

Investigation of Coupled Modal Shapes by Experimental Modal Analysis

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Introduction. This paper deals with investigation of coupled mode shapes by experimental modal analysis method. It is well known that mode shapes of mass and shape symmetric objects are symmetric but such objects also have mode shapes which are mutually quite same but rotated about the symmetry axis. Such mode shapes are called coupled and theoretically they should be belonging to same frequency. In practice, there is always some inhomogeneity of a material or the shape of a specimen is not exactly symmetric so the frequency shift of coupled modes occurs. By experimental modal investigation such shift can be also created by inconvenient support of a tested specimen or by application of transducers at improper locations.

Experimental Modal Analysis of Saw Blade Performed by Pulse System

For experimental investigation of coupled modes the circular-saw blade was used (Fig. 1) which was supported by an elastic very soft foam. There were created 81 measuring points at the saw blade and three accelerometers were evenly fixed along its edge.

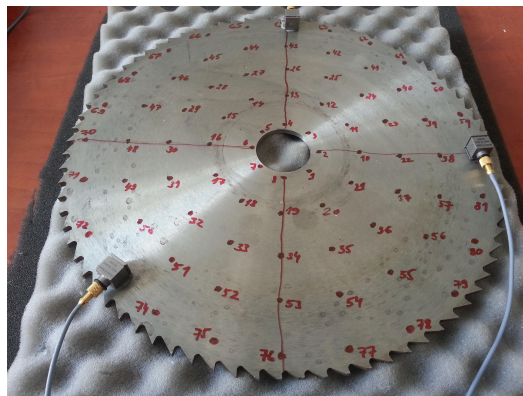


Fig. 1. The tested circular-saw blade.

As a measuring device the Brüel&Kjaer's Pulse system was used. As an exciter the impact hammer Brüel&Kjaer, type 8206 with plastic tip was used. The response was measured with three Brüel&Kjaer's accelerometers, type 4507-B. All the measurement was performed in MTC-Hammer software. Span frequency was set at 5000 Hz and the frequency resolution was 0.7813Hz. For next processing the measured FRFs were exported to Pulse Reflex software and CMIF functions were computed (Fig. 2). Modal parameters of the circular-saw blade were determined on the basis of CMIF with using of rational fraction polynomial (RFP) method.

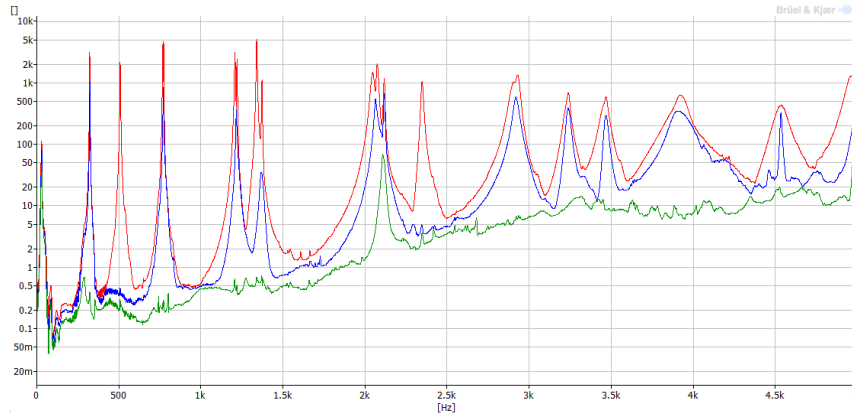


Fig. 2. CMIF functions of exported data.

