A Split Hopkinson Bar Method for Testing Materials with Low Characteristic Impedance

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Abstract: A split Hopkinson pressure bar (SHPB) technique has been developed to study dynamic behaviour of materials having low characteristic impedance. To enable better matching of characteristic impedance with a specimen, polymethyl methacrylate (PMMA) bar is used as the output bar. The viscoelastic properties of PMMA are determined in advance through preliminary experiments. In the present SHPB method, the wave analysis of the stress pulses is executed in the frequency domain. Transmitted pulses on the PMMA output bar resulting from a SHPB test are resolved into frequency components by the Fourier transform, and are corrected to be the waveforms at the specimen-bar interfaces. The corrected waveforms have been used for the evaluation of experimental results on the stress pulse transmission and reflection at the interface between elastic (Aluminium) and viscoelastic bars.

Keywords: Stress Wave Propagation; Viscoelasticity; Reflection; Transmission.

1 Introduction

The split Hopkinson pressure bar (SHPB) technique proposed by Kolsky [1] (1949) was originally developed in order to examine the dynamic plasticity of metal materials and now has been generally recognized as one of the useful experimental methods for determining the dynamic properties of materials. The SHPB technique consists of holding a cylindrical specimen of a test material between input and output bars made of elastic metal. Recently, many researchers have been concerned with the SHPB technique to evaluate properties of low impedance materials. For this purpose, they replace elastic input and output bars with polymeric bars to make better matching of characteristic impedance with specimens. However, a specific problem arises when polymeric materials are used for input and output bars. Since such materials should be considered viscoelastic waveguides, it becomes necessary to correct measured waveforms to take into account the attenuation and dispersion, which are caused while the stress waves propagate between the specimen-bar interfaces and the measuring locations on the input and output bars. The key-point for accurately determining the dynamic properties of the specimen lies in how to correct the waveforms of stress pulses propagating in viscoelastic bars.

The wave propagation in viscoelastic bars is susceptible to both material and geometrical dispersions [2,3]. Due to these effects, the shape of the wave does not remain the same while traveling along a viscoelastic bar. Corrections need to be carried out on recorded strain histories in order to describe those at the bare specimen interface. To carry out such corrections, the wave propagation coefficient of the viscoelastic bar or the material properties of the bar material must be known. Determination of the propagation coefficient for such bars is a critical step for the subsequent evaluation of the material properties of the specimen. The evaluated material properties of the specimen, dynamically tested using viscoelastic SHPB, strongly depend on the accuracy of the propagation coefficient of the both of the bars (incident and transmitter). This propagation coefficient may be determined either using the analytical solutions or through wave propagation experiments. The former method requires that the material properties of the bars be known in advance. While contrary to the case of elastic materials, it is difficult to provide generic properties for such materials as these are dependent on the loading rate, environmental history and manufacturing conditions. The frequency dependent characteristics of the two bars manufactured using the same procedure but with different diameters may be found different e.g. [4, 5]. Such facts necessitate that the published material parameters of viscoelastic materials be used with caution.

In this study PMMA bar is used as the transmitter bar in an SHPB test setup. The incident bar is made from elastic material (Aluminium alloy). The transmitter bar made from PMMA is subjected to the wave propagation testing. Longitudinal strains, generated as a result of axial impact of striker with different striking velocities and recorded at the mid-length of the bars, are used to determine the wave propagation coefficient. The basic theory and experimental results on this topic are presented in the following chapter.

2 Wave Propagation in Viscoelastic Bar

Consider a straight, cylindrical, slender bar made of a linearly viscoelastic material with cross-sectional area and density A and ρ, respectively. It is axially impacted by another bar. If the smallest wavelength of the impact pulse is much greater than the lateral dimensions of the bar, then the lateral motion of the bar can be neglected. The normal stress $\sigma(x, t)$ and the longitudinal strain $\varepsilon(x, t)$ are related to the axial displacement $u(x, t)$ at any cross-section X at time t by

$$\frac{\partial \sigma\left(x.t\right)}{\partial x}=-\rho\frac{\partial^{2}u\left(x,t\right)}{\partial t^{2}}\qquad\varepsilon\left(x,t\right)=\frac{\partial u\left(x,t\right)}{\partial x}\tag{1}$$

