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Application of PhotoStress Method in Visualisation of Stress Fields in Periodically Loaded Structural Elements

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Abstract: The PhotoStress method is a frequently used method for visualisation or quantification of stress fields in structural elements which occur during static or dynamic loading. The presented paper demonstrates a visualisation of stress fields in rotating objects. The stress state in these objects is based on centrifugal forces with dynamic periodic effect. Isoclines and isochromatics of isoclines and isochromatics were recorded with reflection polariscope LF/Z-2 with stroboscopic source of polarised light.

Keywords: Dynamic Photoelasticity; Isoclines; Isochromatics; Reflection Polariscope LF/Z-2; Stroboscopic Light; Photoelastic Coating.

1 Introduction

Based on the factor of time, dynamic effects may initiate periodical or non-periodical stress state of objects or structures. The investigation of periodical dynamic effects within the PhotoStress method is based on records of periodical phenomena which occur with the same time interval and the same oscillation period. The photoelastically coated object in motion, subject to measurements, is illuminated for a short period of time and always in the same position which the object reaches during its motion. Identical images constantly reoccur and create an effect of a static image of the observed object with photoelastic entities – isoclines or isochromatic fringes. In dynamic stress analysis by means of the PhotoStress method stroboscopic light is used as a source of polarised light [3].

2 Stress State During Dynamic Effects

As regards dynamic effects, we distinguish between:

- stress state during dynamic periodical effects,
- stress state during dynamic non-periodical effects.

With dynamic periodical effects, e.g. in a rotated objects, the stress state is initiated by centrifugal forces. If the rotating bodies are small, considering the radius, the stress state can be determined from the similarity of stress states based on individual weight. As regards periodically loaded objects (e.g. rods, rotating discs etc.) with photoelastical coating, the object in motion must be viewed while illuminated with polarised light. As a medium of illumination stroboscopic light is used.

The stress state in dynamic non-periodical effects, which is drawn e.g. from an impact, is recorded with a high-speed camera in colour [2,5].

Both above-mentioned cases, however, require a different methodology and a different experimental method.

In some cases dynamic effect may cause a stable stress state in terms of quantity. Based on time period, this state continuously moves from one position in the object to the other. Such state can be found in lathe turning or material planing, provided that the speed of processing or shift does not change. When a continuously moving photoelastic material is viewed through a reflection polariscope while being processed by a stable (fixed) tool, isoclines and isochromatic fringes remain stable and can easily be recorded with a digital camera. The same state occurs when the machine stops and the tool and material interact in their static position [1,5,6].

Fig. 1 shows an example of isochromatic fringes as viewed through a reflection polariscope LF/Z-2 during turning of a round disc from photoelastic material PS-1A with thickness of 3.125 mm. The round disc is loaded with torsion moment and pressure of the lathe cutting tool.

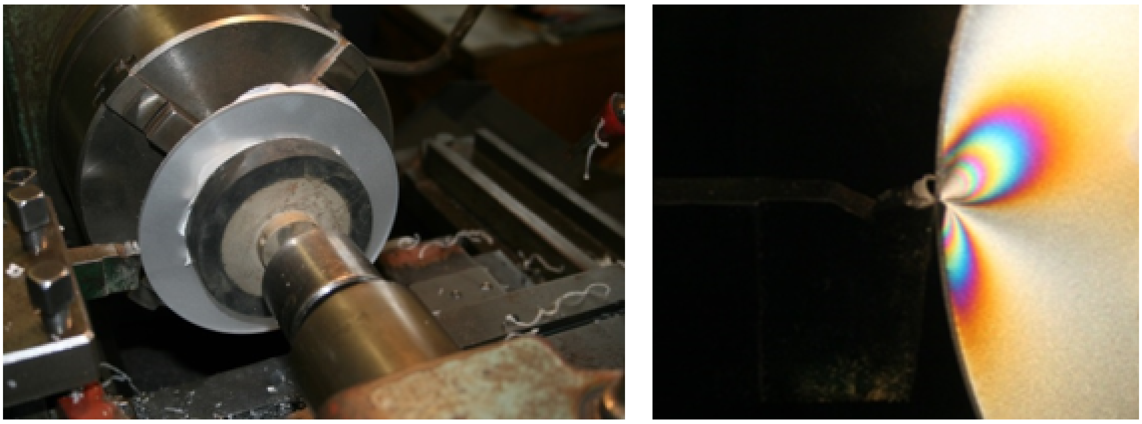

Fig. 1: Isochromatic fringes during lathe turning.

