Experimentální Analýza Napětí 2009
Experimental Stress Analysis

MATERIAL IDENTIFICATION BY THE TENSILE TEST

Karel Vítek[1]

Abstract: The methodology of determining the construction materials' elastic constants is based on the mechanical tests, being mostly tensile tests, of partially loaded specimens (macro-specimens). The elastic constants obtained are applied mainly when a computer-aided modeling structure is used which considerably advanced due to computer techniques development. To obtain a deeper understanding of occurring phenomena, it is necessary to find out the ε-σ distribution as far as to the ultimate strength of specimens taken from various structure localities, instead of the simple knowledge of Young's modulus of elasticity and Poisson's ratio. The random combined loading of the tensile test specimen exist probably. The linear and nonlinear mathematic models of the material identification under the combined loading of the specimen are described in this paper.

1. Introduction

Identification of the mechanical properties of the solid materials via tensile test performed by breaker is based on determination of the specimen deformation under predefined load. Or the other way round, the independent variable is deformation and the determined variable is load required by shredder to achieve required predefined deformation. The tensile test is both by construction layout of the shredders and by the shape of the tested specimen designed for evaluation of single axis tension effects. The single axis tension is thus one of required condition of objective determination of material properties identification.

In case of predefined shredder force load, the deformations of the specimen are usually evaluated by strain gages, which monitor the surface of the sample in one particular region of several millimeters to centimeters. When we place more than one strain gages around the evaluated specimen cross section boundary, the measured deformation can be mutually different although the load tension is supposed to be equivalent in whole cross section. This fact can induce a false impression that the material properties of identical samples can be sometimes even considerably different, see [1]. If we compare the behavior of constructions based on material properties values, the results do not have such a large variance in comparison the dispersion of the tensile test measured values. Thus here arises a question which of the measurement from the set of tensile test results should be taken into account as convenient for material property determination and when the result received from the simple result regression of tensile test data is sufficient for inspected material property determination.

Big variance in the measured values can be caused by combined load of the specimen, which is supposed to be loaded with a single axis pull loading. This can be inflicted by the inappropriate fixation of the sample in the tensile machine. The jaws of the tensile machine can be fixed in tilted position in respect to the load tension axis. The probability of such diverted fixation is high, as depicted in Fig. 1. The axial specimen loading by this non-uniform boundary condition support is really non-uniform loading with unsymmetrical stress in the cross section.

The longitude specimen fibers between the jaws of the tensile machine are no equally long. Also the depth of tensile machine jaw bite is uneven. Numeric simulations of uneven

[1] Dr. Ing. Karel Vítek; Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, CTU in Prague, Technicka 4, 16607 Prague, Czech Republic, E-mail: karel.vitek@fs.cvut.cz

boundary conditions for homogenous linear Hookean material show, that such boundary conditions modify the non-uniform strain distribution over the specimen cross section – the load of cross section is not homogenous. This could be the route cause for the large variance in the tensile test measurements. The practical realization of the tensile test cannot provide the even load over the whole cross section - the partial force homogenous load. Thus, the evaluation of elasticity constant and other material properties can be determined with tensile test only with certain difficulties. The tensile specimen in the tensile machine is always loaded with tension and also random unknown bending moment, which is unmeasured by the tensile machine.

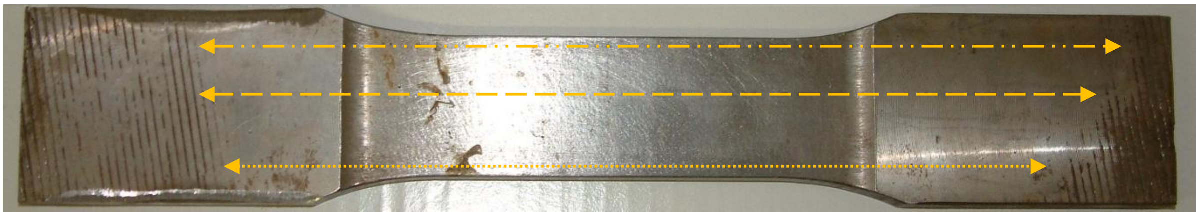

Figure 1: *Tensile specimen and non-uniform boundary support condition do to the shredder jaws*

2. Mathematic model of the data regression

If the specimen has a linear Hookean material with Young's elasticity modulus *E*, the combination of the axis force and the bending moment creates the plane of the stress or the strain above the cross section. Strains measured by the strain gages are tightly related to the plane orientation. The figure 2 depicts the position of six strain gages (1 - 6) placed on the specimen. The gages are placed on boundary of the specimen cross section. The coordinates *z,y* are aligned with the direction of main central axis of the cross section. We suppose the second moments of the specimen cross section area to be J_y, J_z and also the components of the unknown bending moment to be M_y and M_z.