Using Fourier transform, Eq. 2 may lead to one dimensional equation of axial motion in frequency domain, as follows:

$$\frac{\partial^{2}\widehat{\sigma}\left(x,\omega\right)}{\partial t^{2}}=-\rho\omega^{2}\widehat{\varepsilon}\left(x,\omega\right)\tag{2}$$

where $\bar{\sigma}$ and $\bar{\varepsilon}$ denote the Fourier transforms of the stress and strain, respectively. ω is the angular frequency in radians/s.

The linear viscoelastic behaviour of the material can be expressed as follows

$$\widehat{\sigma}\left(x,\omega\right)=E^{*}\left(\omega\right)\widehat{\varepsilon}\left(x,\omega\right)\tag{3}$$

Where E^{*} is the complex modulus of the viscoelastic material. This modulus is given by the frequency dependent propagation coefficient $\gamma(\omega)$:

$$\gamma\left(\omega\right)=\alpha\left(\omega\right)+ik\left(\omega\right)=\alpha\left(\omega\right)+i\frac{\omega}{c\left(\omega\right)}\qquad E^{*}\left(\omega\right)=-\frac{\rho\omega^{2}}{\gamma^{2}\left(\omega\right)}\tag{4}$$

Let us consider a case of axial impact on one end of a slender bar while the other end remains free. If the length of the striker is much shorter than that of the bar under impact, the recorded

incident and reflected strains at the middle of the bar (i.e. × = 0) may be assumed as non-overlapping. In this case $\gamma(\omega)$ may be determined using following relation [5]:

$$\gamma\left(\omega\right)=\left[-\left[\ln\left\{abs\left(\frac{\varepsilon_{R}\left(0,\omega\right)}{\varepsilon_{I}\left(0,\omega\right)}\right)\right\}\right]-i\left[unwrap\left\{angle\left(\frac{\varepsilon_{R}\left(0,\omega\right)}{\varepsilon_{I}\left(0,\omega\right)}\right)\right\}\right]\right]\frac{1}{d}\tag{5}$$

where ε_I(0,ω) and ε_R(0,ω) are the Fourier transforms of the incident and reflected strain signals recorded at the middle of the bar i.e. at × = 0.ln, abs, angle and unwrap represent natural log, absolute value, phase angle (the angle lies between $\pm\pi$), and unwrapping function for phase angles spectra (adds $\pm 2\pi$ to original angle when consecutive phase angles differ by more than π radians) of the complex function, respectively. d is the distance covered by the wave front.

The propagation coefficient has been determined using of an experimental set up where the viscoleastic bar (15 mm in diameter and 1000 mm in the length) made from PMMA was impacted by an elastic ball (6.35 mm in the diameter) made from high strength steel. Incident stress pulse and stress pulse reflected at the free end of the bar were detected by a strain gauge at 410 mm from the impacted end of the bar. Example of the experimental record is shown in the Fig. 1.

The experimental records have been expressed in the frequency domain,ω, using of the Fourier transform. This frequency representation has a general form:

$$\varepsilon_{I}\left(\omega\right)=\left|\varepsilon_{I}\right|e^{i\phi_{I}}\qquad\varepsilon_{R}\left(\omega\right)=\left|\varepsilon_{I}\right|e^{i\phi_{R}}\tag{6}$$

