

Real and Virtual Simple Mechanical Systems for Investigation of the Dynamic Behaviour of Fibres

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Abstract. Fibres, cables and wires can play an important role in design of many machines. One of the most interesting applications is replacement of chosen rigid elements of a manipulator or a mechanism by fibres. Dynamic behaviour of fibres in simple mechanical systems was investigated. In the first step the computational investigation of a fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of a weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models. The simulation aim is to create a phenomenological model of a fibre.

Introduction

The paper was written in the framework of research in mechanisms and manipulators based on parallel kinematic structures, for which a fibre control instead of rigid elements control is designed. The replacement of the chosen rigid elements of manipulators or mechanisms by fibres or cables is advantageous due to the achievement of a lower moving inertia, which can lead to a higher machine speed, and lower production costs. Drawbacks of using the flexible elements like that can be associated with the fact that cables should be only in tension (e.g. [1,2]) in the course of a motion.

Dynamic behaviour of fibres in simplified mechanical systems was investigated. In the first step the computational investigation of fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models. The simulation aim is to create a phenomenological model of a fibre.

Experimental Stands and Computational Models

Originally it was supposed that for the experimental measurement focused on determining dynamic behaviour of fibres an inverted pendulum attached to two fibres and driven by them would be used. Its properties were investigated very thoroughly applying computational

models (see Fig. 1 and [3,4,5,6,7,8,9]). But strength calculation results drew attention to a high loading of fibres which were to be used in the experimental measurements (carbon or watted steel wire) and to the possibility of their breaking [10]. This system was only investigated using computer simulations.

At first experimental measurements focused on the investigation of the fibre behaviour were performed with the simple fibre-mass mechanical system consisted of moving weight coupled with a frame by a carbon fibre (with a silicone coating) (see Fig. 3 and [11,12]). Displacement of the weight and the force acting in the fibre were the measured quantities. The same system was numerically investigated using a multibody model created in the **alaska** simulation tool [13]. The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results were evaluated and the first phenomenological model dependent on the fibre damping coefficient, the fibre stiffness and the friction force between the weight and the prismatic linkage was created.

Then experimental measurements focused on the investigation of the fibre behaviour were performed with the weight-fibre-pulley-drive mechanical system (see Fig. 6, Fig. 7 and [14,15,16,17,18]). In this system the fibre was driven with a drive and was led over a pulley and on its other end there was a prism-shaped steel weight, which moved on an inclined plane. Drive exciting signals could be of different shapes and there was a possibility of variation of a signal rate. Displacement of the weight, displacement of the drive and the force acting in the fibre were the measured quantities. The same system was numerically investigated using a multibody model created in the **alaska** simulation tool, too.

Computational Model of an Inverted Pendulum

An inverted pendulum attached to two fibres and driven by them (see Fig. 1) serves as a typical testing system for the investigation of the fibres influence on the system dynamic response. This system was selected with respect to the fact that it is a simplification of possible cable-based manipulators. In addition it was supposed that the nonlinear system of the inverted pendulum attached to two fibres and driven by them could show an unstable behaviour under specific excitation conditions (e.g. [19]).