The dependence of a strain of the *i*-th strain gauge ε_i on the axis stress $\sigma = F/A$ and on the regression coefficient of the measured data set k_i can be described via equation (1). After substitution of the Hooke's law into equation for the stress (2) for combination of bend and tension in place of i-th strain gauge, we get a linear equation for three unknown variables. Two are the unknown bending moments M_y, M_z and third is the Young's modulus *E*

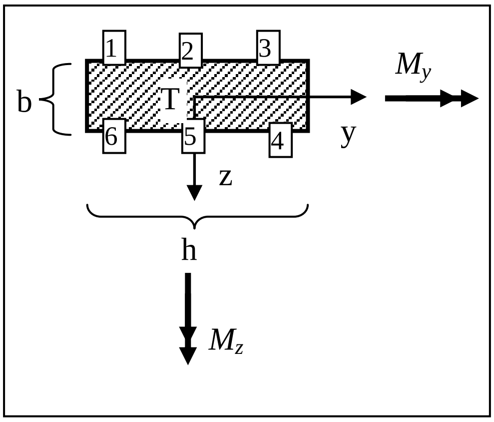

Figure 2: *Assembly of the specimen cross section boundary with the strain gauges 1 to 6. Load of the specimen with the unknown bending moment components* M_y, M_z.

$$\varepsilon_i = k_i * \frac{F}{A} \tag{1}$$

$$\frac{M_z}{J_z} * y_i - \frac{M_y}{J_y} * z_i + \frac{F}{A} = \varepsilon_i * E = k_i * \frac{F}{A} * E \tag{2}$$

$$\frac{M_z}{J_z} * y_i - \frac{M_y}{J_y} * z_i - k_i * \frac{F}{A} * E = -\frac{F}{A} = -\sigma \tag{3}$$

For modulus determination, it is sufficient to apply at least three strain gauges placed on the boundary of the sample cross section. The system of linear equations can be composed for the data measured by these strain gauges, after these values are treated by linear regression according to equation (2). The final linear equation system has three unknown variables E, M_y a M_z. Principal and application was published in [2]. By population of the cross section boundary with greater amount of strain gauges is more costly, but the information received via such measurement from small set of test specimen can be sufficient for identification with good precision. Such evaluation cannot be performed for such tests, where the number of gauges is below the minimum required amount (1 or 2), because these simplified measurement do not contain sufficient amount of information for the real cross section load distribution determination. Such material property identification is not objective (valid) even for large number of specimen as all results can be easily biased.

$$\begin{bmatrix} \cdots & \cdots & \cdots \\ +\frac{y_i}{J_z} & -\frac{z_i}{J_y} & -k_i * \frac{F}{A} \\ \cdots & \cdots & \cdots \end{bmatrix} \times \begin{Bmatrix} M_z \\ M_y \\ E \end{Bmatrix} = \boldsymbol{A} \times \boldsymbol{x} = \boldsymbol{b} \ = \begin{Bmatrix} \cdots \\ -\frac{F}{A} \\ \cdots \end{Bmatrix} = \begin{Bmatrix} \cdots \\ -\sigma \\ \cdots \end{Bmatrix} \qquad (4)$$

$$[\boldsymbol{A}^{\boldsymbol{T}} \times \boldsymbol{A}] \times \{\boldsymbol{x}\} = [\boldsymbol{A}^{\boldsymbol{T}} \times \boldsymbol{b}] \rightarrow \{\boldsymbol{x}\} = [\boldsymbol{A}^{\boldsymbol{T}} \times \boldsymbol{A}]^{-1} \times [\boldsymbol{A}^{\boldsymbol{T}} \times \boldsymbol{b}] \qquad (5)$$