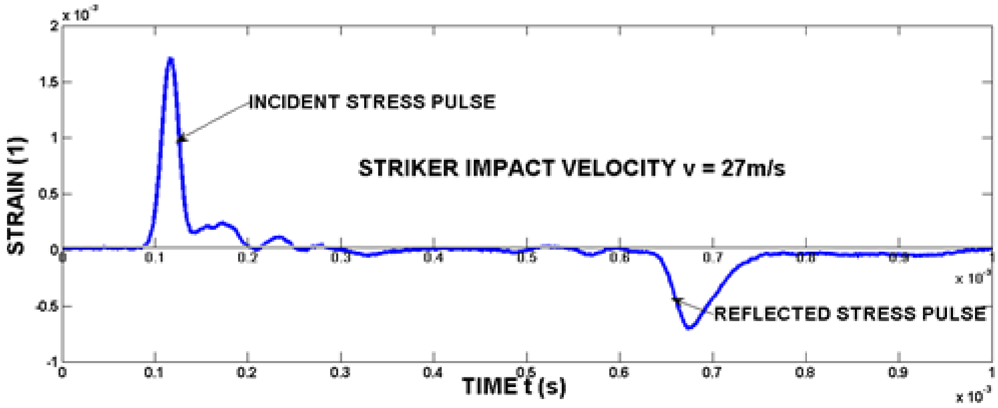

Fig. 1: Example of the experimental record of the incident ε_I and reflected ε_R stress pulse.

Examples of the amplitude and phase frequency dependences are shown in the Figs.2 and 3.

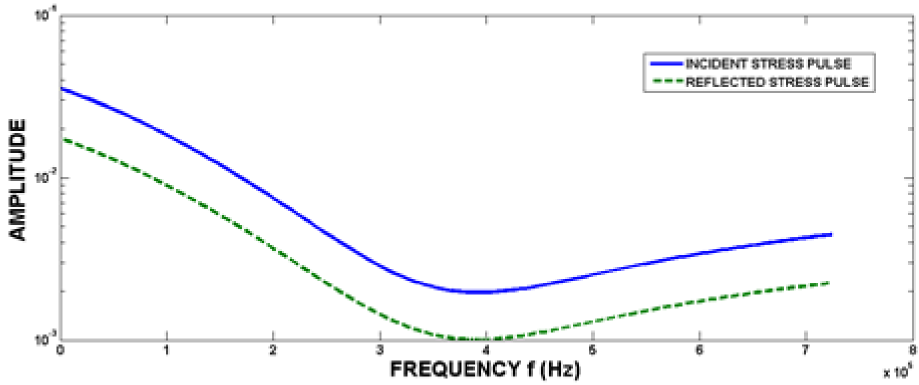

Fig. 2: Frequency dependences of amplitudes.

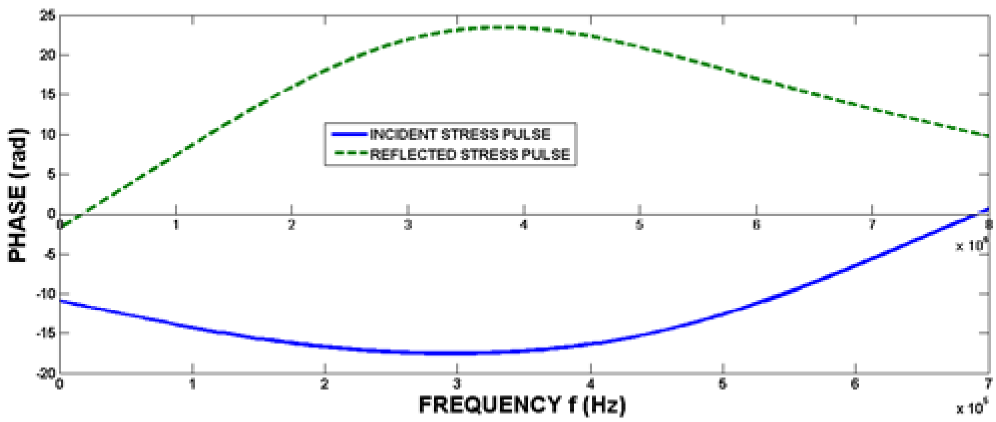

Fig. 3: Frequency dependences of phases.

The knowledge of both quantities enables to evaluate the propagation coefficient $\gamma(\omega)$ using of the Eq. (4). The frequency dependence of the attenuation coefficient is displayed in the Fig. 4.

It is obvious that there is an interval of the frequencies with a minimum attenuation.

Phase velocity c(ω) is shown in the Fig. 5.