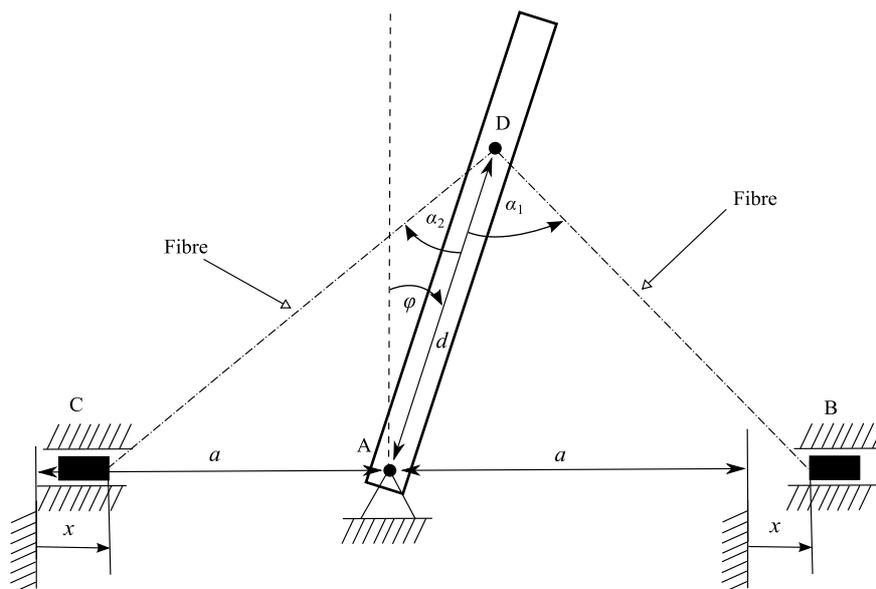


Fig. 1. Inverted pendulum actuated by the fibres.

The motion of the pendulum in this nonlinear system was investigated using the **alaska** simulation tool [13]. The influence of some parameters of the system of inverted pendulum driven by fibres has been investigated.

An inverted pendulum attached to two fibres and driven by them (carbon fibres with a silicone coating or wattled steel wires were considered – see e.g. [3,10]) is affected by a gravitation force. When the pendulum is displaced from the equilibrium position, i.e. from the “upper” position, it is returned back to the equilibrium position by the tightened fibre.

For the investigation of the system of inverted pendulum the massless model or the point-mass model of the fibres were used (e.g. [4,5]). The massless model is presented in this paper (the point-mass model is geometrically identical [4,5] to the massless model given in Fig. 1).

The system kinematics can be described by angle φ of the pendulum angular deflection with respect to its vertical position (one degree of freedom), angular acceleration $\ddot{\varphi}$ and prescribed kinematic excitation $x(t)$. The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_A} \cdot \left(F_{v1} \cdot d \cdot \sin \alpha_1 - F_{v2} \cdot d \cdot \sin \alpha_2 + m \cdot g \cdot \frac{l}{2} \cdot \sin \varphi \right), \quad (1)$$

where I_A is the moment of inertia of pendulum with respect to point A (see Fig. 1), α_1 and α_2 are the angles between the pendulum and the fibres, m is the pendulum mass, F_{v1} and F_{v2} are the forces acting on the pendulum from the fibres, g is the gravity acceleration, l is the pendulum length and d is the distance from the axis in point A to the position of the attachment of fibres to the pendulum (point D). Kinematic excitation acts in the points designated B and C (see Fig. 1).

The forces acting on the pendulum from the fibre are

$$\begin{aligned} F_{v1} &= \left[k_v \cdot (l_{v1} - l_{v0}) + b_v \cdot \frac{dl_{v1}}{dt} \right] \cdot H(l_{v1} - l_{v0}), \\ F_{v2} &= \left[k_v \cdot (l_{v2} - l_{v0}) + b_v \cdot \frac{dl_{v2}}{dt} \right] \cdot H(l_{v2} - l_{v0}), \end{aligned} \quad (2)$$

where k_v is the fibre stiffness, b_v is the fibre damping coefficient, l_{v0} is the original length of the fibres and $H(\cdot)$ is the Heaviside function. It is supposed that forces act in the fibres only when the fibres are in tension.

Actual lengths l_{v1} and l_{v2} of the fibres should be calculated in each time

$$\begin{aligned} l_{v1} &= \sqrt{(d \cdot \cos \varphi)^2 + (a + x(t) - d \cdot \sin \varphi)^2}, \\ l_{v2} &= \sqrt{(d \cdot \cos \varphi)^2 + (a - x(t) + d \cdot \sin \varphi)^2}. \end{aligned} \quad (3)$$

Harmonic kinematic excitation is given by function

$$x(t) = x_0 \cdot \sin(2 \cdot \pi \cdot f \cdot t + \psi), \quad (4)$$

where x_0 is the chosen amplitude of motion, f is the chosen excitation frequency, ψ is the chosen phase shift (in case of symmetric excitation $\psi = 0$) and t is time.