The following part of the paper illustrates an example of dynamic analysis within PhotoStress method which was carried out on the rotating object in a stress state which is based on centrifugal forces – in this case with dynamic periodic effect.

3 Isoclinic and Isochromatic Fringes on Rotating Samples Due to Centrifugal Forces

The stress state in the disc (Fig. 2) with centric hole with radius r_1 and outer radius r_2, rotating with constant angular velocity ω, is given by equations

$$\sigma_r\left(r\right) = \frac{3+\mu}{8}\rho\omega^2\left(r_2^2 + r_1^2 - \frac{r_1^2 r_2^2}{r^2} - r^2\right), \tag{1}$$

$$\sigma_t\left(r\right) = \frac{3+\mu}{8}\rho\omega^2\left(r_2^2 + r_1^2 + \frac{r_1^2 r_2^2}{r^2} - \frac{1+3\mu}{3+\mu}r^2\right), \tag{2}$$

where $\sigma_r(r)$, $\sigma_r(r)$ are radial and circumferential stresses, ρ radius in which stress is to be determined, μ represents Poisson's ratio, ω angular velocity.

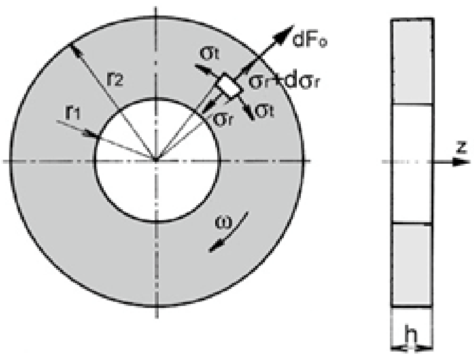

Fig. 2: Rotating disc with constant thickness and centric hole.

Stress state in the rotating disc is symmetrical, independent of angle φ. As shear stress τ_{rt} = 0 stresses $\sigma_r(r)$ and $\sigma_t(r)$ are principal stresses.

The difference of principal stresses $(\sigma_t(r) - \sigma_r(r))$ can be obtained after adjustment of Eq. 1 and 2 in the following form:

$$(\sigma_t(r) - \sigma_r(r)) = \frac{3+\mu}{4}\rho\omega^2\left(\frac{r_1^2 r_2^2}{r^2} + \frac{1-\mu}{3+\mu}r^2\right) \tag{3}$$

The course of difference of principal stresses $(\sigma_t(r) - \sigma_r(r))$ in the rotating disc of a constant thickness is depicted in Fig. 3.

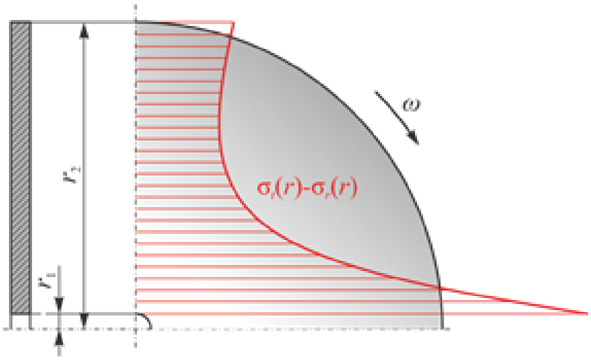

Fig. 3: Course of difference of principal stresses.

