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Experimental Stress Analysis

# EXPERIMENTAL AND NUMERICAL RESEARCH OF MAGNETOSENSITIVE ELASTOMERS

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**Abstract**: This paper presents fabrication, experimental investigation and numerical simulation of magneto-sensitive (MS) elastomers. Aligned magneto-rheological elastomer composites were prepared using silicone elastomer as the matrix material and iron micro-particles as the magnetizable filler. The blend was cured under an external magnetic field. Cylindrical samples consisting of magneto-sensitive elastomers were prepared and tested in quasi-static compression to study the changes in their stiffness characteristics under the influence of a magnetic field. The basic equations of the mechanical equilibrium and Maxwell's equations are briefly summarized and a Helmholtz strain-energy function for MS anisotropic elastomers is shown in order to govern the interaction between a magnetoelastic anisotropic material and a magnetic field. Some FEM solutions of the coupling of MS anisotropic materials and an applied magnetic field are implemented in Comsol Multiphysics.

## 1. Introduction

Magneto-sensitive elastomers belong to the category of smart composite materials. Their mechanical behaviour can be controlled by an applied magnetic field. MS composites typically consist of magnetically polarizable micro or nano particles placed in a non-magnetic solid or gel-like elastomeric matrix. The research of these materials provides large possibilities of industrial applications. For example, a magneto-sensitive elastomer might be used as a field dependent device with a variable stiffness or as a damping system controlled by a magnetic field. However MS materials are well known for several years, it is very important to predict the features by experimental measurement and numerical simulations and this way provides relevant data structures, which could be used in industry.

Recently many experimental researches of magneto-rheological elastomers are published in literature i.e. [1-6] which investigate the possibility of exploitation of extraordinary properties of MS composites in various industrial applications. Concurrently several papers have been devoted to the development of the constitutive theory of isotropic MS composites, such as [7-9]. However, the constitutive theory and numerical simulation of strongly anisotropic MREs need to be further developed.

## 2. Experimental

### 2.1. Preparation of MS composites

We explored behavior of silicon rubber-based elastomers filled by iron particles. The magnetic efficiency to change field dependent mechanical properties might be optimized by choosing the particles with a high magnetic saturation. We used pure iron micro-particles because of their price and availability however carbonyl iron particles which are commonly used have larger saturation. The magnetic saturation of pure iron is 2.1 T which is lower by 0.3 T than the saturation of expensive Cobalt alloys. The isotropic MS composites are prepared like a mixture of filler particles and matrix which is cured without a magnetic field. It is also possible to provide MS composite with aligned structures of iron particles.

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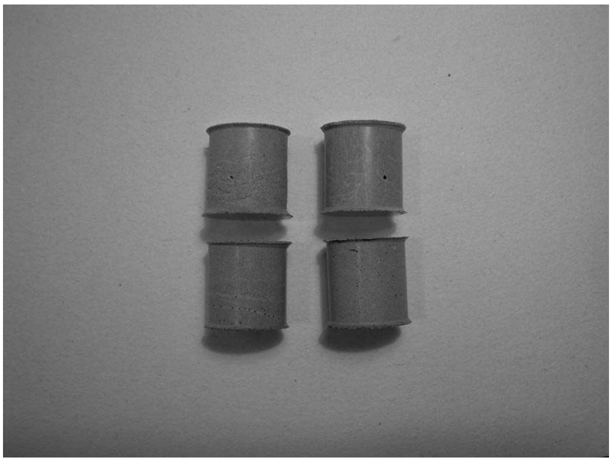


*Figure 1: Specimens*

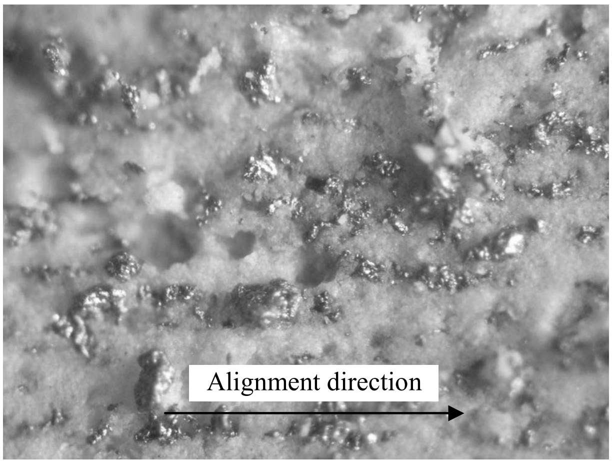



*Figure 2: Internal chain-like microstructure*

We controlled the internal structure of composite by an applied magnetic field in course of the composite polymerization. The magnetic field acted in an axial direction of the sample. The composite samples consisted of the RTV 1532 silicone and 30% volume of pure iron powder. The silicon matrix is suitable because the iron particles are not influenced by polyaddition reaction of products.