The chosen model parameters (see Fig. 1) were: $l = 1$ m, $a = 1.2$ m, $d = 0.75$ m, $I_A = 3.288$ kg·m² (the moment of inertia of pendulum with respect to point A), $m = 9.864$ kg (the pendulum mass). Mass of one fibre is 3.846 grams in case of carbon fibres, mass of one

fibre is 17.783 grams in case of wattled steel wires. Stiffness of the fibre k_v and damping coefficient b_v of the fibre depend on the used type of fibres (carbon or wattled steel wires).

Time histories and extreme values of pendulum angle φ and of the forces acting in the fibres were the monitored quantities. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequencies at kinematic excitation amplitude $x_0 = 0.02$ m are given Fig. 2.

Based on the obtained results it was evident that the pendulum motion is mostly influenced (besides the excitation frequency of the moving fibres) by the fibres preload [6] and by the amplitude of the harmonic kinematic excitation of fibres [7]. At the change of these parameters an unstable behaviour of the studied system was detected. Changes in other investigated parameters of this system – i.e. the change of the phase shift in the case of non-symmetric harmonic excitation [8] and the change in the mass of the fibres [9] – do not cause the unstable behaviour of the pendulum.

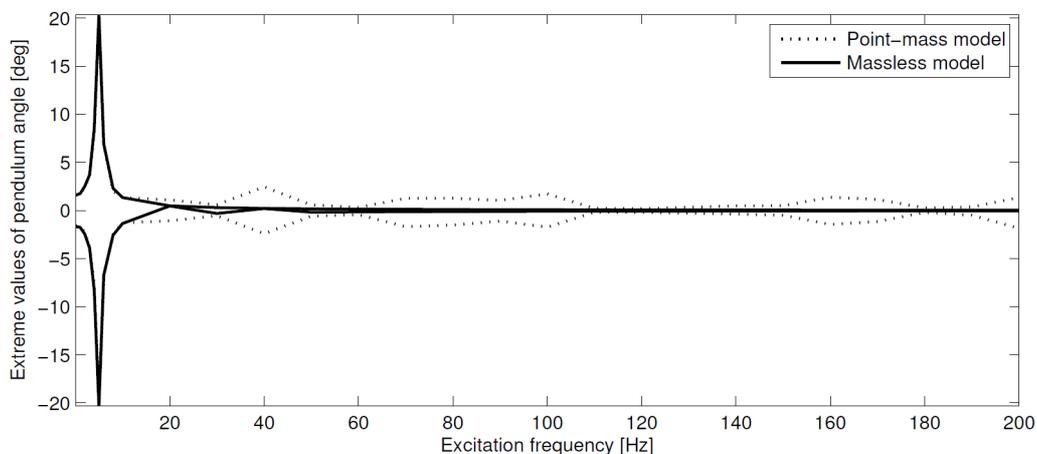


Fig. 2. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequencies [5].

Properties of this inverted pendulum were investigated very thoroughly. But strength calculation results drew attention to a high loading of fibres which were to be used in experimental measurements (carbon or wattled steel wires) and to the possibility of their breaking [10].

Fibre-mass Mechanical System

As it has been already started, at first experimental measurements focused on the investigation of the fibre dynamic behaviour were performed with the simple fibre-mass mechanical system consisting of moving weight coupled with a frame by a carbon fibre (with a silicone coating) (see Fig. 3 and [11,12]).