Equation of isoclines can be derived from relations for directions of principal stresses which arise from the Mohr's circle

$$\text{tg } 2\alpha = \text{const.} \tag{4}$$

$$\text{tg } 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-2(\sigma_t(r) - \sigma_r(r))\cos\varphi\sin\varphi}{(\sigma_t(r)\sin^2\varphi + \sigma_r(r)\cos^2\varphi) - (\sigma_t(r)\cos^2\varphi + \sigma_r(r)\sin^2\varphi)} \tag{5}$$

the adjustment results in the following expression

$$\text{tg } 2\alpha = \text{tg } 2\varphi. \tag{6}$$

As a result, isoclines in the rotating disc of a constant thickness with centric hole are radial straight lines which together with axis x (Fig. 4) form the angle φ that equals angular parameter α of the respective isocline.

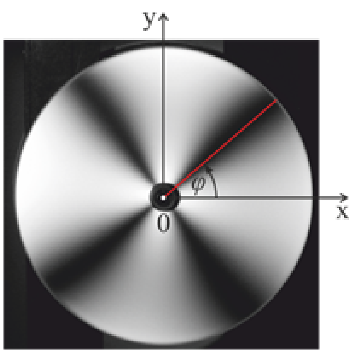

Fig. 4: Definition of angle φ.

Isostatic lines with tangents determining the direction of principal normal stresses can be specified from characteristic equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \text{tg } \alpha \quad \text{- for isostatic lines of the I. type} \tag{7}$$

For the type of isostatic lines, which cross the isostatic lines of the I. type orthogonally, applies

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\text{tg } \alpha} \quad \text{- for isostatic lines of the II. type} \tag{8}$$

Solving differential Eq. 7 and 8 with application of Eq. 6, while $\mathrm{tg}\ \varphi = \frac{y}{x}$, we obtain the equation of isostatic lines of the I. type in the form [4]

$$y_1 = C_1 x \qquad (9)$$

In the rotating disc of a constant thickness with centric hole isostatic lines of the I. type are radial straight lines which cross the centre of this disc (Fig. 7b).

From Eq. 8 after substitution for $\mathrm{tg}\ \alpha$ and integration we obtain the expression fro isostatic lines of the II. type in the form [4]

$$x^2 + y^2 = C_2^2 \qquad (10)$$

Eq. 10 represents the set of concentric circles (Fig. 7b) with midpoint in the centre and radius C_2 [4].

The rotating disc of a constant thickness 3.125 mm with centric hole with radius r_1 = 5 mm and outer radius r_2 = 100 mm, subject to experimental examination, was cut from photoelastic material PS-1A to a required shape. The disc was loaded by centrifugal forces based on the rotation of the disc caused by motor HSM 60 (Fig. 5).

Fig. 5: Measuring and loading apparatus for the rotating disc.

At 5 000 RPM the rotating disc was first illuminated with plane-polarised light from reflection polariscope LF/Z-2 which enabled us to view the isoclinic fringes. The isoclinic fringes appeared as radial straight lines based on rotation angle of the analyser or angular parameter of the isocline α. Fig. 6 depicts isoclinic lines of the rotating disc with angular parameter 0 ° to 90 ° with increments of 10 °.