Complex Mode Indicator Function

Complex mode indicator function (CMIF) is a method which is used for an identification of system modes. Its algorithm is based on singular value decomposition (SVD) of a transfer function complex matrix (Eq. 1) [1 - 3]. Initially, this method was designed for determination of closely spaced modes. Contemporary algorithms of CMIF enable to identify the real and the complex modes and also enable to determine their relative magnitudes and to plot their mode shapes [4].

$$\mathbf{H}(\omega) = \mathbf{U}(\omega) \mathbf{S}(\omega) \mathbf{V}^H(\omega), \quad (1)$$

where $\mathbf{U}(\omega)$ is left singular matrix, $\mathbf{V}(\omega)$ is right singular matrix and $\mathbf{S}(\omega)$ is diagonal matrix of singular values. Mode shapes are represented by left singular vectors of $\mathbf{U}(\omega)$ matrix and modal scale factors are represented by right singular vectors of $\mathbf{V}(\omega)$ matrix. However, it is necessary to note that not all peaks of CMIF indicate the modes. Some peaks are generated as a consequence of noise, leakage or nonlinearity. The indication of coupled modes is given by presence of more peaks of CMIFs on a same frequency.

Experimental Modal Analysis of Saw Blade Performed by DIC Method

Beside the conventional process of modal parameters investigation with the accelerometers and modal hammer the optical method based on digital image correlation (DIC) was used for verification (Fig. 3).



Fig. 3. Process of saw blade measuring with using of digital image correlation method.

Instead of accelerometers two high-speed cameras, Phantom Speed Sense, were used which scanned the surface of saw blade by 5000 fps (measuring the displacements of surface points). By this method the surface of investigated specimen has to be coated with contrast

stochastic pattern [5, 6]. In our instance the stochastic pattern was created with spray paint. The excitation was performed at three different points with same modal hammer as in previous instance. The measured values of displacements were exported for next processing to program MODAN 3D [7, 8] which is being developed by authors^{b,c}. The CMIF functions acquired by MODAN are presented in Fig. 4.

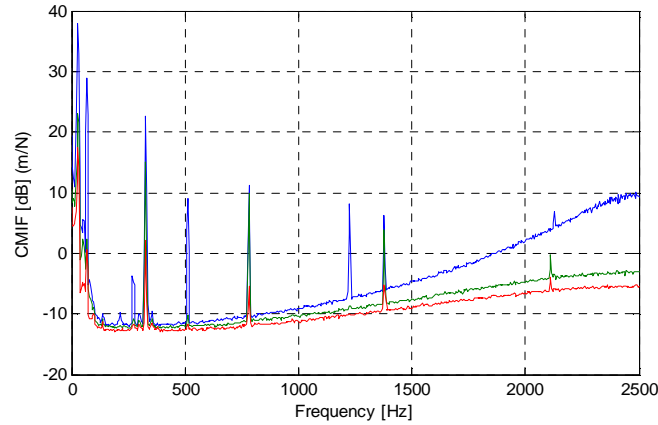


Fig. 4. CMIF functions acquired by MODAN.

Results Discussion

The presence of coupled modes is indicated by the peaks of the same frequency on particular SVDs as we can see in Fig. 2 and Fig. 4. Also from Fig. 2 we can see that for second and eighth mode there will be no coupled mode because only one SVD line has peak on that frequency. The same result we can observe in Fig. 4 for second mode.

The acquired natural frequencies by both methods are presented in Table 1 and comparison of mode shapes is showed in Table 2.

Table 1. Natural frequencies of saw blade determined by two approaches.

		Frequency of a particular mode [Hz]					
method	1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
PULSE	321,7	507,8	767,6	1207	1340	2047	2115
	323,6		772,8	1219	1371	2075	2117
DIC	328.5	512.8	781.3	1226	1378	2111	2127

		Frequency of a particular mode [Hz]					
method	8 th	9 th	10 th	11 th	12 th	13 th	
PULSE	2347	2909	3232	3457	3904	4530	
		2931	3239	3467	3926	4534	

The values of natural frequencies obtained by PULSE are lower than that acquired by DIC by the reason of accelerometers mass. The second reason of frequency difference is caused by the small shift of saw blade on the pad between the particular experiments. Due to the restrained sampling frequency of cameras the maximal evaluated natural frequency by DIC method was 2127 Hz. The high value of solid mode is due to not ideal support of saw blade during the experiment.

Table 2. Comparison of mode shapes acquired by PULSE and DIC method (Part 1).