The fibre (fibre length was 599 mm; fibre mass was 1.63 grams) was fixed on a force gauge of a rectangular thin-wall cross-section profile 15 by 15 mm and thickness of wall 1 mm. In the other end of the fibre the prism-shaped steel weight (i.e. of weight 3.096 kg) was fastened – see Fig. 3. The weight was lifted to a certain height (from 5 to 20 mm) and then let to fall in the vertical direction or to slide down the inclined plane (angle of inclination of the inclined plane could be changed) – see Fig. 3. The weight moved in a prismatic linkage. Time histories of the weight position (in the direction of the inclined plane; measured by means of a dial gauge) and of the force acting in the fibre (measured on a force gauge) were recorded using sample rate of 2 kHz. The examples of time histories of the monitored quantities are given in Fig. 4 and Fig. 5.

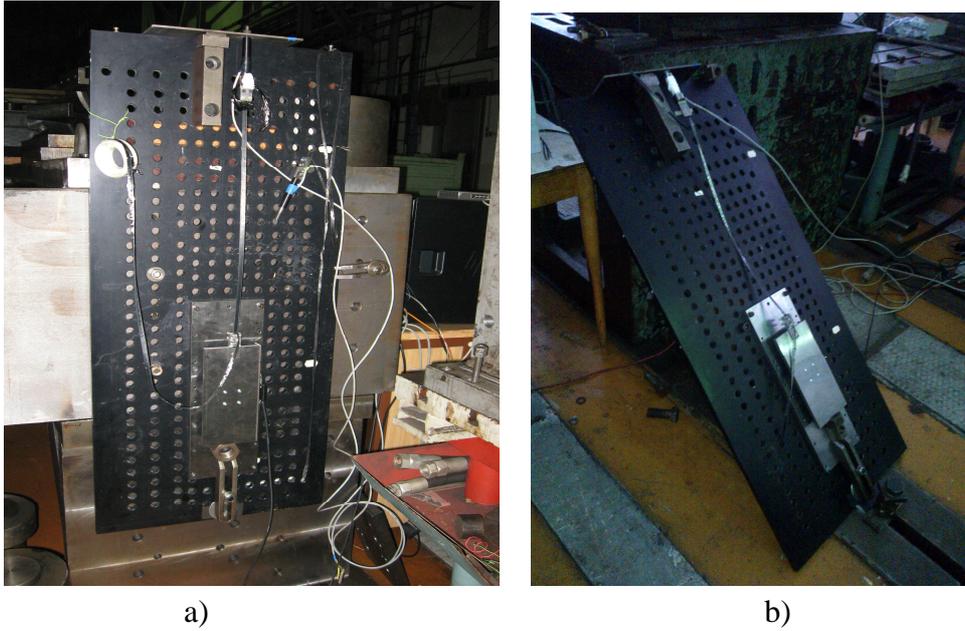


Fig. 3. Weight-fibre mechanical system, a) in a vertical position, b) on an inclined plane.

The same system was numerically investigated using a multibody model created in the **alaska** simulation tool [13]. For the investigation of the fibre-mass mechanical system the massless model or the point-mass model of the fibre were used. The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results were evaluated and the first phenomenological model dependent on the fibre damping coefficient, the fibre stiffness and the friction force between the weight and the prismatic linkage was created.

