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TORSION VIBRATION MEASUREMENTS WITH HIGH RESOLUTION

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Abstract: Torsion vibration is a common problem that occurs in most rotating machines. The calculation of torsion vibration is difficult. It can be performed using motion equations, transmission matrix method or finite elements method. The numerical solution usually requires major simplifications therefore it is not precise enough. For this reason, a measurement is necessary and gives more accurate results. This paper will introduce the evaluation of torsion vibration by an integration of a measured angular speed difference. It is possible with a DMU device which was developed at VÚTS Co. The angular speed measurement with the DMU device is based on time intervals measurements with high resolution so high angle resolution and accuracy is achieved.

1. Introduction

Torsion vibration measurements or generally angular quantities measurements are complicated and belongs to exacting tasks of measurement problems. It is caused by relatively small amplitudes of torsion vibrations and by lack of angular acceleration sensors. The torsion vibrations analysis is complicated because of system non-linearity, especially clearances in mechanisms, variable rigidity and excitation forces, high burdens, influence of the driving motor etc. Torsion vibration affects the whole mechanical system that reacts on the reference shaft and causes unevenness of rotation. Torsion vibrations cause noise, inaccuracies, higher force transmission to the foundation, a reduction in durability of components and increase the risk of fatigue fracture.

For small angular deflection measurements it is necessary to use a sensor with adequate resolution. For commonly used angular sensors especially incremental rotary encoders meet that requirement. Encoders are based on a parallel beam of light that pass through a grid placed on a disc which is mounted on a reference shaft. Raster grid modulated beam of light is converted by photoelectric elements to the corresponding electric signals. The basic characteristic of the encoders is the number of pulses per revolution. There are encoders with the number of pulses from 60 to 90 000 on the market. For small torsion vibrations amplitudes are suitable encoders with a high number of pulses, thus with high resolution. But they are more voluminous, more expensive and have significantly limited the maximum angular speed of the shaft. For common angular speeds, encoders up to 5000 pulses per revolution are typically used. A common approach to measurement using incremental rotary encoders is sensing with constant sampling frequency and then the evaluation of measurement, i.e. with frequency analysis [1]. The angle is measured as a simple reading of the edges of the pulses (4 edges per pulse).

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Because each encoder pulse contains 4 evenly distributed edges, the measurement error is the eighth the size of a single pulse. An encoder with 500 pulses has a measurement error of 0.09°. To measure small torsion vibrations this measurement error is too large. The number of pulses with sinusoidal output can be extended using the interpolator. Their possibilities in the operating angular speeds are limited and are compensated by an increased error resulting from interpolation. Another approach is to determine the angle using the phase demodulation impulse signal received from the encoder. This method was developed by Prof. Tůma [2]. However the measurement error is the same as the previous method [3]. The following text shows the method of determining the angular speed using the measurement of the length of time (period) of encoder pulses.

2. Accuracy of Measurement

Encoders sense with a common angular resolution 0.01° to 0.5°, which is not sufficient enough for some measurements of torsion vibration. As already mentioned, the angle of rotation is determined as the sum of all edges of two lines impulse signals that are read by the encoder from the start (reset counter) to the time measurement. Angular measurement error $\delta\varphi$ is given by the number of pulses p according to the formula

$$\delta\varphi = \frac{360}{8p} \ \left[\text{deg}\right]. \tag{1}$$

The measurement error decreases with the increasing number of encoder pulses. The common angular speed for example 2000 RPM and sampling frequency 100 kHz require an encoder with a minimum number of pulses $p = 1500$. Then the deviation of the measured angle from the real angle can be up to 0.03°. The amplitudes of torsion vibration in orders of hundredths of a degree or less cannot be measured. Usually the angular speed and acceleration are needed to assess the torsion vibrations. The angular speed and acceleration evaluated by the derivative of measured angle by this method are poor.