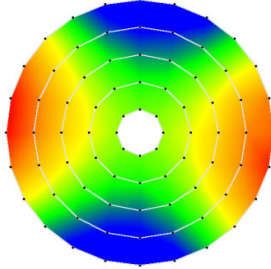
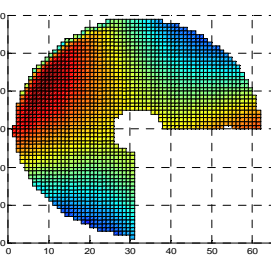
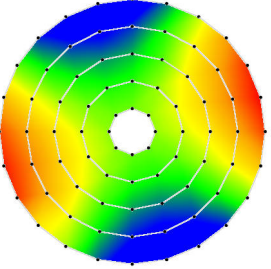
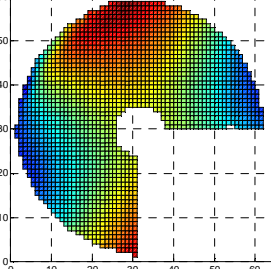
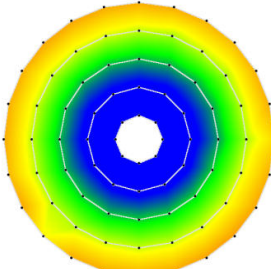
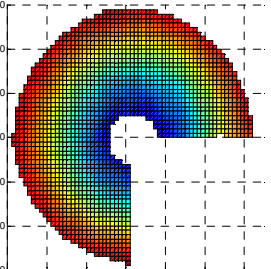
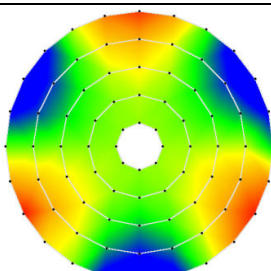
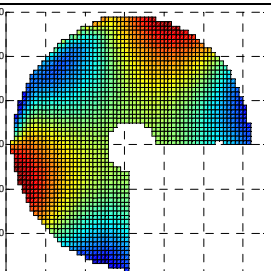
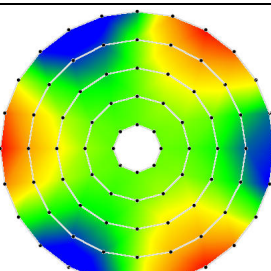
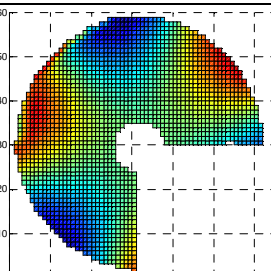
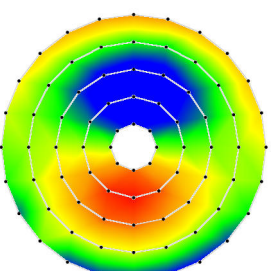
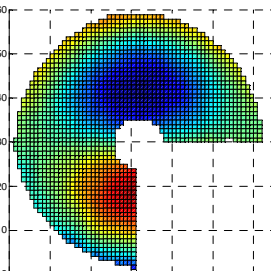
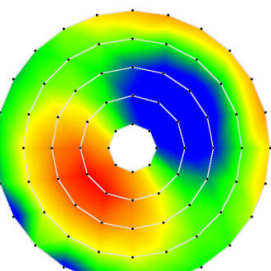
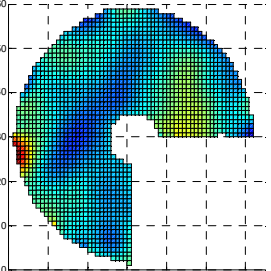
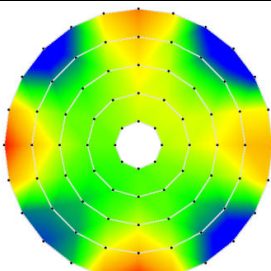
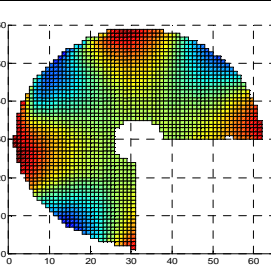
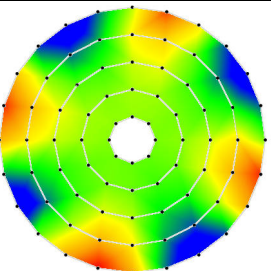
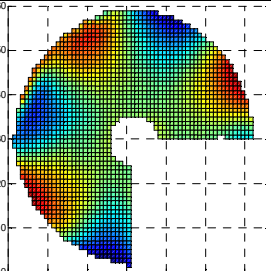
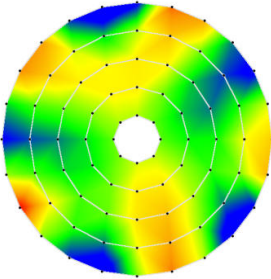
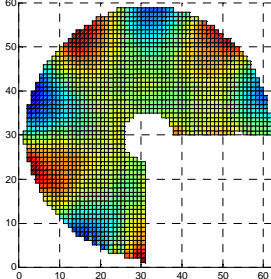
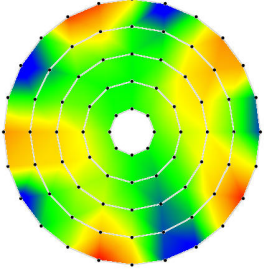
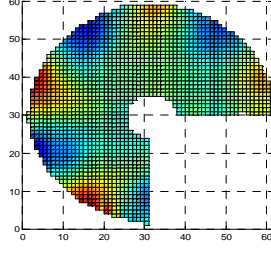
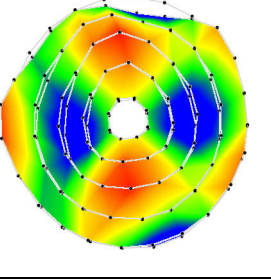
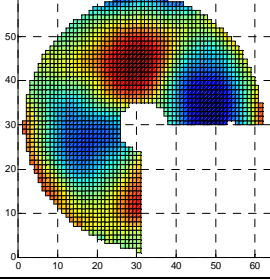
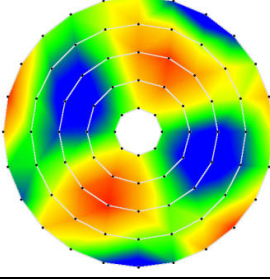
PULSE shapes	DIC shapes	PULSE shapes (coupled)	DIC shapes (coupled)
1 st mode	1 st mode	1 st mode	1 st mode
			