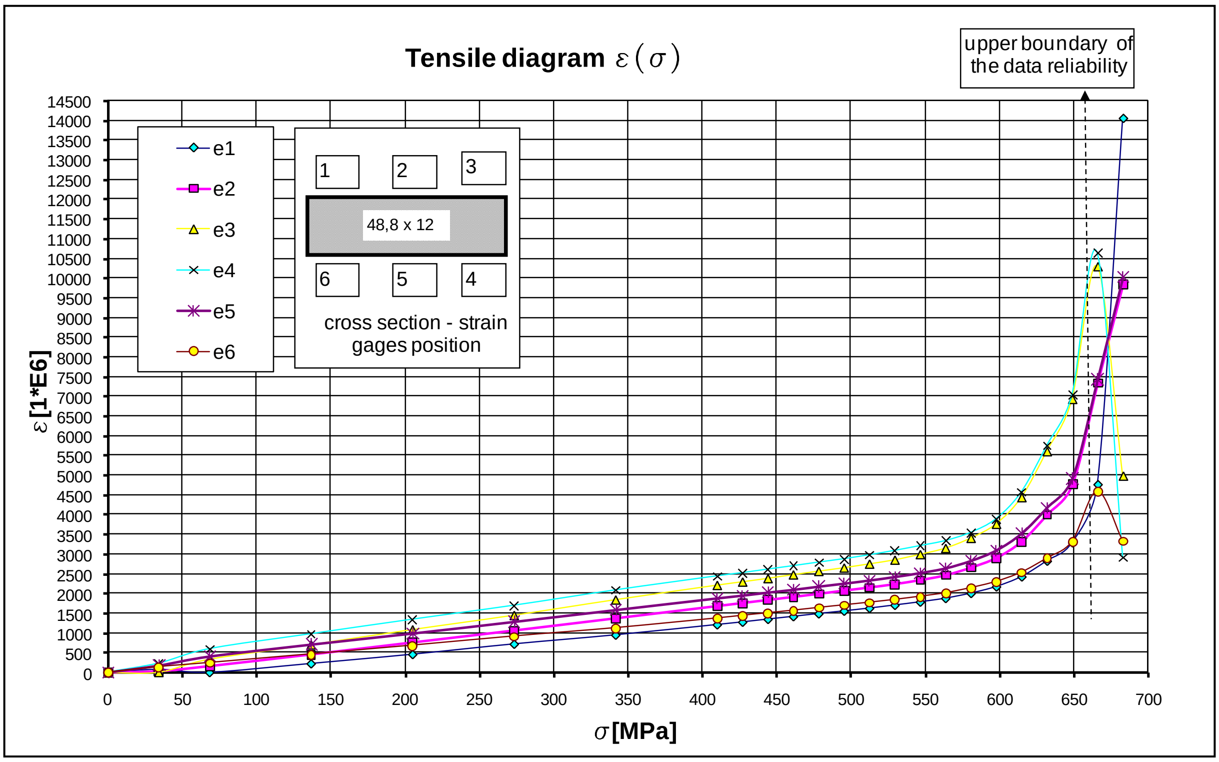

Figure 3: *Representative tensile diagram of the specimen with sixth strain gages.*

In cases that the larger number of strain gauges is used (for Hookean materials only), the linear regression of individual gauges data can be used. This allows us to modify the linear equation system (2) with unknown variable vector $\boldsymbol{x}(E, M_y$ a $M_z,)$ into the equation (3). If we transform the linear equations for individual strain gauges (2) into the matrix form with the coefficient matrix $\boldsymbol{A}$ with unknown variables vector $\boldsymbol{x}$ of deformations and right hand side $\boldsymbol{b}$, we get the new linear system (4).

