

## Real and Virtual Simple Mechanical Systems for Investigation of the Dynamic Behaviour of Fibres

POLACH Pavel<sup>1,a</sup> and VÁCLAVÍK Jaroslav<sup>2,b</sup>

<sup>1</sup>Výzkumný a zkušební ústav Plzeň s.r.o., Section of Materials and Mechanical Engineering Research, Tylova 1581/46, 301 00 Plzeň, Czech Republic

<sup>2</sup>Výzkumný a zkušební ústav Plzeň s.r.o., Section of Testing and Calibration Laboratories, Tylova 1581/46, 301 00 Plzeň, Czech Republic

<sup>a</sup>polach@vzuplzen.cz, <sup>b</sup>vaclavik@vzuplzen.cz

**Keywords:** Fibre, mechanical system, dynamics, experiment, simulation.

**Abstract.** Fibres, cables and wires can play an important role in design of many machines. One of the most interesting applications is replacement of chosen rigid elements of a manipulator or a mechanism by fibres. Dynamic behaviour of fibres in simple mechanical systems was investigated. In the first step the computational investigation of a fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of a weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models. The simulation aim is to create a phenomenological model of a fibre.

### Introduction

The paper was written in the framework of research in mechanisms and manipulators based on parallel kinematic structures, for which a fibre control instead of rigid elements control is designed. The replacement of the chosen rigid elements of manipulators or mechanisms by fibres or cables is advantageous due to the achievement of a lower moving inertia, which can lead to a higher machine speed, and lower production costs. Drawbacks of using the flexible elements like that can be associated with the fact that cables should be only in tension (e.g. [1,2]) in the course of a motion.

Dynamic behaviour of fibres in simplified mechanical systems was investigated. In the first step the computational investigation of fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models. The simulation aim is to create a phenomenological model of a fibre.

### Experimental Stands and Computational Models

Originally it was supposed that for the experimental measurement focused on determining dynamic behaviour of fibres an inverted pendulum attached to two fibres and driven by them would be used. Its properties were investigated very thoroughly applying computational

models (see Fig. 1 and [3,4,5,6,7,8,9]). But strength calculation results drew attention to a high loading of fibres which were to be used in the experimental measurements (carbon or watted steel wire) and to the possibility of their breaking [10]. This system was only investigated using computer simulations.

At first experimental measurements focused on the investigation of the fibre behaviour were performed with the simple fibre-mass mechanical system consisted of moving weight coupled with a frame by a carbon fibre (with a silicone coating) (see Fig. 3 and [11,12]). Displacement of the weight and the force acting in the fibre were the measured quantities. The same system was numerically investigated using a multibody model created in the **alaska** simulation tool [13]. The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results were evaluated and the first phenomenological model dependent on the fibre damping coefficient, the fibre stiffness and the friction force between the weight and the prismatic linkage was created.

Then experimental measurements focused on the investigation of the fibre behaviour were performed with the weight-fibre-pulley-drive mechanical system (see Fig. 6, Fig. 7 and [14,15,16,17,18]). In this system the fibre was driven with a drive and was led over a pulley and on its other end there was a prism-shaped steel weight, which moved on an inclined plane. Drive exciting signals could be of different shapes and there was a possibility of variation of a signal rate. Displacement of the weight, displacement of the drive and the force acting in the fibre were the measured quantities. The same system was numerically investigated using a multibody model created in the **alaska** simulation tool, too.

### Computational Model of an Inverted Pendulum

An inverted pendulum attached to two fibres and driven by them (see Fig. 1) serves as a typical testing system for the investigation of the fibres influence on the system dynamic response. This system was selected with respect to the fact that it is a simplification of possible cable-based manipulators. In addition it was supposed that the nonlinear system of the inverted pendulum attached to two fibres and driven by them could show an unstable behaviour under specific excitation conditions (e.g. [19]).

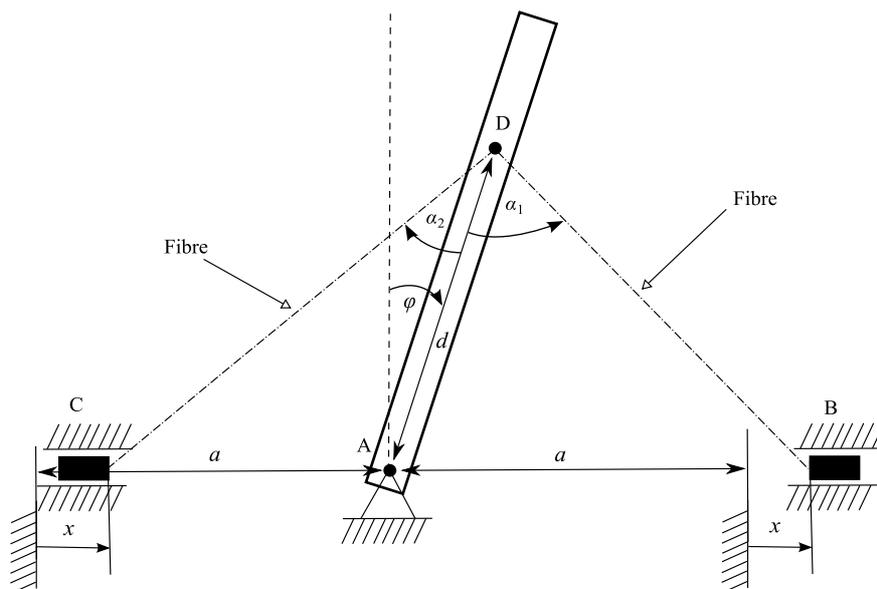


Fig. 1. Inverted pendulum actuated by the fibres.