A propagation coefficient that is determined experimentally using Eq. (19) contains all material and geometric effects on both dispersion and attenuation. Generally, it is recommended that a number of tests are carried out in order to reduce errors. The results presented in the Figs.4 and 5 represent an average from 10 measurements for different striking velocities.

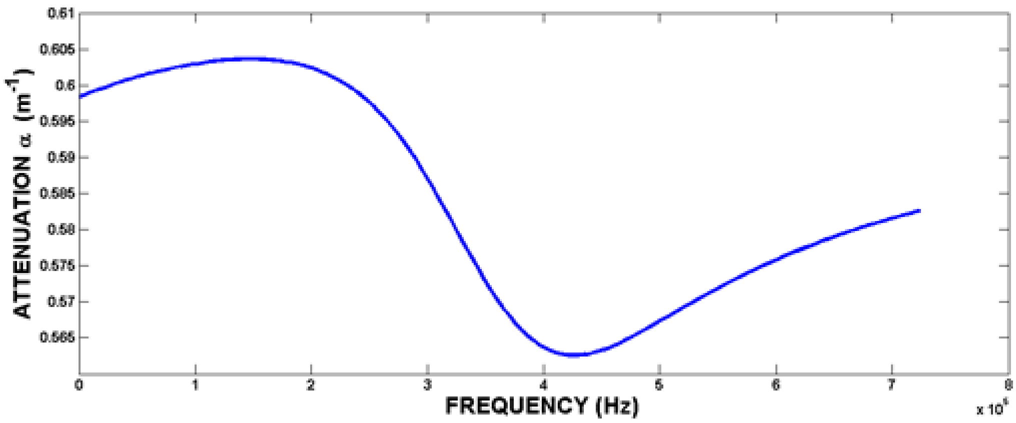

Fig. 4: The effect of the frequency on the attenuation coefficient.

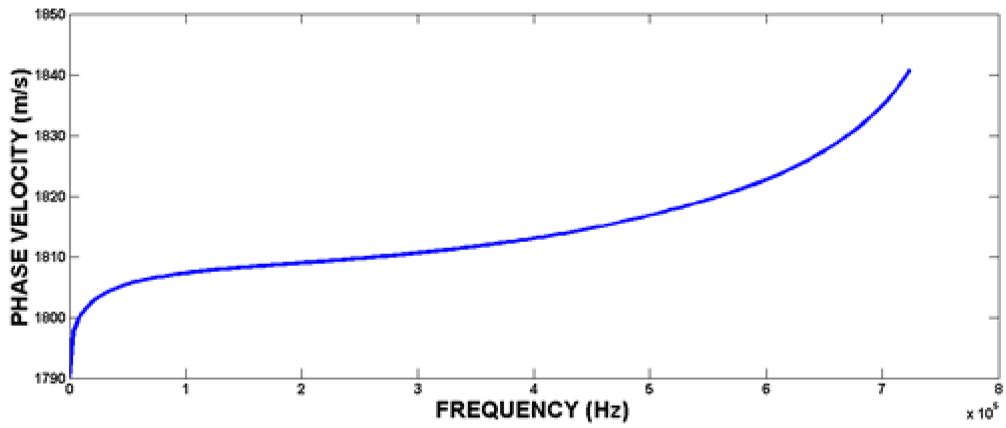

Fig. 5: Phase velocity in the viscoelastic PMMA bar.

The propagation coefficient was then used for the evaluation of the complex Young modulus E*. The amplitude and phase of this modulus are displayed in the Figs. 6 and 7.