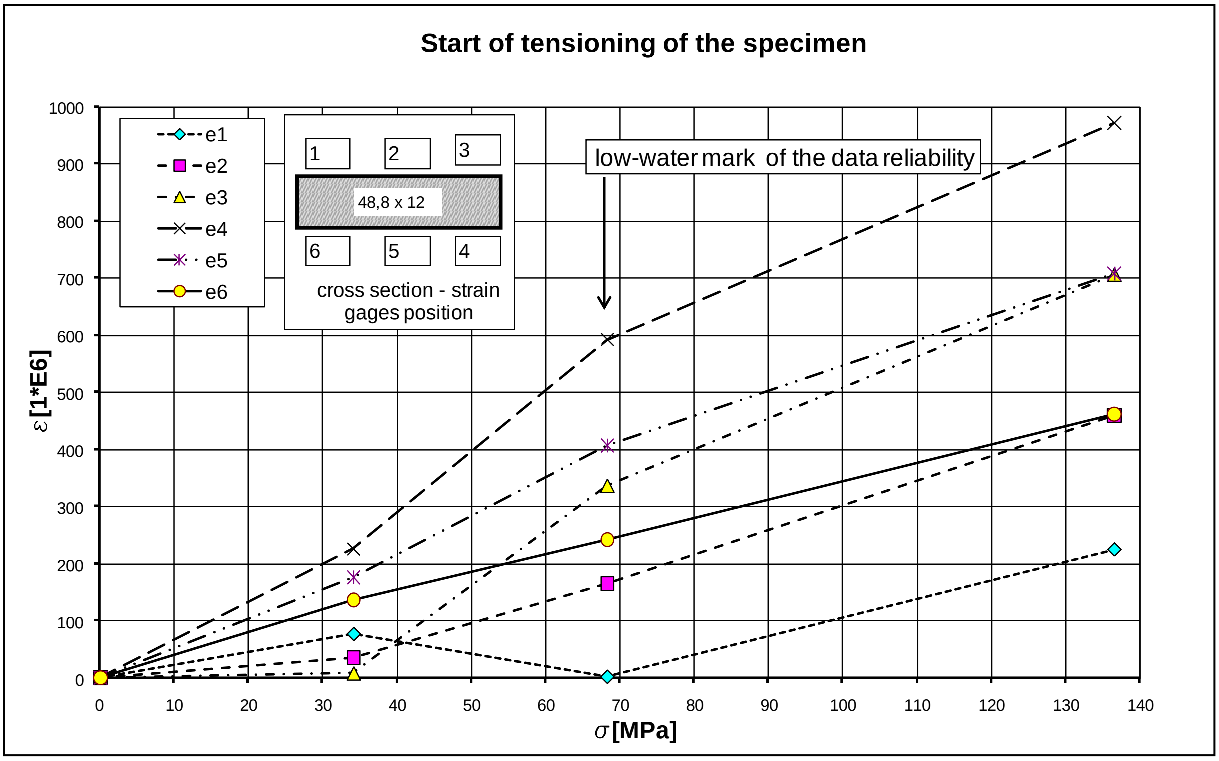

Figure 4: *The first part of the tensile diagram of the specimen with six strain gages.*