The motion of the pendulum in this nonlinear system was investigated using the **alaska** simulation tool [13]. The influence of some parameters of the system of inverted pendulum driven by fibres has been investigated.

An inverted pendulum attached to two fibres and driven by them (carbon fibres with a silicone coating or wattled steel wires were considered – see e.g. [3,10]) is affected by a gravitation force. When the pendulum is displaced from the equilibrium position, i.e. from the “upper” position, it is returned back to the equilibrium position by the tightened fibre.

For the investigation of the system of inverted pendulum the massless model or the point-mass model of the fibres were used (e.g. [4,5]). The massless model is presented in this paper (the point-mass model is geometrically identical [4,5] to the massless model given in Fig. 1).

The system kinematics can be described by angle  $\varphi$  of the pendulum angular deflection with respect to its vertical position (one degree of freedom), angular acceleration  $\ddot{\varphi}$  and prescribed kinematic excitation  $x(t)$ . The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_A} \cdot \left( F_{v1} \cdot d \cdot \sin \alpha_1 - F_{v2} \cdot d \cdot \sin \alpha_2 + m \cdot g \cdot \frac{l}{2} \cdot \sin \varphi \right), \quad (1)$$

where  $I_A$  is the moment of inertia of pendulum with respect to point A (see Fig. 1),  $\alpha_1$  and  $\alpha_2$  are the angles between the pendulum and the fibres,  $m$  is the pendulum mass,  $F_{v1}$  and  $F_{v2}$  are the forces acting on the pendulum from the fibres,  $g$  is the gravity acceleration,  $l$  is the pendulum length and  $d$  is the distance from the axis in point A to the position of the attachment of fibres to the pendulum (point D). Kinematic excitation acts in the points designated B and C (see Fig. 1).

The forces acting on the pendulum from the fibre are

$$F_{v1} = \left[ k_v \cdot (l_{v1} - l_{v0}) + b_v \cdot \frac{dl_{v1}}{dt} \right] \cdot H(l_{v1} - l_{v0}), \quad (2)$$

$$F_{v2} = \left[ k_v \cdot (l_{v2} - l_{v0}) + b_v \cdot \frac{dl_{v2}}{dt} \right] \cdot H(l_{v2} - l_{v0}),$$

where  $k_v$  is the fibre stiffness,  $b_v$  is the fibre damping coefficient,  $l_{v0}$  is the original length of the fibres and  $H(\cdot)$  is the Heaviside function. It is supposed that forces act in the fibres only when the fibres are in tension.

Actual lengths  $l_{v1}$  and  $l_{v2}$  of the fibres should be calculated in each time

$$l_{v1} = \sqrt{(d \cdot \cos \varphi)^2 + (a + x(t) - d \cdot \sin \varphi)^2}, \quad (3)$$

$$l_{v2} = \sqrt{(d \cdot \cos \varphi)^2 + (a - x(t) + d \cdot \sin \varphi)^2}.$$

Harmonic kinematic excitation is given by function

$$x(t) = x_0 \cdot \sin(2 \cdot \pi \cdot f \cdot t + \psi), \quad (4)$$

where  $x_0$  is the chosen amplitude of motion,  $f$  is the chosen excitation frequency,  $\psi$  is the chosen phase shift (in case of symmetric excitation  $\psi = 0$ ) and  $t$  is time.

The chosen model parameters (see Fig. 1) were:  $l = 1$  m,  $a = 1.2$  m,  $d = 0.75$  m,  $I_A = 3.288$  kg·m<sup>2</sup> (the moment of inertia of pendulum with respect to point A),  $m = 9.864$  kg (the pendulum mass). Mass of one fibre is 3.846 grams in case of carbon fibres, mass of one

fibre is 17.783 grams in case of wattled steel wires. Stiffness of the fibre  $k_v$  and damping coefficient  $b_v$  of the fibre depend on the used type of fibres (carbon or wattled steel wires).

Time histories and extreme values of pendulum angle  $\varphi$  and of the forces acting in the fibres were the monitored quantities. Extreme values of time histories of pendulum angle  $\varphi$  in dependence on the excitation frequencies at kinematic excitation amplitude  $x_0 = 0.02$  m are given Fig. 2.