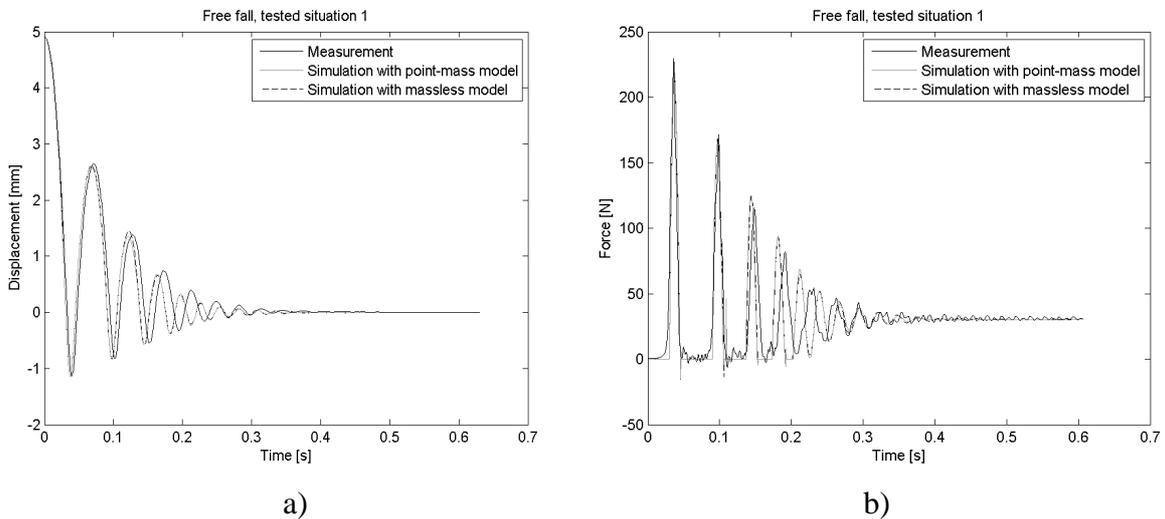


Fig. 4. Example of time histories at free fall, a) the weight position, b) the force acting in fibre [11].

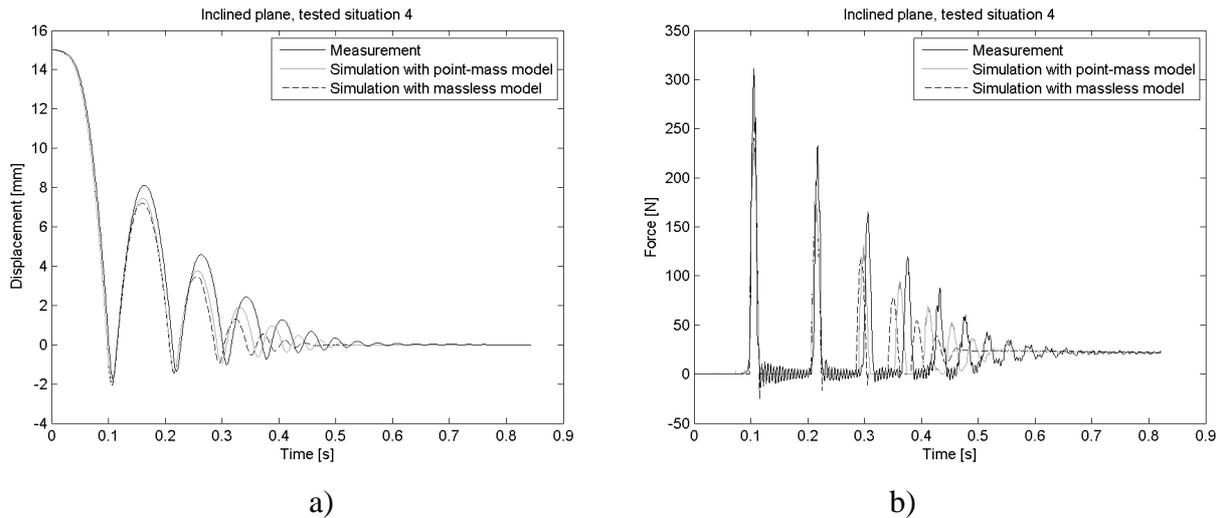


Fig. 5. Example of time histories at sliding down the inclined plane, angle of inclined plane 45 degrees, a) the weight position, b) the force acting in fibre [11].

Weight-fibre-pulley-drive Mechanical System

As it has been already stated the following experimental measurements focused on the investigation of the fibre behaviour were performed with the weight-fibre-pulley-drive mechanical system (see Fig. 6, Fig. 7 and [14,15,16,17,18]).