The DMU device is based on time interval measurement of encoder pulses [4]. This makes possible to measure instantaneous angular speed as the reciprocal value of the length of the time interval, therefore with a high (but variable) sampling frequency. Angular accuracy of encoder signal is large (angle intervals between pulses of quality encoders differ by less than 0.1%) and period Δt between the edges of the signal is measured with high resolution 10 ns. Angular speed is calculated as

$$\omega(\varphi) = \frac{\Delta\varphi}{\Delta t(\varphi)} = \frac{360 \cdot i_{\omega}}{p} \frac{1}{t(\varphi) - t(\varphi - \Delta\varphi)} \ \left[\frac{\text{deg}}{s}\right], \tag{2}$$

where $\Delta\varphi$ is the angle interval which is measured over a period of time. The angle interval is defined by the number of pulses of the encoder per revolution p and by quantity i_{ω} which indicates how many pulses of the encoder is measured through the time. The evaluated angular speed ω is not an instantaneous angular speed but the mean of the angular speed in time interval Δt.

DMU therefore measures primarily angular speed. The measured values are very accurate according to the high frequency used for time measurement $f_m = 100$ MHz. The angle of rotation is then obtained by an integration of measured angular speed. The calculated angle is to several orders more accurate than angle given by the sum of the impulse signal edges as is shown below. Angular speed at the ends of the shaft is defined in accordance to the previous as

$$\omega_A\left[\frac{rad}{s}\right]=\frac{2\pi\cdot f_m\cdot i_\omega}{p\cdot N_A}, \tag{3}$$

$$\omega_B\left[\frac{rad}{s}\right]=\frac{2\pi\cdot f_m\cdot i_\omega}{p\cdot N_B}, \tag{4}$$

where the values N_A, N_B indicate the number of period of measuring frequency f_m which is found between the two measured pulses of encoder A, resp. B. The angle of rotation of the twisting shaft is then

$$\Delta\varphi_i[rad]=\int_o^{t_i}(\omega_A-\omega_B)\cdot dt\cong\Delta\varphi_{i-1}+\frac{2\pi\cdot f_m\cdot i_\omega}{p}\left(\frac{1}{N_A}-\frac{1}{N_B}\right)\Delta t=$$

$$=\Delta\varphi_{i-1}+\frac{2\pi\cdot f_m\cdot i_\omega}{p}\frac{N_B-N_A}{N_BN_A}\Delta t=\Delta\varphi_{i-1}+\omega_A\frac{N_B-N_A}{N_B}\Delta t. \tag{5}$$

It is expressed from the condition of the measurement

$$\Delta t=\frac{N_B}{f_m} \tag{6}$$

and after the substitution to the relation (5) formula for the angle difference $\Delta\varphi_i$-$\Delta\varphi_{i-1}$ is obtained

$$\delta\varphi\ [rad]=\frac{\omega_A}{f_m}(N_B-N_A). \tag{7}$$

For the smallest non-zero measured deviation the difference N_B-N_A has value 1. Then the formula is simplified and expresses a maximum theoretical resolution (measurement error) of this method

$$\delta\varphi\ =\frac{\omega}{f_m}. \tag{8}$$

As we see from the formula, this measurement error does not depend on the number of encoder pulses and linearly worsens with an increasing angular speed [5], [6]. For an angular speed of 2000 RPM a maximum resolution of 0.00002° is achieved. That is three orders better result than measurement of angle by sum of impulse signal edges with the same encoder.

3. Application

A practical demonstration of this method is the measurement of torsion vibration of power generator shaft used for the production of electricity and heat. The machine consists of two six-cylinder combustion engines connected together using a torsion stiff clutch and synchronous generator, connected directly to the second engine balance wheel (with no bearing to the side of the engine). The engines and generator are fixed to a machine frame which is elastically mounted on silent-blocks.

The purpose of the measurement was the evaluation of torsion vibrations at the ends of the main shaft and comparing the results with the values obtained by calculations. According to the relatively compliant frame and its possible effect on the shaft torsion vibrations, the motion of the frame was also measured. It was necessary to measure it very precisely because of small expected vibrations.

3.1. Description of measurement

From the description of the machine it is clear that it is a very complicated torsion system. The difficulty of the measurement increased the power regulation of the machine which drove the fuel supply. Random variations were usually in the range greater than ±2%. This caused a fluctuation of the mean value of the angular speed during one cycle (720°) although the mean value of the angular speed of the whole measurement was very accurate (given by the frequency of the electric network). Vibrations therefore were not stationary.

According to the conditions of measurement the angular speed was measured only at the ends of the shaft. To place the encoders inside would have been too difficult and expensive. However this method was satisfactory for the comparison of results with the calculation. Several modes of measurement were chosen to assess the influence of power. Each mode was measured five times.