Based on the obtained results it was evident that the pendulum motion is mostly influenced (besides the excitation frequency of the moving fibres) by the fibres preload [6] and by the amplitude of the harmonic kinematic excitation of fibres [7]. At the change of these parameters an unstable behaviour of the studied system was detected. Changes in other investigated parameters of this system – i.e. the change of the phase shift in the case of non-symmetric harmonic excitation [8] and the change in the mass of the fibres [9] – do not cause the unstable behaviour of the pendulum.

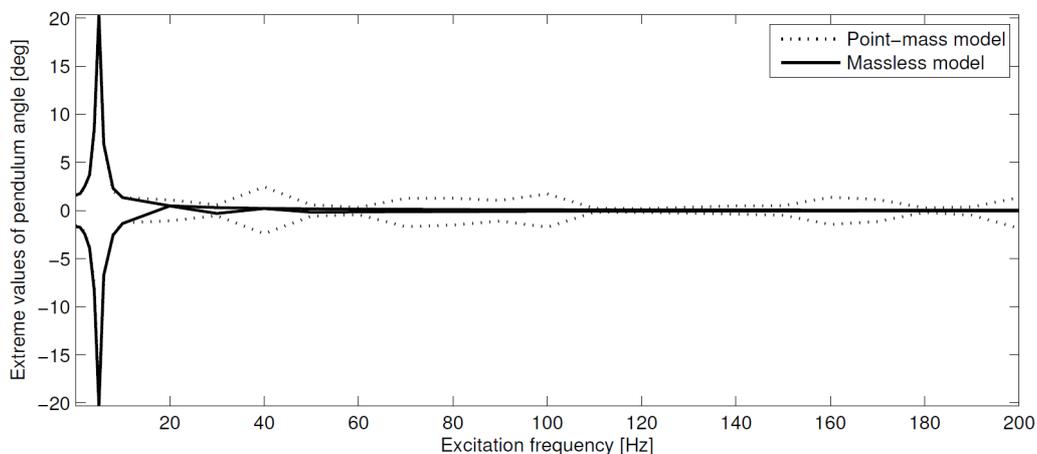


Fig. 2. Extreme values of time histories of pendulum angle  $\varphi$  in dependence on the excitation frequencies [5].

Properties of this inverted pendulum were investigated very thoroughly. But strength calculation results drew attention to a high loading of fibres which were to be used in experimental measurements (carbon or wattled steel wires) and to the possibility of their breaking [10].

### Fibre-mass Mechanical System

As it has been already started, at first experimental measurements focused on the investigation of the fibre dynamic behaviour were performed with the simple fibre-mass mechanical system consisting of moving weight coupled with a frame by a carbon fibre (with a silicone coating) (see Fig. 3 and [11,12]).

The fibre (fibre length was 599 mm; fibre mass was 1.63 grams) was fixed on a force gauge of a rectangular thin-wall cross-section profile 15 by 15 mm and thickness of wall 1 mm. In the other end of the fibre the prism-shaped steel weight (i.e. of weight 3.096 kg) was fastened – see Fig. 3. The weight was lifted to a certain height (from 5 to 20 mm) and then let to fall in the vertical direction or to slide down the inclined plane (angle of inclination of the inclined plane could be changed) – see Fig. 3. The weight moved in a prismatic linkage. Time histories of the weight position (in the direction of the inclined plane; measured by means of a dial gauge) and of the force acting in the fibre (measured on a force gauge) were recorded using sample rate of 2 kHz. The examples of time histories of the monitored quantities are given in Fig. 4 and Fig. 5.

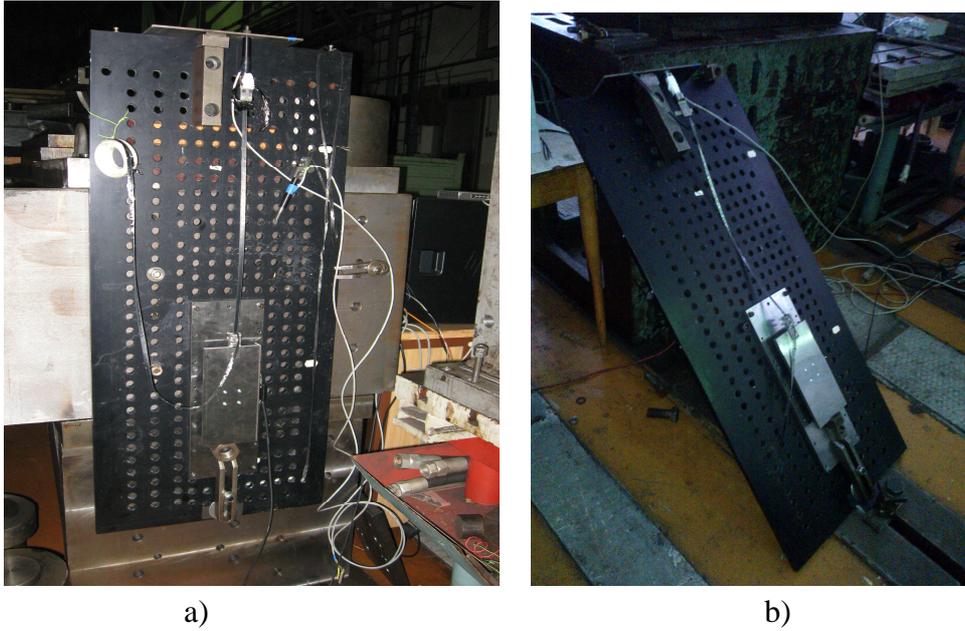


Fig. 3. Weight-fibre mechanical system, a) in a vertical position, b) on an inclined plane.