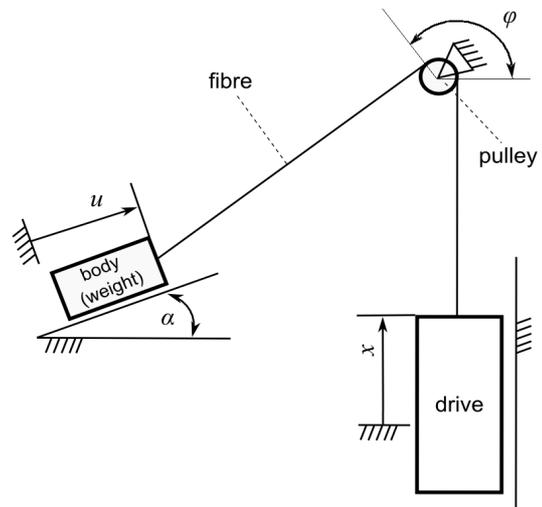
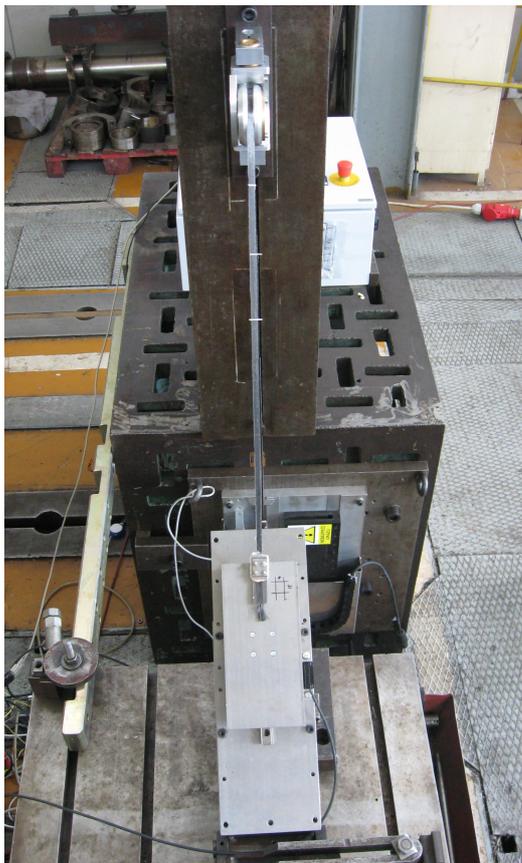


Fig. 6. A real weight-fibre-pulley-drive mechanical system (symmetric position of the weight) and its scheme.

A carbon fibre (with a silicone coating) driven with one drive was led over a pulley. The fibre length was 1.82 meters (fibre weight was 4.95 grams), the pulley diameter was 80 mm. The weight position could be symmetric (see Fig. 6, [14,15]) or asymmetric (see Fig. 7, [16,17]) with respect to the vertical plane of drive-pulley symmetry. Distance of the weight from the vertical plane of drive-pulley symmetry was $d = 280$ mm in the case of the asymmetric weight position (see Fig. 7). At the drive the fibre was fixed on a force gauge. On the other end of the fibre there was a prism-shaped steel weight (weight 3.096 kg), which moved in a prismatic linkage on an inclined plane. It was possible to add an extra mass (of the weight 5.035 kg) to the weight [15,17]. The angle of inclination of the inclined plane could be changed, but this possibility was not at experimental measurements utilized (in the case of the symmetric weight position the angle of inclination was $\alpha = 30$ degrees and the pulley-fibre angle was $\varphi = 150$ degrees, in the case of the asymmetric weight position the angle of inclination was $\alpha = 30.6$ degrees and the pulley-fibre angle was $\varphi = 146$ degrees). Drive exciting signals could be of a rectangular, a trapezoidal and a quasi-sinusoidal shape and there was a possibility of variation of a signal rate. The amplitudes of the drive displacements were up to 90 mm. Time histories of the weight position u (in direction of the inclined plane; measured by means of a dial gauge), of the drive position x (in vertical direction) and of the force acting in the fibre (measured on a force gauge at drive) were recorded using sample rate of 2 kHz.