Cylindrical specimens (Figure 1) were cured in an external homogenous magnetic field induced by two permanent magnets. The value of induction was 0.455 Tesla which was measured by a teslameter. The magnetic field caused a chain-like structure of iron particles (Figure 2) and the resulting mechanical and magnetic anisotropy of the material. The chain-like particle structure enhances mechanical and magnetic properties of MS composite.

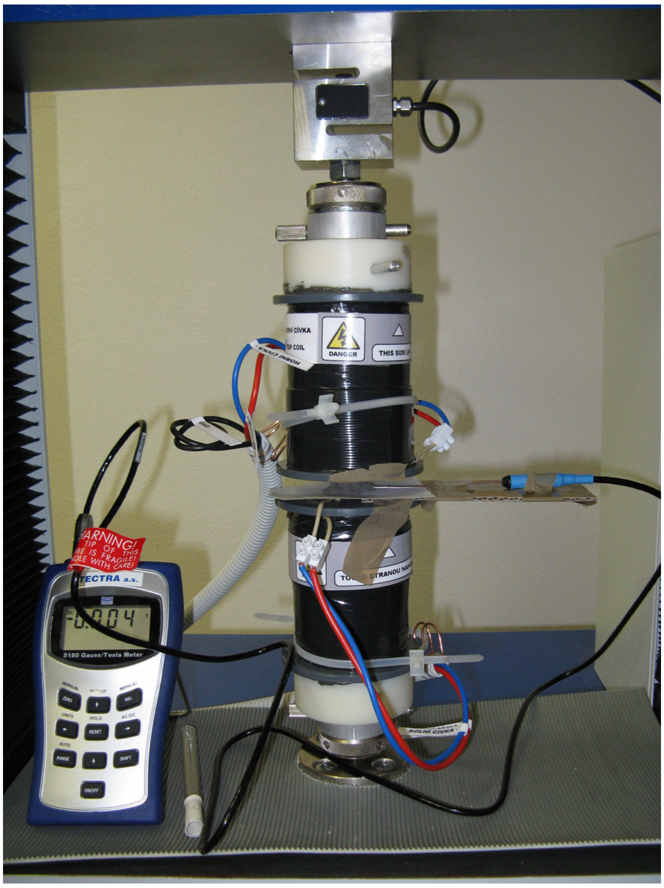


***Figure 3:** Magnetic and loading device*

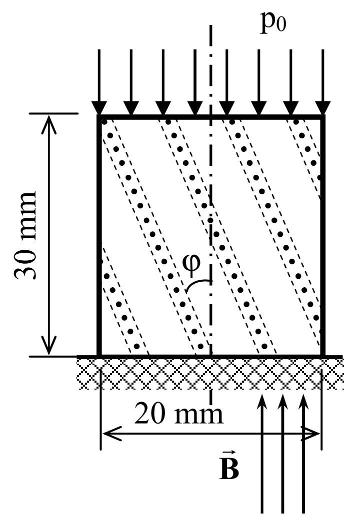



***Figure 4:** MS block in magnetic field*

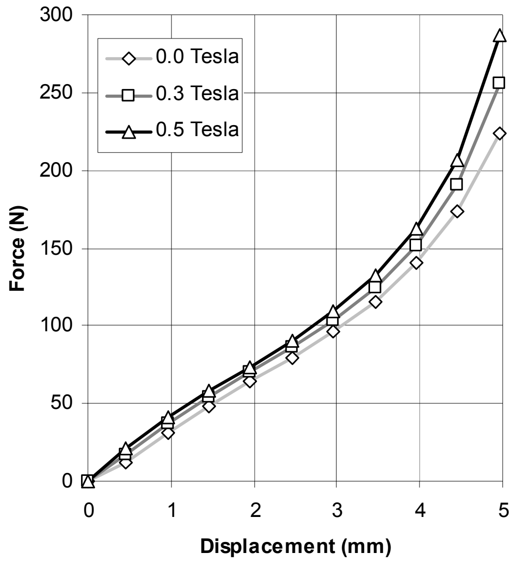



*Figure 5:* *Dependence of force on displacement at different values of magnetic induction*