The same system was numerically investigated using a multibody model created in the **alaska** simulation tool [13]. For the investigation of the fibre-mass mechanical system the massless model or the point-mass model of the fibre were used. The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results were evaluated and the first phenomenological model dependent on the fibre damping coefficient, the fibre stiffness and the friction force between the weight and the prismatic linkage was created.

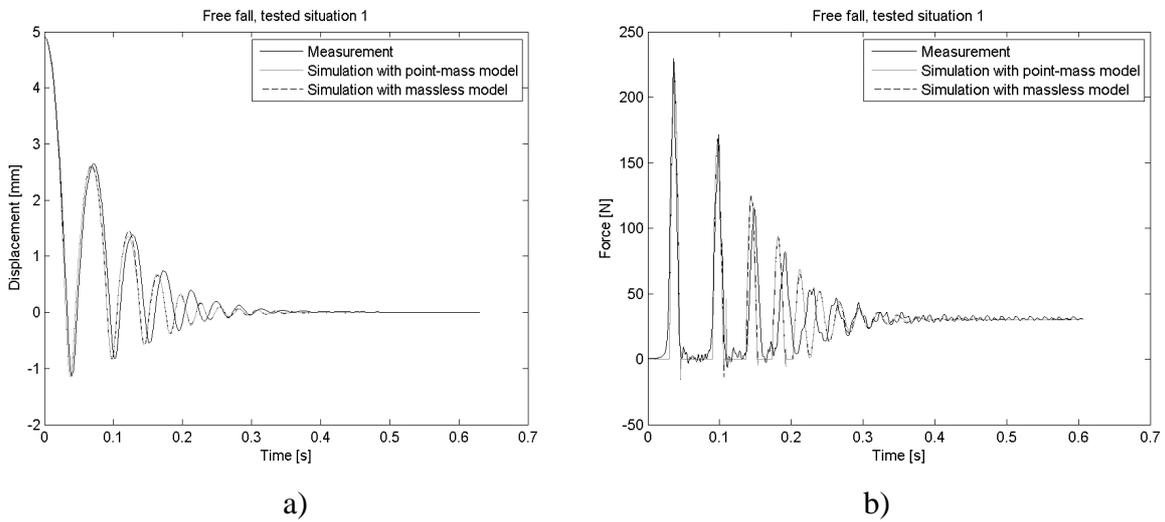


Fig. 4. Example of time histories at free fall, a) the weight position, b) the force acting in fibre [11].

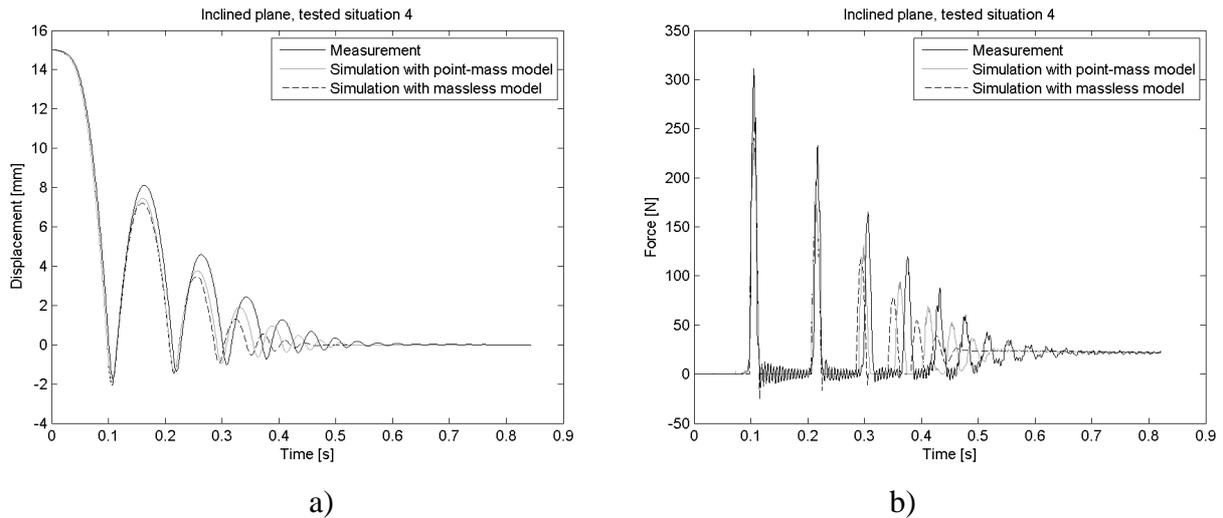


Fig. 5. Example of time histories at sliding down the inclined plane, angle of inclined plane 45 degrees, a) the weight position, b) the force acting in fibre [11].