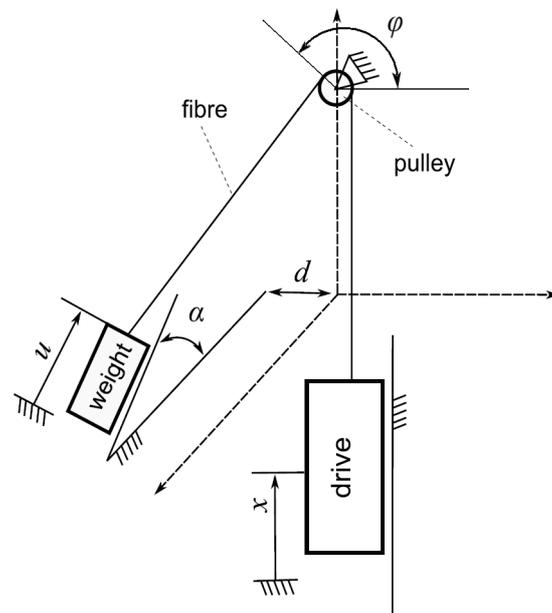
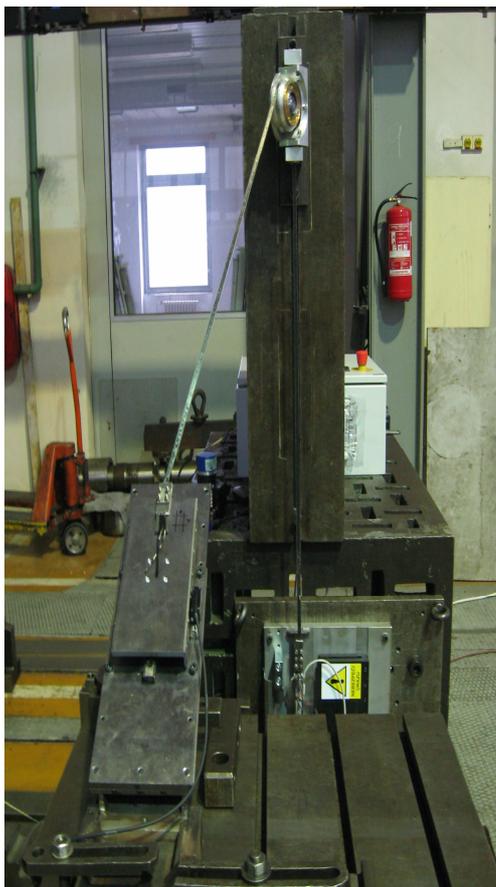


Fig. 7. A real weight-fibre-pulley-drive mechanical system (asymmetric position of the weight) and its scheme.

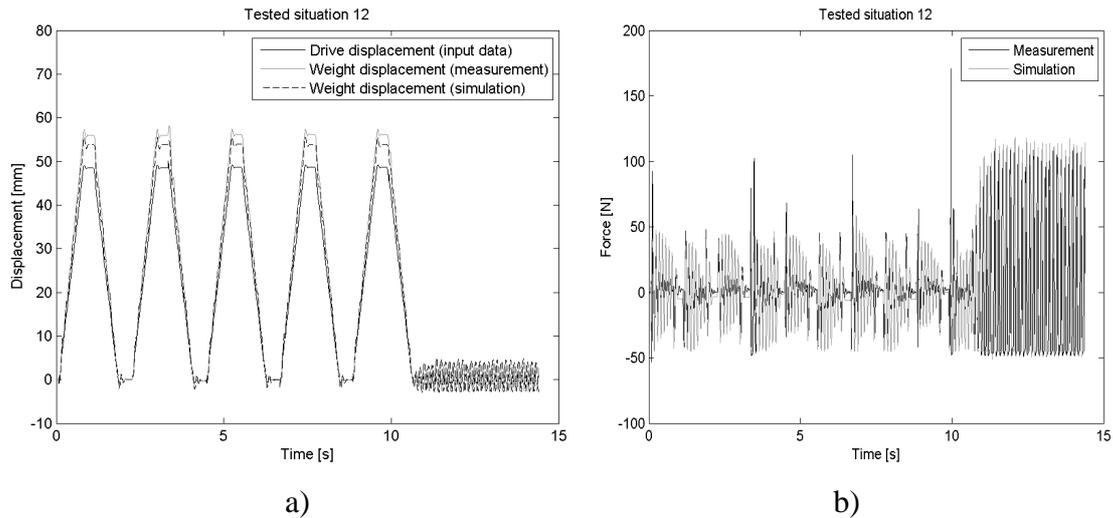


Fig. 8. Example of time histories of the monitored quantities, a) weight displacement, b) dynamic force acting in a fibre (asymmetric position of the weight, the weight with added mass).