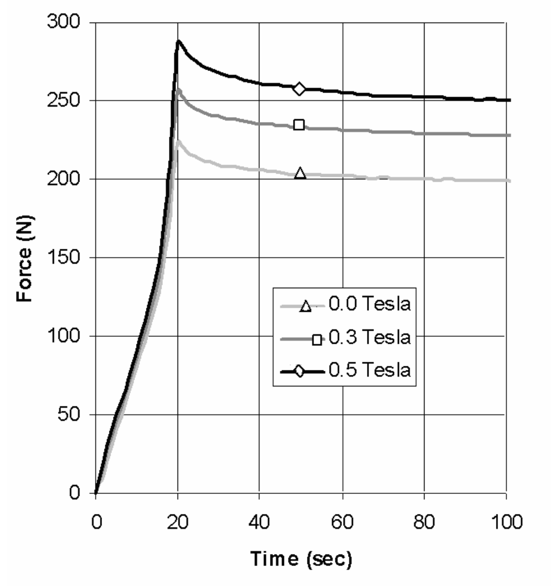



*Figure 6:* *Evolution of force in time at different values of mg induction. Stretch of the cylindrical sample $\lambda$=0.75.*

### 2.2. Experimental device

Two coils magnetic system was used in the experiments (Figure 3), which provided the magnetic induction value **B** in range from 0 to 1.0 Tesla. The largest value of magnetic field intensity **H** was 20 kA/m. The final strength of induced magnetic field depends on the permeability, shape and volume of specimen. In compressive testing of the material value of magnetic induction was measured by the transversal probe (Tectra).

All of the experiments were executed with TIRA-test universal testing machine, firmly connected to the magnetic device. The mechanical properties of the MS composite were tested in compression passively (without any magnetic field) and with increasing magnetic flux density. The load and the magnetic field direction were applied parallel to the direction of the chains of particles. We measured the dependence of the compressive force on displacement (Figure 5) and an evolution of measured force in time (Figure 6) at constant stretch $\lambda=0.75$ first without the magnetic field to determine so called zero-field stiffness. Then the magnetic field was switched on and the response was measured at different values of magnetic field induction.

We see that the value of force, which is needed to maintain the constant displacement, is increasing with the value of magnetic induction. On the right-hand graph we may recognize that the force is decreasing with time because of a relaxation. From (Figure 6) we see that at the magnetic induction of 0.5 T the stiffness increased by 25% with respect to zero-field stiffness and that the compressive stiffness of the aligned MS composite was increased proportionally to increasing magnetic flux density.

## 3. Fundamental theory of MS elastomers

The balance equations for nonlinear magnetoelastic elastomers in a static magnetic field, as developed generally in [7], are summarized concisely in the following equations

$$div\mathbf{B} = 0, \qquad curl\mathbf{H} = \mathbf{0} \tag{1}$$

$$\rho_0 = J\rho, \qquad div\boldsymbol{\tau} = \mathbf{0} \tag{2}$$

incorporated with the boundary continuity conditions between the magnetic and mechanical fields

$$[\mathbf{B}].\mathbf{n} = 0, \qquad [\mathbf{H}]\times\mathbf{n} = \mathbf{0} \tag{3}$$

$$[\boldsymbol{\tau}]\mathbf{n} = \mathbf{0} \tag{4}$$

where **B** and **H** are the magnetic induction and the magnetic field respectively, [•] signifies a discontinuity across the boundary, **n** is its outward unit normal and $\boldsymbol{\tau}$ denotes the resulting total Cauchy stress tensor with advantage of being symmetric including a magnetic stress tensor (see [7-9]), in which the Maxwell stress outside the material, denoted $\boldsymbol{\tau}_M$, is given by

$$\boldsymbol{\tau}_M = \mathbf{H}^* \otimes \mathbf{B}^* - \frac{1}{2}\left(\mathbf{H}^*.\mathbf{B}^*\right)\mathbf{I} \tag{5}$$