### Weight-fibre-pulley-drive Mechanical System

As it has been already stated the following experimental measurements focused on the investigation of the fibre behaviour were performed with the weight-fibre-pulley-drive mechanical system (see Fig. 6, Fig. 7 and [14,15,16,17,18]).

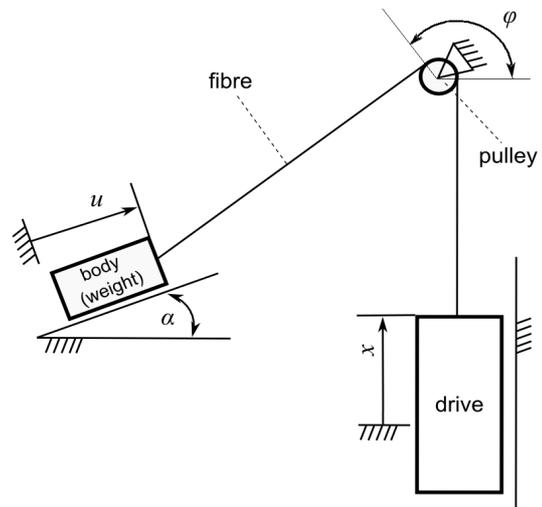
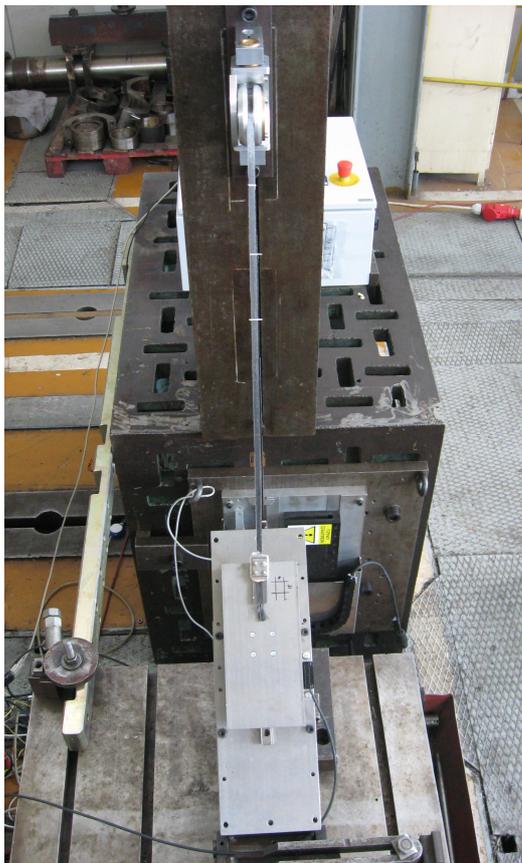


Fig. 6. A real weight-fibre-pulley-drive mechanical system (symmetric position of the weight) and its scheme.

A carbon fibre (with a silicone coating) driven with one drive was led over a pulley. The fibre length was 1.82 meters (fibre weight was 4.95 grams), the pulley diameter was 80 mm. The weight position could be symmetric (see Fig. 6, [14,15]) or asymmetric (see Fig. 7, [16,17]) with respect to the vertical plane of drive-pulley symmetry. Distance of the weight from the vertical plane of drive-pulley symmetry was  $d = 280$  mm in the case of the asymmetric weight position (see Fig. 7). At the drive the fibre was fixed on a force gauge. On the other end of the fibre there was a prism-shaped steel weight (weight 3.096 kg), which moved in a prismatic linkage on an inclined plane. It was possible to add an extra mass (of the weight 5.035 kg) to the weight [15,17]. The angle of inclination of the inclined plane could be changed, but this possibility was not at experimental measurements utilized (in the case of the symmetric weight position the angle of inclination was  $\alpha = 30$  degrees and the pulley-fibre angle was  $\varphi = 150$  degrees, in the case of the asymmetric weight position the angle of inclination was  $\alpha = 30.6$  degrees and the pulley-fibre angle was  $\varphi = 146$  degrees). Drive exciting signals could be of a rectangular, a trapezoidal and a quasi-sinusoidal shape and there was a possibility of variation of a signal rate. The amplitudes of the drive displacements were up to 90 mm. Time histories of the weight position  $u$  (in direction of the inclined plane; measured by means of a dial gauge), of the drive position  $x$  (in vertical direction) and of the force acting in the fibre (measured on a force gauge at drive) were recorded using sample rate of 2 kHz.

