. Huňady: Vibration and modal analysis of mechanical systems (in Slovac), TU v Košiciach, (2012).

[4] R. J. Allemang, D. L. Brown: A Complete Review of the Complex Mode Indicator Function with Applications. International Conference on Noise and Vibration Engineering, ISMA, 2006.

[5] S. J. Shelley: Investigation of Discrete Modal Filters for Structural Dynamics Applications, Doctoral Dissertation, University of Cincinnati, 1991, 269 pp.

[6] W. Leurs, F. Deblauwe, F. Lembregts: Modal Parameter Estimation Based Upon Complex Mode Indicator Functions. IMCA XI 1993.