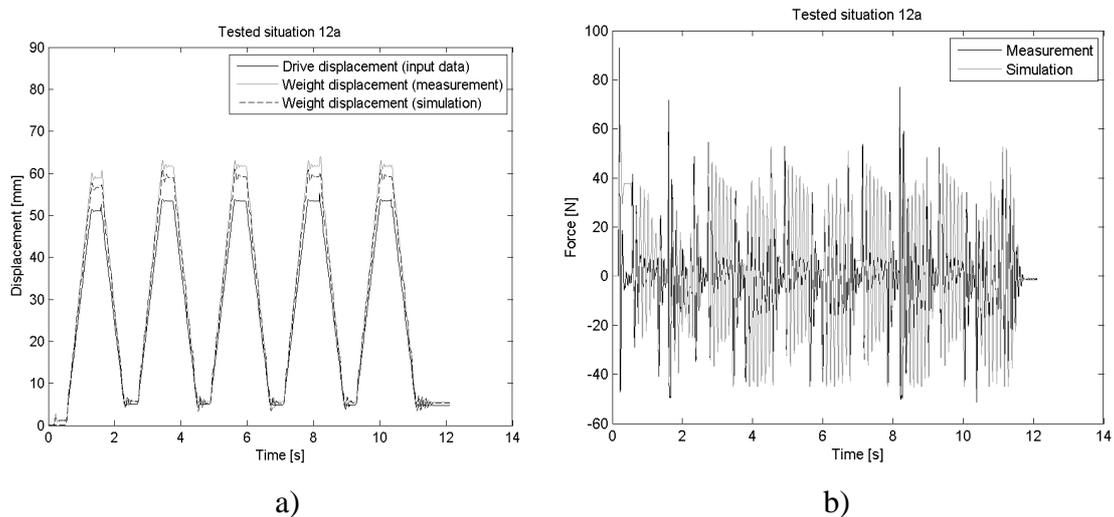


Fig. 9. Example of time histories of the monitored quantities, a) weight displacement, b) dynamic force acting in a fibre (asymmetric position of the weight, the weight with added mass) [17].

The same system was numerically investigated using multibody models created in the **alaska** simulation tool [13]. The massless fibre model was considered in the presented phase of the weight-fibre-pulley-drive system investigation. The influence of the model parameters on the coincidence of the results of experimental measurements and the simulations results was evaluated. The simulation aim is to create a phenomenological model of the fibre, which will be utilizable in fibre dynamic behaviour modelling in the case of more complicated mechanical or mechatronic systems.

The example of time histories of the monitored quantities is given in Fig. 8 (in this case spontaneous vibrating of drive occurred after finishing the control of the drive motion at trapezoidal exciting signal; spontaneous vibrating is given by electromechanical properties of drive and mechanical properties of the weight-fibre-pulley-drive system) and in Fig. 9 (the same case was measured again in order to eliminate the spontaneous vibrating of drive).

Conclusions

Dynamic behaviour of fibres in simplified mechanical systems was investigated. In the first step the computational investigation of the fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of the weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models.

The aim of the presented investigation is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the case of mechanical or mechatronic systems. Development of the fibre phenomenological model will continue. From the obtained results of measurements and computer simulations (summarized in [18]) it is evident that parameters of the fibre phenomenological model must, in addition, be considered dependent on the speed of the weight motion. The question is if it is possible to create the phenomenological model like that.

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