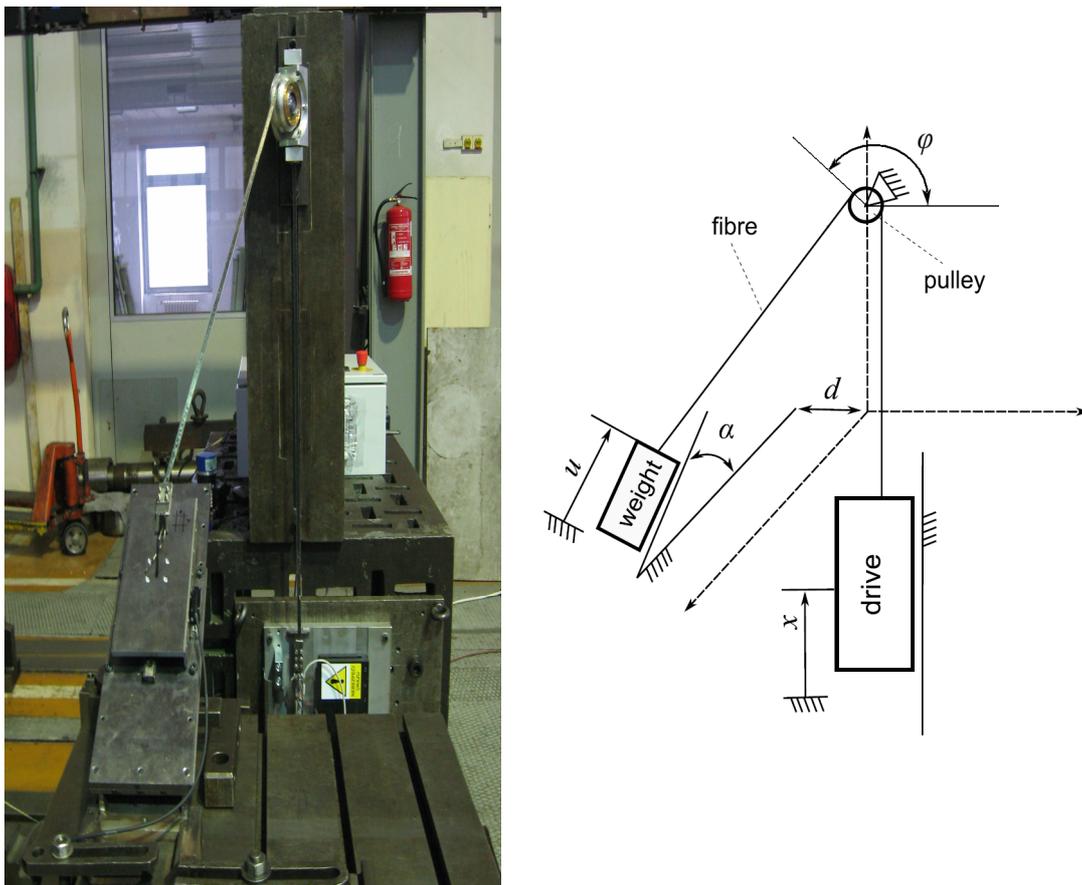


Fig. 7. A real weight-fibre-pulley-drive mechanical system (asymmetric position of the weight) and its scheme.

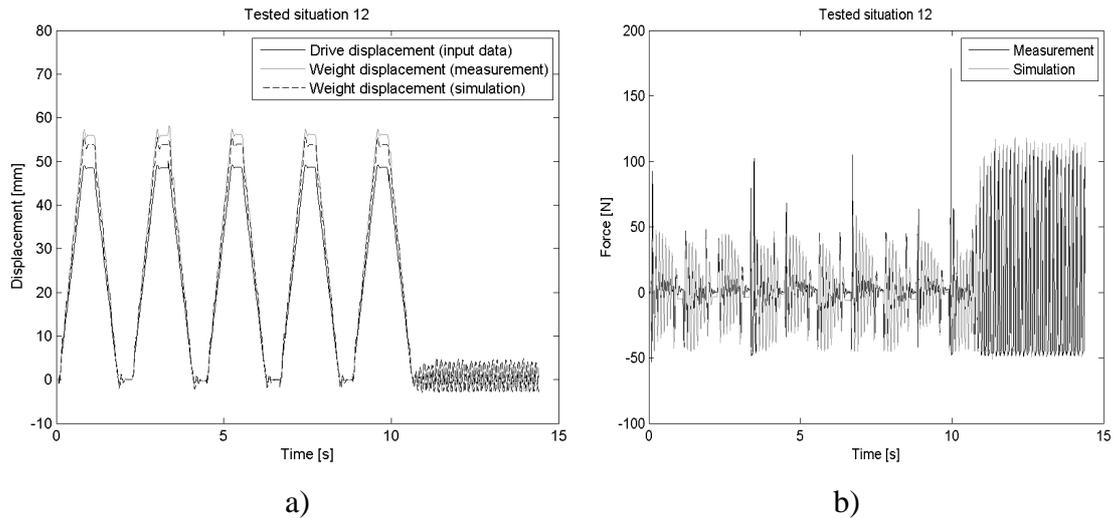


Fig. 8. Example of time histories of the monitored quantities, a) weight displacement, b) dynamic force acting in a fibre (asymmetric position of the weight, the weight with added mass).