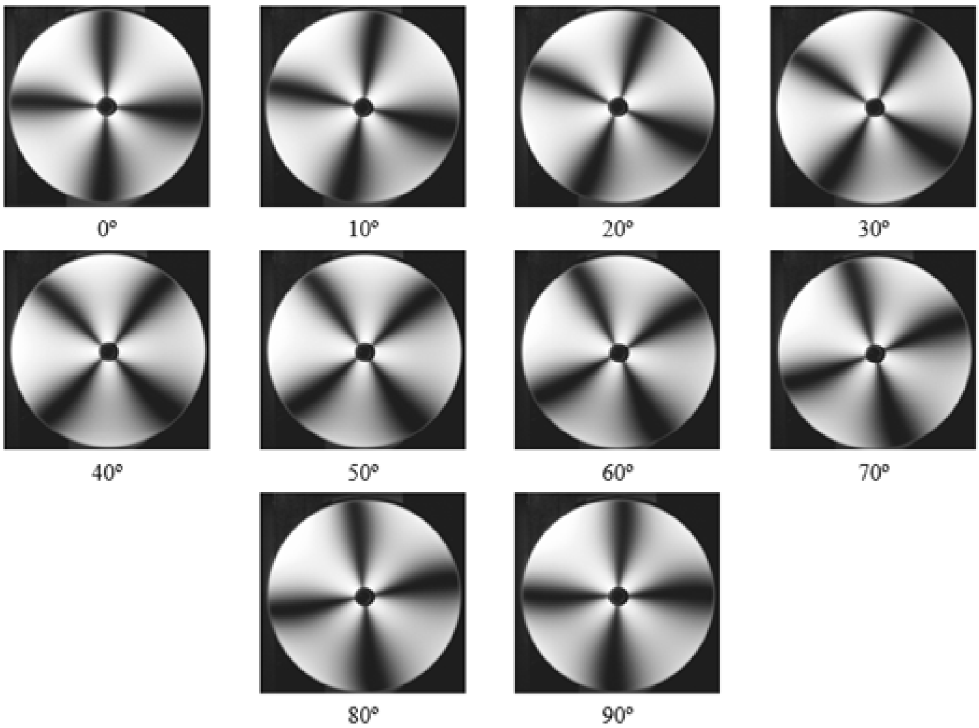

Fig. 6: Isoclinic fringes of the rotating disc loaded by centrifugal forces.

A set of isostatic lines of the I. type and the II. type (Fig. 7b) was generated from the set of isoclinic lines (Fig. 7a) [3].

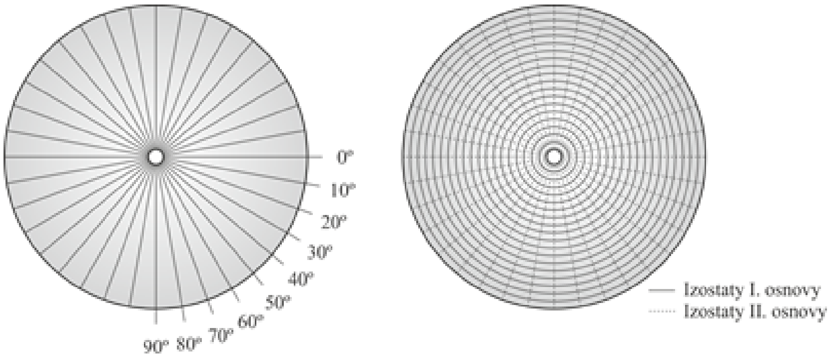

Fig. 7: The set of a) isoclines; b) isostatic lines of the I. type and the II. type of the rotating disc loaded by centrifugal forces.

Considering the Fig. 7b it is obvious that the isostatic lines of the I. type are radial straight lines crossing the centre of the rotating disc and the isostatic lines of the II. type are concentric circles with centre in the disc axis. Both types of isostatic lines are crossing each other at right angle.

The rotating disc of a constant thickness is loaded by centrifugal force during its rotation. During rotation of the disc isochromatic fringes observed under circular polarised light form concentric circles. These are depicted in Fig. 8 for 10 000 RPM to 15 000 RPM [3].