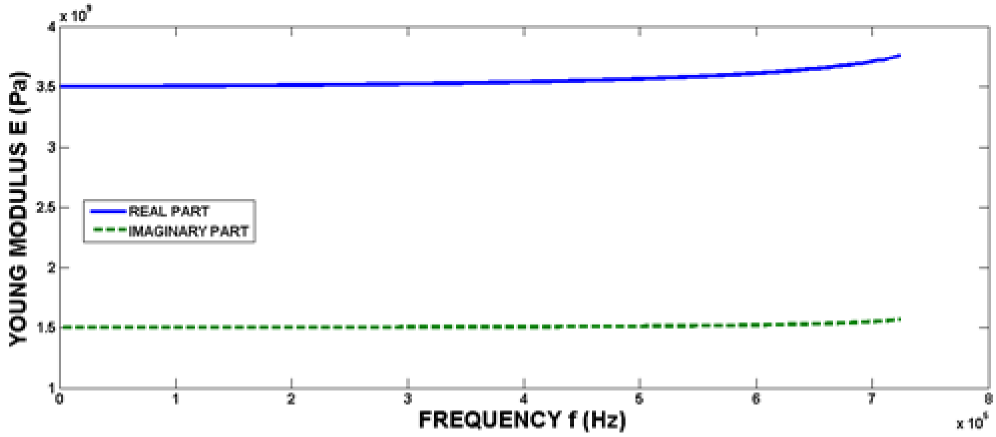

Fig. 6: Real and imaginary part of the Young modulus.

The obtained results have been used for the evaluation of some experiments focused on the transmission and reflection of the stress pulse at the interface between bars made from the different materials.

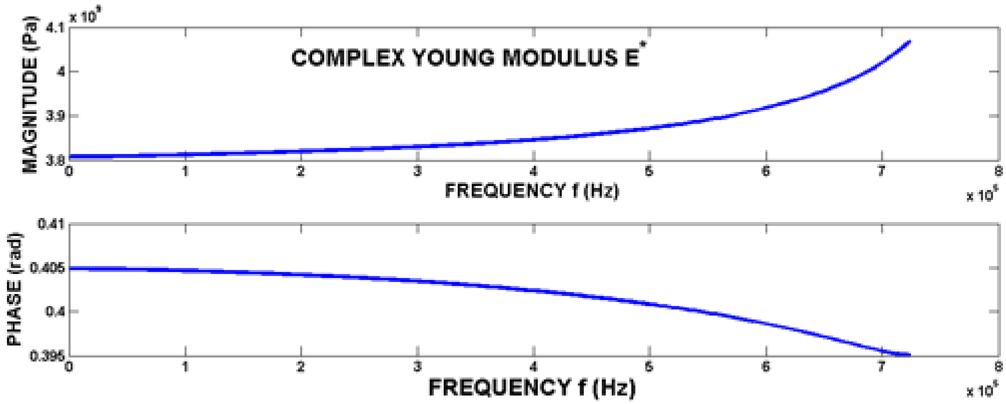

Fig. 7: Amplitude and phase of the complex modulus E*.

3 Stress Pulse Behaviour at the Interface of Two Bars

The experiments were conducted on an arrangement of two long bars. The incident and transmission bars utilized were 1 m in length, 15 mm in diameter. The impacted bar has been made from 7075 T640 aluminium alloy which is considered as elastic. The transmission bar has been made from PMMA and considered as viscoelastic. Two strain gauges were located on each bar 500 mm from the bars interface and arranged in a diametrically opposed configuration so as to cancel out any effects of bending. The striker bar used with the aluminium – PMMA bars systems was made from the beech wood, 200 mm in length and 6 mm in diameter. Different striking velocities raging from about 2 mm/s up to about 60 mm/s were used.

The incident,ε_I, reflected,ε_R and transmitted,ε_T strain pulses have been recorded. If we limit our consideration to one dimensional wave theory the corresponding stress pulses in the elastic bar is evaluated as:

$$\sigma_I = E_b \varepsilon_I \qquad \sigma_R = E_b \varepsilon_R \tag{7}$$

Where Z_b is the acoustic impedance of the elastic bar. The acoustic impedance of the used bar is 13.95 MPas/m.