Incremental rotary encoders had 3600 pulses per revolution, i.e. an angular resolution of 0.025°. In contrast, the minimum measurement error of angle evaluated by integration from the measured angular speed has for 1500 RPM value 0.00009° (approximately 280x better).

The accuracy of the measurement was increased by the elimination of a systematic error that was caused mainly by imperfect centring of the encoder against the measured shaft. During the manual rotation with open cylinders (with no compression), when only low passive resistances acted, the difference of angles was measured. The difference of angles was added as a correction to the measured torsion vibration for the assessment of the measurement. The correction was in the range approximately from 0° to 0.03°.

3.2. Assessment of measurement

The measurements were assessed statistically and with frequency analysis. The absolute torsion vibration of the angular speed of both ends of the shaft and their difference were evaluated. The angular torsion vibration of the whole shaft was calculated by the integration of the angular vibration difference. The influence of the twisted frame was indicated as negligible due to the measured torsion vibrations.

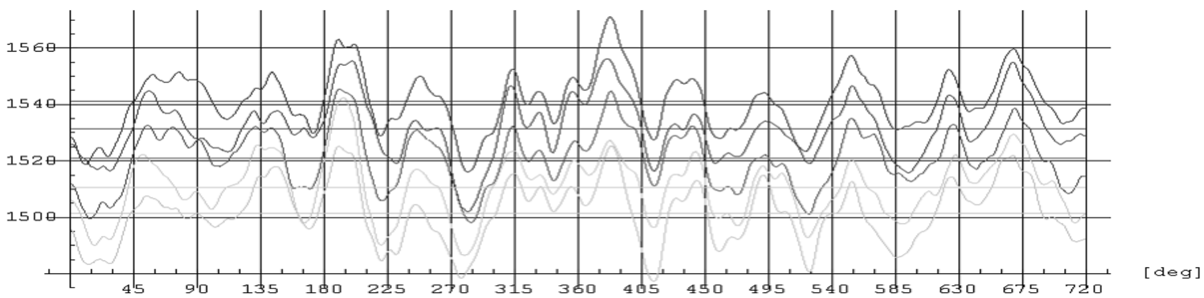

Figure 1: *Angular speed of the 1st engine shaft [RPM] for power equal to 300 kW (5 different measurements which are mutually shifted for a better reading). The maximum peak to peak values are 48.1, 56, 47.5, 54.3 and 55 RPM. The mean values of the angular speed are also shown.*

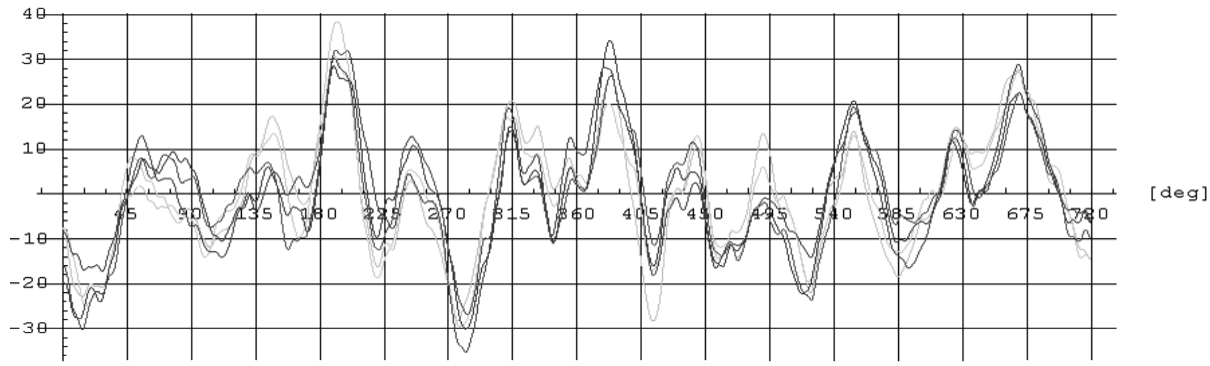

Figure 2: *The angular speed difference of engine minus generator [RPM] for the power equal to 300 kW. Maximum peak to peak values are 59, 68, 60.5, 67.2 and 64.2 RPM.*