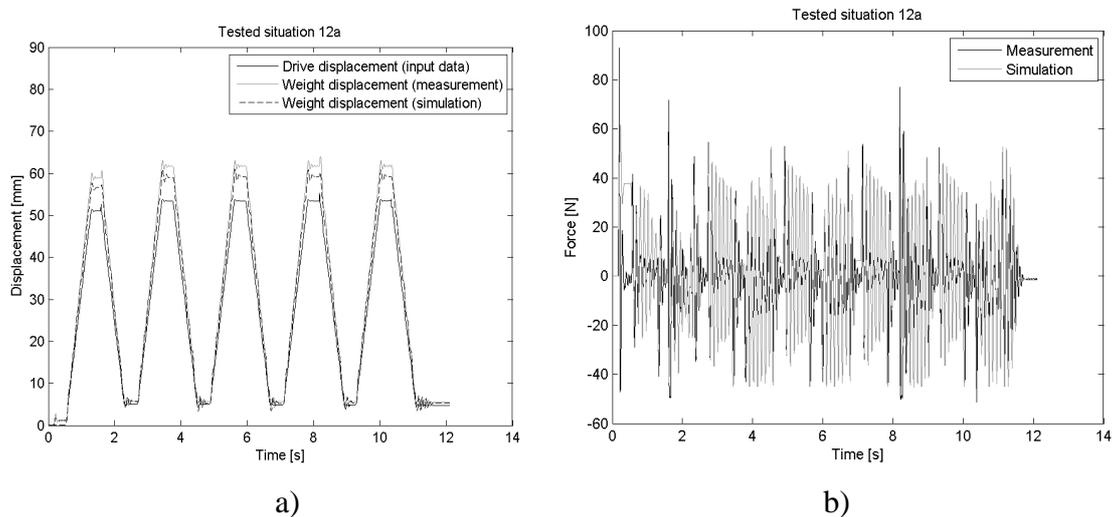


Fig. 9. Example of time histories of the monitored quantities, a) weight displacement, b) dynamic force acting in a fibre (asymmetric position of the weight, the weight with added mass) [17].

The same system was numerically investigated using multibody models created in the **alaska** simulation tool [13]. The massless fibre model was considered in the presented phase of the weight-fibre-pulley-drive system investigation. The influence of the model parameters on the coincidence of the results of experimental measurements and the simulations results was evaluated. The simulation aim is to create a phenomenological model of the fibre, which will be utilizable in fibre dynamic behaviour modelling in the case of more complicated mechanical or mechatronic systems.

The example of time histories of the monitored quantities is given in Fig. 8 (in this case spontaneous vibrating of drive occurred after finishing the control of the drive motion at trapezoidal exciting signal; spontaneous vibrating is given by electromechanical properties of drive and mechanical properties of the weight-fibre-pulley-drive system) and in Fig. 9 (the same case was measured again in order to eliminate the spontaneous vibrating of drive).

## Conclusions

Dynamic behaviour of fibres in simplified mechanical systems was investigated. In the first step the computational investigation of the fibre dynamic behaviour in the system of an inverted pendulum attached to two fibres and driven by them was performed. Then experiments and computer simulations with two simple laboratory mechanical systems followed. The first system consisted of the weight moving on an inclined plane and of one fibre, the second one contained a drive and a pulley in addition to it. The results of experimental measurements serve for tuning the computational models.

The aim of the presented investigation is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the case of mechanical or mechatronic systems. Development of the fibre phenomenological model will continue. From the obtained results of measurements and computer simulations (summarized in [18]) it is evident that parameters of the fibre phenomenological model must, in addition, be considered dependent on the speed of the weight motion. The question is if it is possible to create the phenomenological model like that.

The paper has originated in the framework of solving No. P101/11/1627 project of the Czech Science Foundation entitled “Tilting Mechanisms Based on Fibre Parallel Kinematical Structure with Antibacklash Control”.

## References

- [1] C. Gosselin, M. Grenier, On the determination of the force distribution in overconstrained cable-driven parallel mechanisms, *Meccanica* 46 (2011) 3-15.
- [2] M. Valášek, M. Karásek, HexaSphère with Cable Actuation, in: T. Brezina, R. Jablonski (Eds.), *Recent Advances in Mechatronics: 2008-2009*, Springer-Verlag, Berlin, 2009, pp. 239-244.
- [3] P. Polach, M. Hajžman, Z. Šika, Influence of crucial parameters of the system of an inverted pendulum driven by fibres on its dynamic behaviour, *Applied and Computational Mechanics* 6 (2012) 173-184.
- [4] P. Polach, M. Hajžman, Investigation of dynamic behaviour of inverted pendulum attached using of fibres, in: J. Awrejcewicz, M. Kaźmierczak, P. Olejnik, J. Mrozowski (Eds.), *Proceedings of the 11th Conference on Dynamical Systems – Theory and Applications, Nonlinear Dynamics and Control, Department of Automatics and Biomechanics, Technical University of Łódź, Łódź, 2011*, pp. 403-408.
- [5] P. Polach, M. Hajžman, O. Tuček, Computational analysis of dynamic behaviour of inverted pendulum attached using fibres, *Differential Equations and Dynamical Systems* 21 (2013) 71-81.
- [6] P. Polach, M. Hajžman, Effect of Fibre Preload on the Dynamics of an Inverted Pendulum Driven by Fibres, in: P. Eberhard, P. Ziegler (Eds.), *Proceedings of the 2nd Joint International Conference on Multibody System Dynamics, University of Stuttgart, Institute of Engineering and Computational Mechanics, Stuttgart, 2012*, USB flash drive.
- [7] P. Polach, M. Hajžman, Influence of the excitation amplitude on the dynamic behaviour of an inverted pendulum driven by fibres, *Procedia Engineering* 48 (2012) 568-577.
- [8] P. Polach, M. Hajžman, Investigation of dynamic behaviour of inverted pendulum attached using fibres at non-symmetric harmonic excitation, in: J.B. Jonker, W. Schiehlen, J.P. Meijaard, R.G.K.M. Aarts (Eds.), *Book of abstracts of the EUROMECH Colloquium 524*