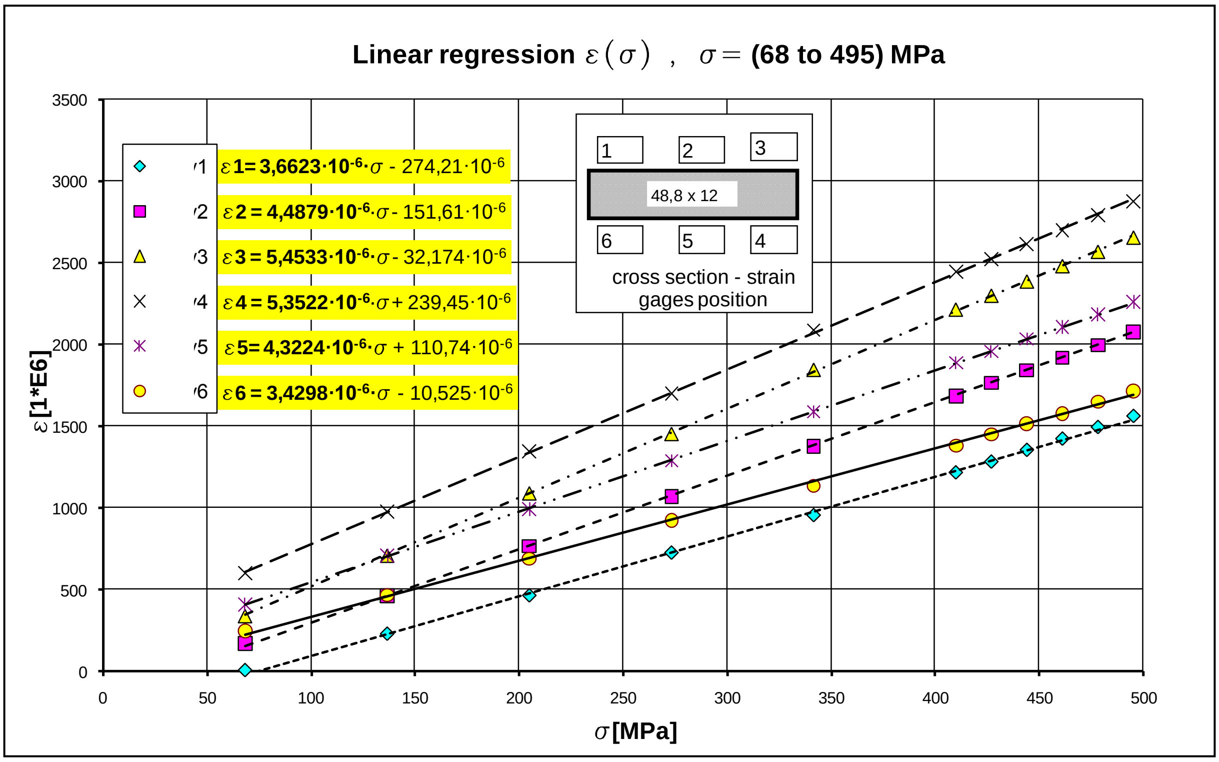

Figure 5: *The data part of the linear regression.*

The number of equations can be generally larger than number of unknown variables, which is three. The system can be solved as (5). Equations (4), (5) can be successively normalized in respect to nominal stress by divide of the value $\sigma = F/A$. The value of Young modulus will not be influenced, but the components of bending moment are then normalized to unitary nominal tension stress.

Practical illustration of the measurement of the tensile test specimen with six strain gauges is depicted in Fig. 3. Axis of the stress σ stands for nominal axis stress, which is determined as a fraction of tension force *F* inflicted by tensile machine and the cross section surface *A*. This σ is not equivalent to stress in the places, where the strain gauges measure its deformation values. In first phase of specimen load (under threshold value of $\sigma=70\ MPa$) the jaws are grabbing the specimen more and more firmly, however, the fixation in the tensile machine jaws is not yet stabilized. Measured data are unstable and they have no relation to stabilized construction behavior. Thus, for the low data reliability, this data set is excluded from the final evaluation. The detail can be seen in Fig. 4.

Up to the stress value of $\sigma=500\ MPa$ the measured data can be considered to be highly linear. Equations of regression lines for the range with the linear material behavior are depicted in Fig. 5. Adaptation of these equations to helps to overcome the initial transition instability in the measured data from Fig. 4. Thus, we shift the regression lines into the origin of the diagram and we centralize the measurements to this new origin (by subtracting the corresponding absolute coefficients, so that the measurement starts with the data from the linear area).

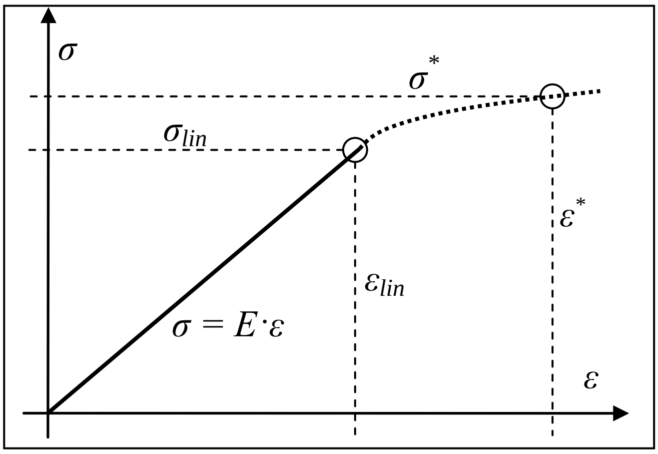

Figure 6: *The sequence regressions principle*

By transformation of the linear computation model of the tensile test with use of equations (4) and (5) we receive unbiased linear regression of this part of the tensile test. The function values are predetermined by Young's modulus *E* as shown in Fig. 6, which can be subsequently determined from the measured values. Each level of load, which is in reality decomposed into the tension and bending components, is represented in this final line by several deformation values of the measurement strain gauges. The linear area of deformation is smoothly followed by nonlinear deformation part in point ($\varepsilon_{lin},\sigma_{lin}$). The nonlinear part of regression curve is depicted as $\sigma^*(\varepsilon^*)$. The range, where the strain stress relationship can be regressed linearly continues smoothly in point ($\varepsilon_{lin},\sigma_{lin}$) into the nonlinear dependency $\sigma^*(\varepsilon^*)$, which is generally in respect to the assumption that the dependency is a smooth function.

From the diagram depicted in Fig. 3, we further analyze the possibility to identify a relationship in the measured data in the range of nominal stress levels from 495 to 649 MPa. This is depicted in Fig. 7. This data can be fitted via the polynomial regression function of 4th order. The dependency can be expressed both in direct $\varepsilon(\sigma)$ and in inverse form $\sigma(\varepsilon)$. The equations for all 6 measurement gauges are formulized in equations (6) and (7).