Lagrangian counterparts of **B** and **H**, denoted $\mathbf{B}_l$ and $\mathbf{H}_l$, respectively, are only considered in material domains to be given by [7]

$$\mathbf{B}_l = J\mathbf{F}^{-1}\mathbf{B}, \quad \mathbf{H}_l = \mathbf{F}^T\mathbf{H} \tag{6}$$

To describe the behavior of isothermal MS anisotropic materials we postulate the existence of a Helmholtz free energy function $\Psi$, which depends not only on a deformation gradient tensor **F** and a preferred direction of ferrous particles represented by $\mathbf{a}_0$ but also on a magnetic induction $\mathbf{B}_l$, denoted in the Lagrangian configuration, and the total free energy function is expressed as

$$\Psi = \Psi\left(\mathbf{F}, \mathbf{a}_0, \mathbf{B}_l\right) \tag{7}$$

From Clausius–Duhem inequality for electro-magnetic media we can deduce constitutive equations for the total nominal stress tensor **T** and the magnetic field $\mathbf{H}_l$ inside the deformed magnetic material as

$$\mathbf{T} = \frac{\partial\Psi}{\partial\mathbf{F}}, \qquad \mathbf{H}_l = \frac{\partial\Psi}{\partial\mathbf{B}_l} \tag{8}$$

In order to simulate behaviors of incompressible MS anisotropic elastomers, we choose the free energy function similar to Ottenio et al., see [9]

$$\begin{gathered} \Psi = \Psi_{iso} + \Psi_{ani} - p(J-1) \\ \Psi_{iso} = \frac{G}{4}\left[(1+\gamma)\left(\bar{I}_1 - 3\right) + (1-\gamma)\left(\bar{I}_2 - 3\right)\right] \\ \Psi_{ani} = \frac{k}{2}\left(\bar{I}_4 - 1\right)^2 + \frac{1}{\mu_0}\left(\alpha I_6 + \beta\bar{I}_7\right) \end{gathered} \tag{9}$$

where $G = G_0\left(1+\eta_G I_6\right)$ is the shear modulus in the reference configuration, $G_0$ is the field independent shear modulus (or zero-field modulus) and $k = k_0\left(1+\eta_k I_6\right)$ represents the anisotropic characteristic of MS elastomers. The material parameters $\alpha$, $\beta$ and $\gamma$ are non-dimensional material constants and $\eta_G$ and $\eta_k$ are material constants involving the magnetic strength, these parameters need to be determined by coupling magnetic and mechanical experiments, and $p$ is the hydrostatic pressure. $\bar{I}_1$, $\bar{I}_2$, $\bar{I}_4$, $I_6$ and $\bar{I}_7$ are invariants involving to the deformation gradient, the preferred direction and the magnetic field and determined by

$$\begin{gathered} I_1(\mathbf{C}) = tr\mathbf{C}, \quad I_2(\mathbf{C}) = \frac{1}{2}\left[(tr\mathbf{C})^2 - tr\mathbf{C}^2\right] \\ I_4(\mathbf{C}, \mathbf{a}_0) = \mathbf{a}_0.\mathbf{C}\mathbf{a}_0, \quad I_6(\mathbf{B}_l) = \mathbf{B}_l.\mathbf{B}_l \\ I_7(\mathbf{C}, \mathbf{B}_l) = \mathbf{B}_l.\mathbf{C}\mathbf{B}_l \end{gathered} \tag{10}$$

and $\bar{I}_\alpha = J^{-2/3} I_\alpha \left(\alpha = 1,4,7\right)$, $\bar{I}_2 = J^{-4/3} I_2$, more details look at [8].

The influence of the magnetic field on the surface of magnetoelastic body is calculated via magnetic tractions given by this expression, refer to [7]

$$\mathbf{t}_a = J\boldsymbol{\tau}_M \mathbf{F}^{-T}\mathbf{N} \tag{11}$$

where $\boldsymbol{\tau}_M$ is the Maxwell stress defined by equation (5) and **N** is a unit outer normal vector on the boundary of the undeformed body.