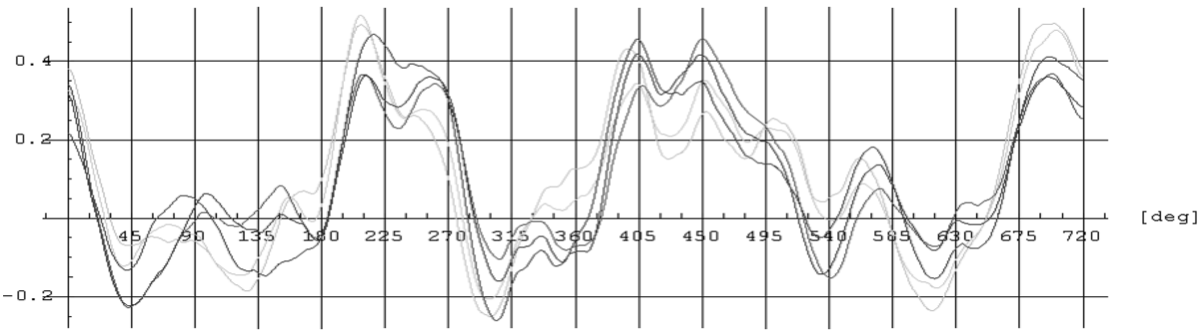

Figure 3: *The angular difference of engine minus generator [deg] for the power equal to 300 kW. Maximum peak to peak deviations are 0.75, 0.72, 0.69, 0.73 and 0.64 deg.*

The mean value of the angular speed of one cycle (720°) fluctuated about 1.96 RPM at the maximum (peak to peak). Maximum values of instantaneous angular speed deviations (Figure 1) were less than 56 RPM which represents a fluctuation of about 2%. It appeared that vibration of the 1st engine shaft was considerably larger than of the generator shaft. Prevailing angular speed vibrations corresponded to the ignition of each cylinder (12 per cycle).

The amplitudes of the angular speed difference of engine minus generator (Figure 2) were more significant, up to 68 RPM, and again the 12th harmonic amplitude dominated. The angular torsion vibrations were different (Figure 3), the maximum value of deviation was 0.75° and the third harmonic amplitude dominated. The non-stationary process is evident from the statistics of angular torsion vibrations on Figure 4 compared with Figure 3.

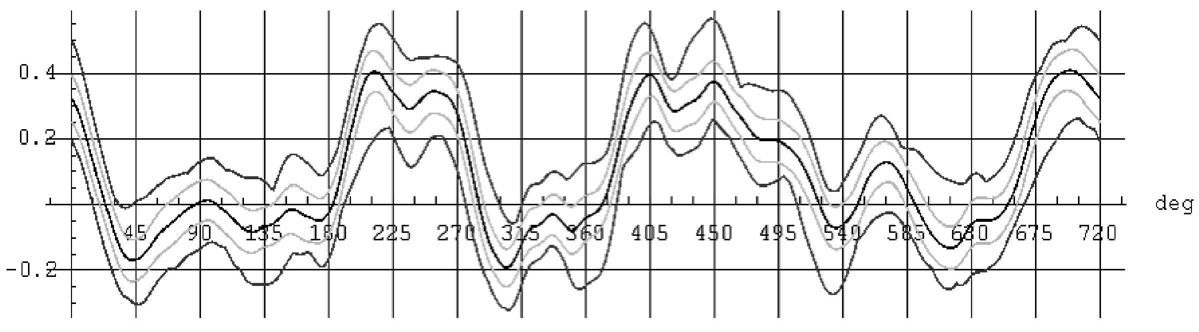

Figure 4: *The statistics of the angle of rotation difference engine minus generator [deg]. The middle curve represents mean value, the neighbouring curves are added standard deviation and the ultimate curves are maximum and minimum values. Evaluated from 42 cycles. The overall mean value of the middle curve 0.098 deg (mean twisting from the acting torques), the peak to peak deviation of mean value is 0.601 deg and the maximum peak to peak deviation is 0.886 deg.*

A good assessment is given by the frequency spectra. The biggest angular amplitude, 0.192°, had a third harmonic component that corresponded to the frequency of 37.52 Hz. The frequency of 150 Hz corresponded to cylinder ignitions, caused the biggest amplitude in the angular speed spectra, 8.37 RPM.