2 nd mode	2 nd mode	2 nd mode	2 nd mode
		Not exist	Not exist
3 rd mode	3 rd mode	3 rd mode	3 rd mode
			
4 th mode	4 th mode	4 th mode	4 th mode
			
5 th mode	5 th mode	5 th mode	5 th mode
			

Table 2. Comparison of mode shapes acquired by PULSE and DIC method (Part 2).

PULSE shapes	DIC shapes	PULSE shapes (coupled)	DIC shapes (coupled)
6 th mode	6 th mode	6 th mode	6 th mode
			
7 th mode	7 th mode	7 th mode	7 th mode
			Not found

As we can see in Table 2, there are only presented three quarters of particular mode shapes obtained by DIC method. The remaining quarter served for excitation of saw blade. Also the shape of solid mode is not presented here. The resemblance of mode shapes obtained by both methods is apparent. In case of 7th mode the coupled mode shape was not found because of increased noise at higher frequencies by DIC method, see Fig. 4.

Conclusions

The investigation of coupled modes is a difficult process because the noise-free measurement has to be performed and proper extraction method has to be used. In our case, when relatively stiff saw blade was used, the classical approach with using of PULSE system showed to be a better approach. More natural frequencies and mode shapes can be acquired by this method. On the other hand digital image correlation is contactless method, so it does not influence the natural frequencies of investigated specimen.

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