## 4. FEM simulation of MS anisotropic elastomers

In our numerical simulation we implemented the free energy function (9) into the open FE code Comsol Multiphysics. The material parameters are listed in Table 1.

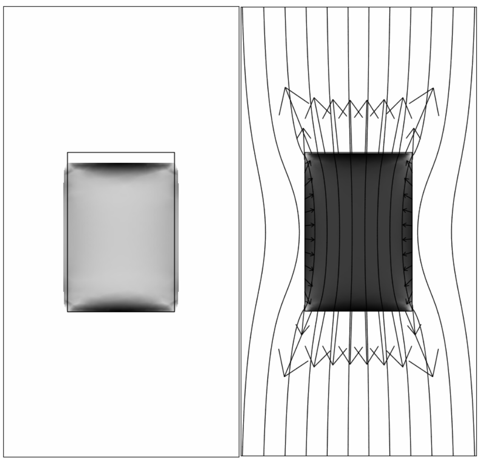
***Table 1**. Material parameters of a MS anisotropic composite*

| $G_0$ [MPa] | $\eta_G$ [$T^{-2}$] | $k_0$ [MPa] | $\eta_k$ [$T^{-2}$] |
|---|---|---|---|
| 1.8 | 0.6 | 5.0 | 0.9 |

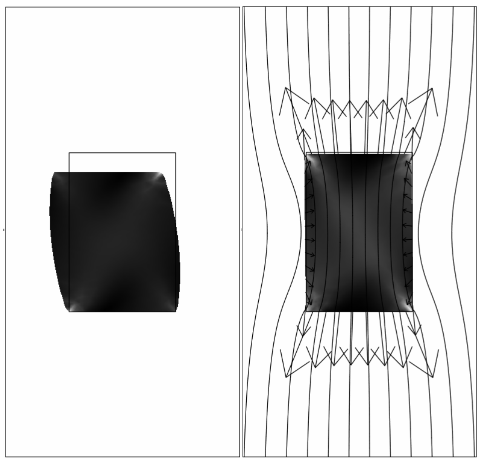
| $\alpha$ [-] | $\beta$ [-] | $\gamma$ [-] |
|---|---|---|
| 0.05 | 0.1 | 0.6 |

### 4.1. Compression of a block

First we consider an MS anisotropic block with different aligned directions of ferrous particle chains embedded in a static uniform magnetic field parallel to its axis and simultaneously subjected to a constant pressure $p_0$=1*MPa* see (Figure 4).
We have obtained series of stress and strain responses of the block as well as the distribution of the magnetic field interior and exterior domains of the MS anisotropic material. (Figure 7) illustrates some achieved results of deformations and equivalent stress distributions corresponding to ferrous particle orientations by $0^0$ and $30^0$ without and with the magnetic field applied by B=1T, moreover the direction of the magnetic traction vectors implies the body tends to lengthen along the direction of the applied field. In (Figure 8) we represent a distribution of the magnetic field and a magnetization of the body with the chain orientation by $30^0$ and the magnetic flux density at that time as 0.5T. It can be seen that owing to the change of the domains induced by the deformation of the block the distribution of magnetic field does not remain to be symmetric and a strong dependency of the magnetic field on the deformation of the material is still denoted by the distribution of the magnetization inside the material.