Multibody system modelling, control and simulation for engineering design, University of Twente, Enschede, 2012, pp. 42-43.

[9] P. Polach, M. Hajžman, Z. Šika, J. Mrštík, P. Svatoš, Effects of fibre mass on the dynamic response of an inverted pendulum driven by fibres, *Engineering Mechanics* 19 (2012) 341-350.

[10] P. Polach, J. Václavík, M. Hajžman, Load of fibres driving an inverted pendulum system, in: M. Růžička, K. Doubrava, Z. Horák (Eds.), *Proceedings of 50th Annual international conference on Experimental Stress Analysis, CTU in Prague, Tábor, 2012*, pp. 337-344.

[11] P. Polach, M. Hajžman, J. Václavík, Experimental and Computational Investigation of a Simple Fibre-mass System, in: I. Zolotarev (Ed.), *Proceedings of 19th International Conference Engineering Mechanics 2013, IT AS CR, Svratka, 2013*, CD-ROM.

[12] P. Polach, M. Hajžman, J. Václavík, Simple fibre-mass model and experimental investigation, in: L. Pešek (Ed.), *Proceedings of the National Colloquium with International Participation Dynamics of Machines 2013*, Institute of Thermomechanics, Academy of Sciences of the Czech Republic, Prague, 2013, pp. 79-84.

[13] P. Maißer, C.-D. Wolf, A. Keil, K. Hendel, U. Jungnickel, H. Hermsdorf, P.A. Tuan, G. Kielau, O. Enge, U. Parsche, T. Härtel, H. Freudenberg, **alaska**, User Manual, Version 2.3, Institute of Mechatronics, Chemnitz, 1998.

[14] P. Polach, M. Hajžman, J. Václavík, Z. Šika, P. Svatoš, Model parameters influence of a simple mechanical system with fibre and pulley with respect to experimental measurements, in: Z. Terze, M. Vrdoljak (Eds.), *Proceedings of ECCOMAS Thematic Conference Multibody Dynamics 2013*, University of Zagreb, Zagreb, 2013, CD-ROM.

[15] P. Polach, M. Hajžman, J. Václavík, Z. Šika, M. Valášek, Influence of the mass of the weight on the dynamic response of the simple mechanical system with fibre, in: V. Adámek, M. Zajíček, A. Jonášová (Eds.), *Proceedings of the 29th Conference with International Participation Computational Mechanics 2013*, University of West Bohemia in Plzeň, Špičák, 2013, CD-ROM.

[16] P. Polach, M. Hajžman, J. Václavík, Z. Šika, M. Valášek, Experimental and Computational Investigation of a Simple Mechanical System with Fibre and Pulley, in: J. Awrejcewicz, M. Kaźmierczak, P. Olejnik, J. Mrozowski (Eds.), *Proceedings of the 12th Conference on Dynamical Systems – Theory and Applications, Dynamical Systems – Applications Department of Automation, Biomechanics and Mechatronics, Łódź University of Technology, Łódź, 2013*, pp. 717-728.

[17] P. Polach, M. Hajžman, J. Václavík, O. Červená, Dynamics of the Weight-fibre-pulley-drive Mechanical System: Influence of Mass of Weight at System Asymmetry, in: L. Pešek (Ed.), *Proceedings of the National Colloquium with International Participation Dynamics of Machines 2014*, Institute of Thermomechanics AS CR, v.v.i., Prague, 2014, pp. 115-124.

[18] P. Polach, M. Hajžman, Utilizing of a Weight-fibre-pulley-drive Mechanical System for the Investigation of a Fibre Behaviour, in: V. Fuis (Ed.), *Proceedings of 20th International Conference Engineering Mechanics 2014*, Institute of Solid Mechanics, Mechatronics and Biomechanics, FME, BUT Brno, Svratka, 2014, pp. 500-503.

[19] M.H. Wei, Y.Q. Xiao, H.T. Liu, Bifurcation and chaos of a cable-beam coupled system under simultaneous internal and external resonances, *Nonlinear Dynamics* 67 (2012) 1969-1984.