The real stress in the locations of measurement gauges are known and, similarly as in the linear case, these do not match the nominal stress brought to the specimen via breaker. However, the nominal stress takes a part in equations (6) and (7). Because the relationship between the deformation and stress is here nonlinear, deforms the strain the cross section area.

$$\left.\begin{aligned}
\varepsilon_1(\sigma) &:= 4.85299\times 10^{4} + -4.80304\times 10^{2}\cdot\sigma + 1.70593\times 10^{0}\cdot\sigma^{2} + -2.57334\times 10^{-3}\cdot\sigma^{3} + 1.41859\times 10^{-6}\cdot\sigma^{4}\\
\varepsilon_2(\sigma) &:= 5.68188\times 10^{4} + -6.32712\times 10^{2}\cdot\sigma + 2.40624\times 10^{0}\cdot\sigma^{2} + -3.8018\times 10^{-3}\cdot\sigma^{3} + 2.16729\times 10^{-6}\cdot\sigma^{4}\\
\varepsilon_3(\sigma) &:= 1.998\times 10^{5} + -1.88134\times 10^{3}\cdot\sigma + 6.41943\times 10^{0}\cdot\sigma^{2} + -9.44123\times 10^{-3}\cdot\sigma^{3} + 5.10213\times 10^{-6}\cdot\sigma^{4}\\
\varepsilon_4(\sigma) &:= -7.58598\times 10^{4} + 2.33242\times 10^{1}\cdot\sigma + 1.50565\times 10^{0}\cdot\sigma^{2} + -3.83066\times 10^{-3}\cdot\sigma^{3} + 2.70869\times 10^{-6}\cdot\sigma^{4}\\
\varepsilon_5(\sigma) &:= -1.25989\times 10^{5} + 6.80057\times 10^{2}\cdot\sigma + -1.12061\times 10^{0}\cdot\sigma^{2} + 3.97229\times 10^{-4}\cdot\sigma^{3} + 2.9811\times 10^{-7}\cdot\sigma^{4}\\
\varepsilon_6(\sigma) &:= 5.63147\times 10^{4} + -5.09349\times 10^{2}\cdot\sigma + 1.70668\times 10^{0}\cdot\sigma^{2} + -2.47814\times 10^{-3}\cdot\sigma^{3} + 1.33121\times 10^{-6}\cdot\sigma^{4}
\end{aligned}\right\}(6)$$

$$\left.\begin{aligned}
\sigma_1(\varepsilon) &:= -1.3742\times 10^{3} + 2.17357\times 10^{0}\cdot\varepsilon + -8.88038\times 10^{-4}\cdot\varepsilon^{2} + 1.6017\times 10^{-7}\cdot\varepsilon^{3} + -1.05028\times 10^{-11}\cdot\varepsilon^{4}\\
\sigma_2(\varepsilon) &:= -1.99683\times 10^{3} + 2.5862\times 10^{0}\cdot\varepsilon + -9.71722\times 10^{-4}\cdot\varepsilon^{2} + 1.63278\times 10^{-7}\cdot\varepsilon^{3} + -1.02435\times 10^{-11}\cdot\varepsilon^{4}\\
\sigma_3(\varepsilon) &:= -1.48363\times 10^{3} + 1.61826\times 10^{0}\cdot\varepsilon + -4.69594\times 10^{-4}\cdot\varepsilon^{2} + 6.03869\times 10^{-8}\cdot\varepsilon^{3} + -2.87324\times 10^{-12}\cdot\varepsilon^{4}\\
\sigma_4(\varepsilon) &:= -1.66695\times 10^{3} + 1.80863\times 10^{0}\cdot\varepsilon + -5.3742\times 10^{-4}\cdot\varepsilon^{2} + 7.0586\times 10^{-8}\cdot\varepsilon^{3} + -3.42673\times 10^{-12}\cdot\varepsilon^{4}\\
\sigma_5(\varepsilon) &:= -1.86112\times 10^{3} + 2.52011\times 10^{0}\cdot\varepsilon + -9.7392\times 10^{-4}\cdot\varepsilon^{2} + 1.6829\times 10^{-7}\cdot\varepsilon^{3} + -1.08488\times 10^{-11}\cdot\varepsilon^{4}\\
\sigma_6(\varepsilon) &:= -1.51341\times 10^{3} + 2.5775\times 10^{0}\cdot\varepsilon + -1.18654\times 10^{-3}\cdot\varepsilon^{2} + 2.45741\times 10^{-7}\cdot\varepsilon^{3} + -1.90135\times 10^{-11}\cdot\varepsilon^{4}
\end{aligned}\right\}(7)$$