*a) Orientation of particle chains: $\varphi=0^0$ at B=0T and B=1T*



*b) Orientation of particle chains: $\varphi=30^0$ at B=0T and B=1T*

***Figure 7:** Deformation of the MS anisotropic block without/with a uniform magnetic field*

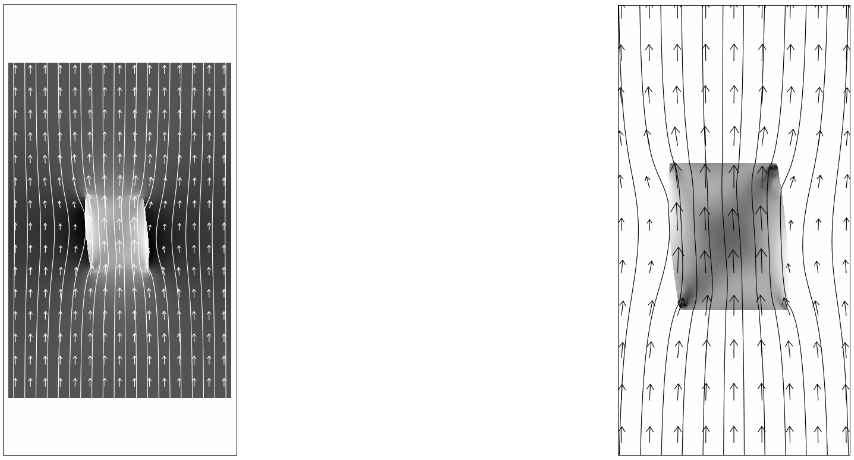
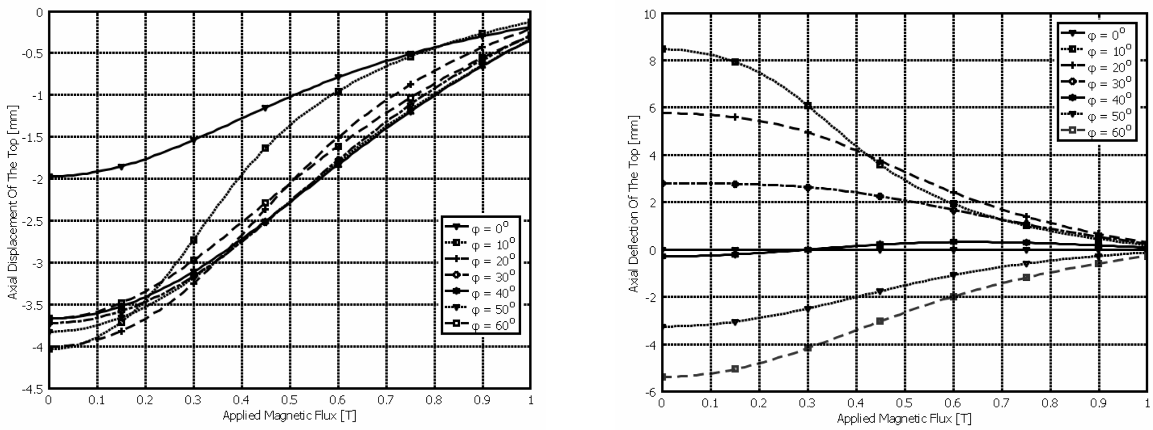


***Figure 8:*** *Distribution of the magnetic field and magnetization of the MS anisotropic material*




*a) Vertical displacement of the top surface* *b) Horizontal displacement of the top surface*

***Figure 9:****Displacements of the top surface of the block versus the magnetic flux density*

Dependencies of horizontal and vertical displacements of the top surface on orientations of particle chains and on an applied magnetic filed are shown at (Figure 9). The stiffness of the body increases fast that is demonstrated by the deformed block recovered nearly complete when the magnetic field reaches to 1T.

### 4.2. Simple shear

A rectangular MS anisotropic composite plate subjected to a simple shear state is investigated in altered directions of an applied magnetic. Here we assume ferrous particles are oriented in a vertical direction, only the modification of the applied magnetic field is considered. The geometry of the plate and loading conditions are represented in (Figure 10). The direction of the magnetic field is defined by θ compared to the vertical direction. The external loading is set up by a constant traction as $\tau_0 = 1MPa$.