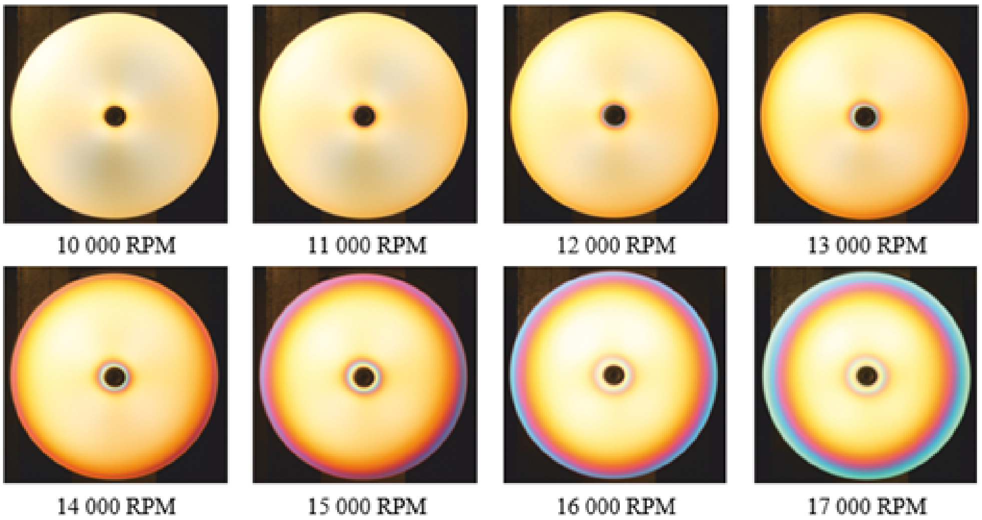

Fig. 8: Rotating disc during continuous increase of revolutions.

Fig. 9 depicts a detail of isochromatic fringes near the hole of the rotating disc at 15 000 RPM [3, 7].

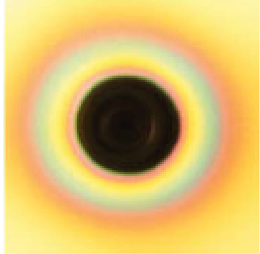

Fig. 9: Detail of isochromatic fringes.

Fig. 10 depicts an example of isochromatic fringes viewed under circular polarised light in other rotating samples of a different shape. For recording of photoelastic entities we used reflection polariscope LF/Z-2 with stroboscopic light source STROBEX/135M-12.

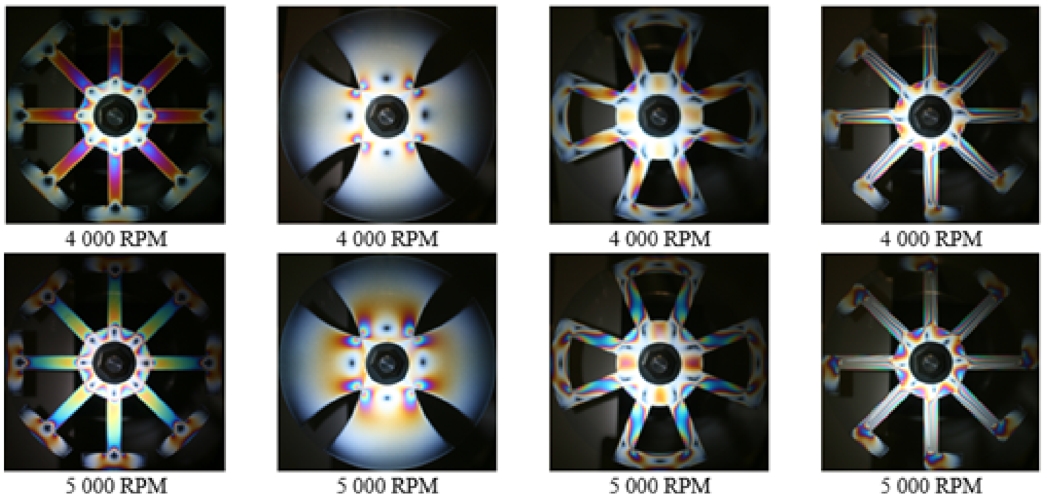

Fig. 10: Example of distribution of isochromatic fringes in rotating samples at 4,000 RPM and 5000 RPM.

4 Conclusion

PhotoStress method as one of optical experimental methods of mechanics is today irreplaceable in the evaluation of deformation and stress fields. In comparison to other experimental methods, the advantage of PhotoStress method lies in a quick visual identification of deformation or stress field distribution and concentration along the whole photoelastically coated area of the structural element subject to examination. The above-mentioned was proved in the presented paper. Based on photoelastic entities (isoclinic and isochromatic fringes) the method enables us to quantify directions as well as magnitudes of principal strains and principal stresses in every point of a photoelastically coated element. Quantitative analyses which arise from dynamic measurements by means of PhotoStress method will be published in future papers.

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