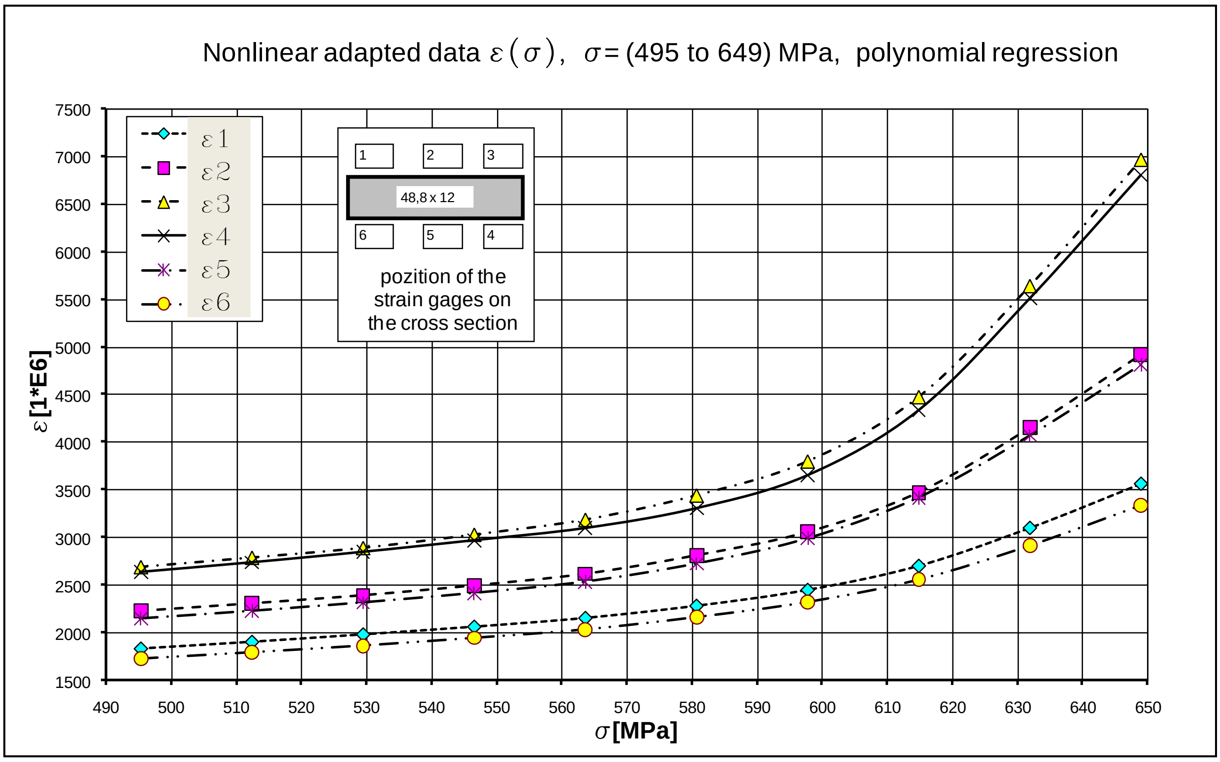

Figure 7: *The data part available for the nonlinear regression model*

The assumption for the next step is, that we start from the certain level of nominal stress σ, which value is close to linear threshold $(\varepsilon_{lin}, \sigma_{lin})$ as depicted in Fig 6. If there are at least six measurement gauges available for the axial deformation measurement on the boundary of the specimen cross section, it is possible to form a quadratic regression function for strain as in equation (8). This function contains six parameters d_0 to d_5, which can be computed from the deformations measured by the gauges at certain nominal stress level.

$$\varepsilon(z,y) = d_0 + d_1{\cdot}z + d_2{\cdot}z^2 + d_3{\cdot}y + d_4{\cdot}y^2 + d_5{\cdot}z{\cdot}y \tag{8}$$

Normalized regression function for the $\sigma^*(\varepsilon)$ is assumed to be in the same format as the regression functions for the nominal (virtual) stress given by (7) - in the format of 4th order polynomial (9).

$$\sigma^*(\varepsilon) = c_0 + c_1{\cdot}\varepsilon + c_2{\cdot}\varepsilon^2 + c_3{\cdot}\varepsilon^3 + c_4{\cdot}\varepsilon^4 \tag{9}$$