The stress in the viscoelastic bar is given by the Eq. (3). The relation between incident, reflected and transmitted pulses is given as [6]

$$\sigma_R = \frac{Z - Z_b}{Z + Z_b} \sigma_I \qquad \sigma_T = \frac{2Z}{Z + Z_b} \sigma_I \tag{8}$$

Where Z denotes the acoustic impedance of the transmission bar. The acoustic impedance of the viscoelastic bar is given as:

$$Z = \sqrt{\rho E^*} = \rho \frac{\omega}{\gamma} \tag{9}$$

The density of the used PMMA is 1189 kg/m^3. The PMMA is sometimes taken as an elastic material, where Z = 2.153MPas/m. Generally the transmission and reflection are complex numbers. Their frequency dependences are shown in the Figs. 8 and 9.

The viscoelasticity generally leads to smaller amplitudes of the transmitted pulses. This procedure has been used for the evaluation of the transmitted and reflected stress pulses. The maximum values of transmitted pulse, σ_{Tmax} and minimum values of reflected pulses,σ_{Rmin}, are shown in the Fig. 9. In this figure the values of these quantities evaluated under assumption of the elastic behaviour of PMMA bar are also plotted. It seems that the differences for low values of the amplitude of incident stress pulse are not too meaningful.

4 Conclusion

Wave propagation coefficients are determined experimentally for viscoelastic bar made of PMMA, employed as transmitter bars in SHPB test setup. The stress pulse propagation in system of two bars was studied. Two processing methods were compared. The first processing takes into account the viscoelastic behaviour

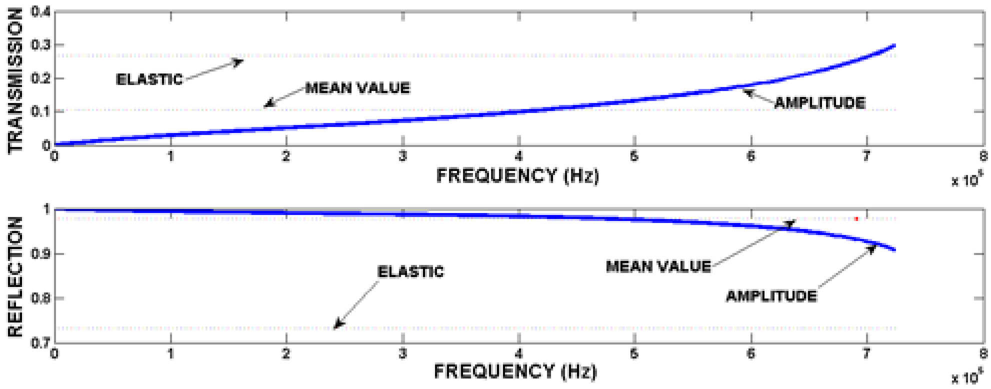

Fig. 8: Amplitude of the transmission and reflection characteristics.

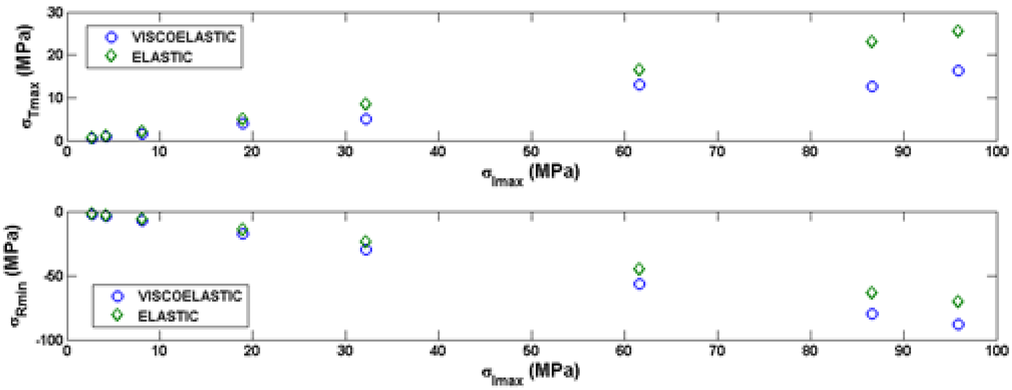

Fig. 9: Amplitudes of the reflected and transmitted stress pulses.

of bars whereas the second uses an elastic stiffness modulus for polymeric bars. It was shown that the elastic processing induces errors that increase with increasing strain rate.

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