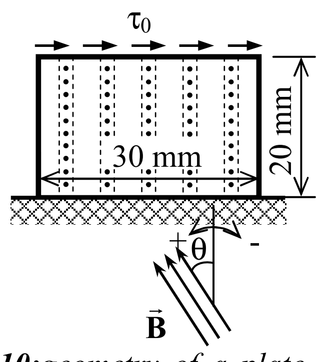



***Figure 10:****geometry of a plate subjected a simple shear state is embedded in a static uniform magnetic field*

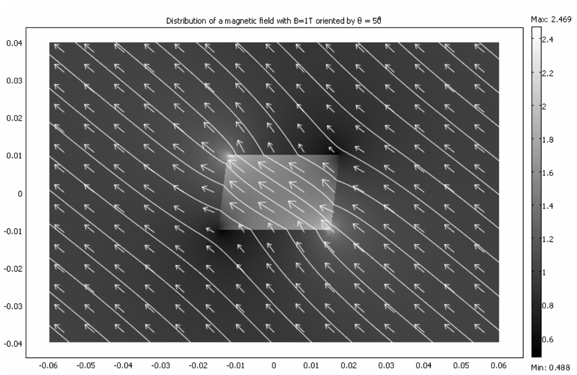
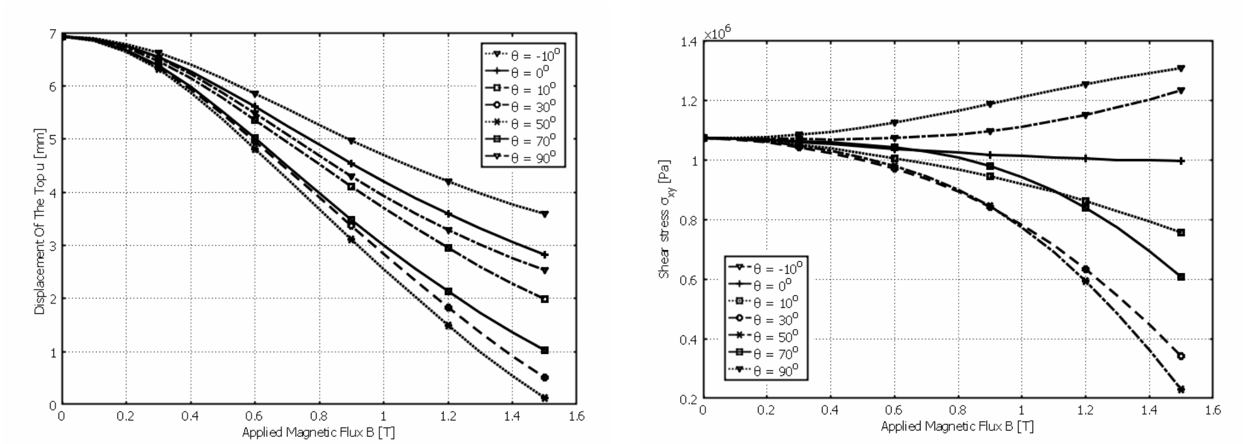



***Figure 11:****Distribution of the magnetic field with* $\theta = 50^0$




*a) Displacement of the top surface versus different directions of the applied magnetic field*

*b) Shear stress at the center of the body versus different directions of the applied magnetic field*

***Figure 12:*** *Dependencies of a displacement and a shear stress on the applied magnetic field*

In order to verify effects of the applied magnetic field on the MS body under a simple shear state a direction of the magnetic field is changed and the deformation is calculated. We can conclude that the magnetic forces lengthen the MS body and enhance strongly the stiffness of the material. A high concentration and a large alteration of the magnetic field through the interface of the material and the surrounding space are shown in (Figure 11). Inside a dense material the field is stronger otherwise it still depends on the deformation of the body.

Dependencies of a displacement of the top surface caused by the external loading and a shear stress at the central position of the plate on the altered direction of the applied magnetic field are investigated in (Figure 12). Combination of both the displacement and the shear stress results state that we should apply the opposite direction of the magnetic field against the direction of the deformation in order to enhance a recovery back to the initial form of the deformed MS material.

## 5. Conclusion

In this paper we showed that the magneto-sensitive elastomers might be used as device with a variable stiffness which is controlled by the magnetic field. The two coils device was developed for testing the MS cylindrical samples in compression. The aligned MS composite samples were fabricated and tested. We measured the force-displacement characteristics of

MS samples in the changing magnetic field and the value of sample stiffness was shown to increase with the magnetic field intensity.

We have summarized concisely constitutive equations and shown a particular Helmholtz strain-energy function to represent an interaction between an MS anisotropic elastomer and an applied magnetic field. A coupling of the magnetic field and the mechanical problems for MS anisotropic materials was solved successfully in Comsol Multiphysics. Reciprocal effects of MS materials and the magnetic field are examined via two standard examples: A compression and a simple shear. Achieved FEM results agree with practical experimental results and with the predictions in the mentioned papers.

We pursued the experimental and numerical research of magneto-sensitive elastomers in our previous works [10-12]. We showed that MS elastomers as smart controllable materials are suitable for new industrial applications and provide an impetus for continued research in this area.

## Acknowledgement

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