Distribution of the real axis stress load over the measured cross section $\sigma^*(z,y)= \sigma^*(\varepsilon(z,y),c_0, c_1, c_2, c_3, c_4)$ is then determined by the substitution of the equations (8) and (9), as the individual threads in the material have different deformation and to different deformation corresponds different stress level. For the unknown coefficients c_0, c_1, c_2, c_3, c_4 there are several conditions to satisfy. First of all there are boundary requirement that the transition from linear area must be smooth – here formed as equations (10) and (11) and depicted in Fig. 6. Addition ally the force equilibrium condition (12) for the cross section area and moment equilibrium condition for the measured cross section must be fulfilled. The moment conditions is composed in the independent direction of coordinates z and y of the cross section.

$$\sigma^*(\varepsilon_{lin}) = c_0 + c_1{\cdot}\varepsilon_{lin} + c_2{\cdot}\varepsilon_{lin}{}^2 + c_3{\cdot}\varepsilon_{lin}{}^3 + c_4{\cdot}\varepsilon_{lin}{}^4 = E{\cdot}\varepsilon_{lin} = \sigma_{lin} \tag{10}$$

$$\frac{d(\sigma^*(\varepsilon_{lin}))}{d\varepsilon_{lin}} = c_1 + 2{\cdot}c_2{\cdot}\varepsilon_{lin}{}^1 + 3{\cdot}c_3{\cdot}\varepsilon_{lin}{}^2 + 4{\cdot}c_4{\cdot}\varepsilon_{lin}{}^3 = E \tag{11}$$

$$\iint_A \sigma^*(\varepsilon){\cdot}dA = \sigma{\cdot}A = F \tag{12}$$

$$\iint_A \sigma^*(\varepsilon){\cdot}y{\cdot}dA = M_z \tag{13}$$

$$\iint_A \sigma^*(\varepsilon){\cdot}z{\cdot}dA = M_y \tag{14}$$

In this system of equations, the known value is the stress on boundary of the linearized data part σ_{lin}, the next variable is the Young's modulus of this data part, the cross section A and the nominal stress σ – determined by the loading force F induced by the tensile machines. Additional requirement is to measure via experiment both moment components M_z, M_y, which are not measured with standard tensile machines. With unknown moment components, the problem of material property evaluation would be infeasible.

$$\begin{aligned}\sigma^*(\varepsilon_k) &= c_0 + c_1{\cdot}\varepsilon_k + c_2{\cdot}\varepsilon_k{}^2 + c_3{\cdot}\varepsilon_k{}^3 + c_4{\cdot}\varepsilon_k{}^4 = \\ = \sigma^*(\varepsilon_{k+1}) &= c_0 + c_1{\cdot}\varepsilon_{k+1} + c_2{\cdot}\varepsilon_{k+1}{}^2 + c_3{\cdot}\varepsilon_{k+1}{}^3 + c_4{\cdot}\varepsilon_{k+1}{}^4\end{aligned} \tag{15}$$

$$\frac{d\left(\sigma^*(\varepsilon_k)\right)}{d\varepsilon_k} = c_1 + 2 \cdot c_2 \cdot {\varepsilon_k}^1 + 3 \cdot c_3 \cdot {\varepsilon_k}^2 + 4 \cdot c_4 \cdot {\varepsilon_k}^3 =$$
$$= \frac{d\left(\sigma^*(\varepsilon_{k+1})\right)}{d\varepsilon_{k+1}} = c_1 + 2 \cdot c_2 \cdot {\varepsilon_{k+1}}^1 + 3 \cdot c_3 \cdot {\varepsilon_{k+1}}^2 + 4 \cdot c_4 \cdot {\varepsilon_{k+1}}^3 \qquad (16)$$

Analogist approach could be used for the next step of nonlinear identification only when using equations (10) and (11) to modify equations (15),(16), which express the properties of the regression boundary(where k is the subscript of the measured data part). After a level by level mapping of the nonlinear properties behavior of the tensile diagram, the constants c_0, c_1, c_2, c_3, c_4 for the individual stress levels are computed. These enable for values of individual strain ε_i, which are measured by strain gauges, record into table value the corresponding loading stress. The resulting data set then can be finally approximate by function, which expresses the partial material behavior during the tensile test.

Acknowledgement

This research was supported by the grant of GA CR_101/09/1492:"The experimental method for the stress state identification".

References

[1] Vítek, K.- Mechanical properties of steel X60, 32th International conference Experimental Stress Analysis, EAN-1994, Janov nad Nisou, 1994, 201-203.
[2] Vítek, K., Holý S.- Identification of the tensile material test, 37th International conference Experimental Stress Analysis, EAN-1999,, Frenštát pod Radhoštěm, 1999, pp. 213-216.