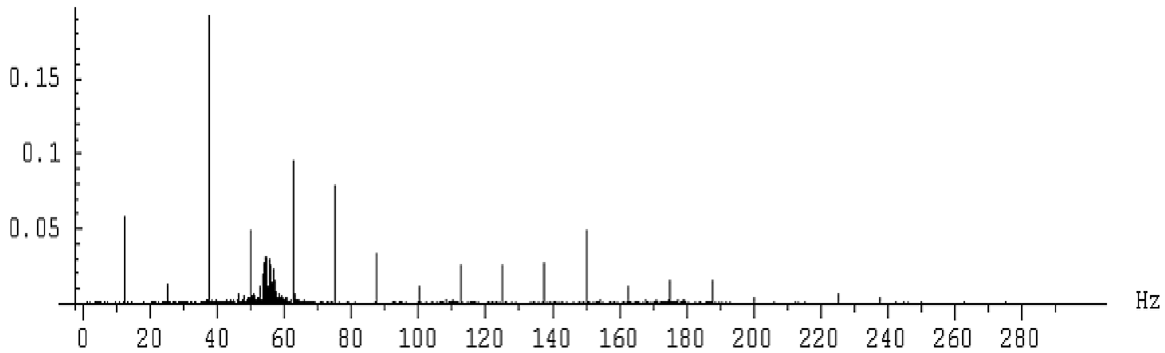

Figure 5: *The amplitude spectra of the angular difference engine minus generator [deg] from 42 cycles for the power equal to 300 kW.*

Interesting vibrations were located approximately in the range of 50 to 60 Hz in the angular difference spectrum (Figure 5). Although the biggest amplitudes of these vibrations were only several hundredths of a degree, in this sum they gave strong vibrations with beats. They achieved a high deviation in orders of a tenth of a degree and well documented total complexity of vibrations (Figure 6).

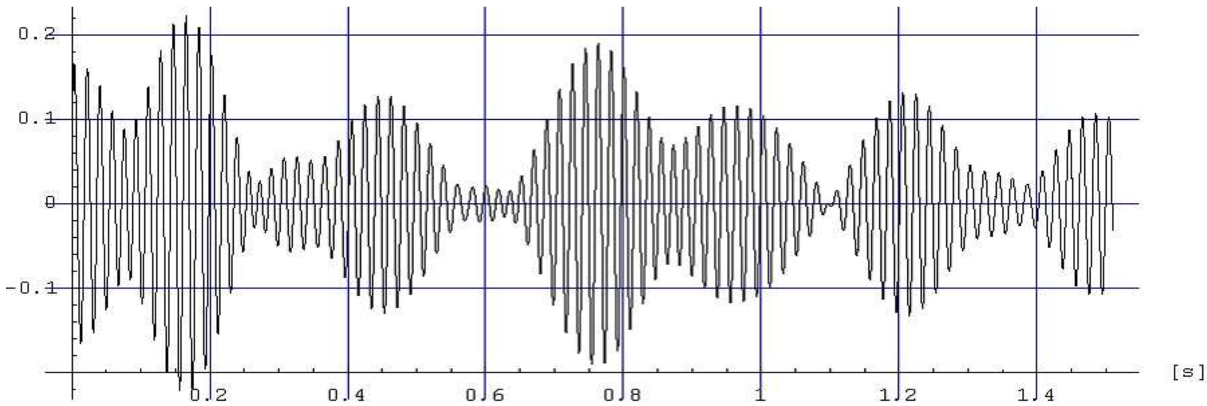

Figure 6: *The angle of rotation difference of engine minus generator [deg] in the frequency range of 50.1 to 60 Hz. The first 20 cycles (40 revolutions). The maximum deviation is 0.22°, minimal deviation is -0.22°. The biggest vibrations have a frequency of around 55 Hz.*

4. Conclusion

From the theoretical consideration of the various methods of torsion vibration measurement using the incremental rotary encoders and from the example of measurement, results show a large advantage of angular speed measurements based on determination of time intervals between the encoder pulses. According to the high accuracy and resolution of time measurement and with high quality encoders, high accuracy and resolution is achieved in angular speed measurements. The angle is evaluated by an integration of angular speed, which means an increase of resolution in orders about several orders compared to the angular measurement performed by a summation of impulse signal edges. High quality angular acceleration is given by derivative of measured angular speed that is unobtainable by a two-ply derivative of a directly measured angle.

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