Experimentální Analýza Napětí 2009
Experimental Stress Analysis

EXPERIMENTAL MEASUREMENT OF THE J INTEGRAL

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Abstract: Experimental measurement of the J integral on the thin planar specimen will be presented. It is possible measure J integral directly as curve intgral as it is defined. This aproach is not as comfortable as standard measurement on CT specimen but it is more appropriate for thin planar specimens.

1. Introduction

It is not appropriate to characterize the fracture behavior in meaning of Linear Elastic Fracture Mechanics (LEFM) and an alternative fracture mechanics model is required for majority of ductile metal materials. Elastic-plastic fracture mechanics applies to materials that exhibit time-independent nonlinear behavior (i.e., plastic deformation). *J* contour integral is widely used as fracture criterion for elastic-plastic materials. Critical value of J_c give nearly size-independent measure of fracture toughness, even for relatively large amounts of crack tip plasticity. There are limits to the applicability of *J* but these limits are much less restrictive than validity requirements of LEFM. *J* integral is equal to the energy release rate in a nonlinear elastic body that contains a crack. For the special case of a linear elastic material, $J=G$.

Let's suppose power law relationship between plastic strain and stress. At distances very close to the crack tip, well within the plastic zone, elastic strains are small in comparison to the total strain, and the stress-strain behavior reduces to a simple power law. A structure in small-scale yielding has two singularity-dominated zones; one in the elastic region, where stress varies as $1/r^{1/2}$ (*K* dominant region) and one in the plastic zone where stress varies as $r^{-1/(n+1)}$ (so called HRR singularity; named by Hutchinson Rice an Rosenberg). The *J* integral defines the amplitude of the HRR singularity, just as the stress intensity factor characterizes the amplitude of the linear elastic singularity:

$$J = \int_{\Gamma} \left(w dy - \sigma_{ij} n_{ij} \frac{\partial u_i}{\partial x} ds \right) \tag{1}$$

where w is the strain energy density, n_{ij} are components of the unit vector normal to Γ, u_i are the displacement vector components, and ds is a length increment along the contour Γ. The strain energy density is defined as

$$w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \tag{2}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, respectively. In the case of the small scale yielding, both *K* and *J* characterize crack tip conditions. Assuming monotonic, quasistatic loading, a *J* dominated region occurs in the plastic zone, where the elastic singularity no longer applies. Well inside of the plastic zone, the HRR solution is approximately valid. As the plastic zone increases in size, the *K* dominant zone disappears, but the *J* dominant zone persists in some geometries. With large scale yielding, there is no longer a region uniquely

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characterized by J and it's critical value exhibit a size and geometry dependence. Consequently fracture toughness depends on the size and geometry of the test specimen. [1].

The path independence of J can be established when the strain energy density of the material is a single-valued function of strain and the material is appropriately homogenous and the body forces are zero. In a deformation theory of plasticity, which is valid for proportional monotonic loading, but precludes unloading and thus is essentially and mathematically equivalent to a non-linear theory of elasticity), J still characterizes the crack-tip fields. However, in this case J does not have the meaning of an energy release rate; it is simply the total potential-energy difference between identical and identically (monotonically) loaded cracked bodies which differ in crack lengths by a differential amount [2]. For a stationary crack, the integral in (1) remains finite. For the elasto-plastic body, w is the total stress-work at a material point (per unit volume) as defined in (2).

As the crack begins to growth, there is no reason to expect that the crack-tip integral as defined in Eq. (1), which had been finite until growth initiation, would not continue remain finite. On the other hand, with elastic unloading, accompanying large amounts of growths and the consequent invalidation of a deformation theory of plasticity, one would expect that crack-tip integral, as defined in (1) would not remain path-independent by itself, without the presence of a domain integral [3].

2. Fracture Toughness Testing

A fracture toughness test measures the resistance of a material to crack extension. Such a test may yield either a single value of fracture toughness or a resistance curve, where a toughness parameter such as *K*, *J* or *CTOD* is plotted against crack extension. A single toughness value is usually sufficient to describe a test that fails by cleavage, because this fracture mechanics is typically unstable. There are several standardized specimens that characterize fracture initiation and crack growth: in the ASTM standard there are the compact specimen (CT), the single edge notched bend (SENB) geometry, the arc-shaped specimen, the disk specimen, and the middle tension (MT) panel [4]. Fracture toughness is calculated indirectly using one experimental parameter, such as COD for instace, and a simple linear equation which is based on quite strong assumptions. This approach is comfortable for engineering practice but can be risky if test results are extrapolated for different geometric and loading conditions presented in real structures, especially if a ductile material is used.

An evaluation of the fracture toughness from experimental data is done rather rarely; see for instance [5], [6]. This direct evaluation comes out from theoretical assumptions of selected fracture criteria and it is based on the full field measurement of the displacement/strain field and material constitutive equations. The evaluation is usually verified/supported by FEM simulations. The reason for the rare employment of this approach is its time consumption and high experimental demands. The advantage of this approach is that theoretical assumptions of selected fracture criteria are kept in mind and fundamental results can be obtained. Main advantage of this approach is possibility to verify whether theoretical assumptions of FM selected are fulfilled such as path independence of J integral for instance.

3. Direct J Integral Calculation Based on Experimental Data

Nowadays experimental methods provide full field measurement of the displacement/strain fields of the specimen analyzed. Stress fields and strain energy density can be calculated from the strain fields under knowledge of the material constitutive equations. Hook law for plane stress state utilizing tensile stress-stain record was used for this purpose in our work.

The digital image correlation method [7, 9] (DIC) was utilized for displacement field measurement. We are looking for self-similar places in a sequence of images acquired during the experiment utilizing this method. The DIC uses the following general procedure: a template surrounding the control point is extracted in the reference image for each control-point pair and in the target image at the same coordinates. A normalized cross-correlation of the templates is calculated for this start position and for positions surrounding this point. Finally, the absolute sub-pixel peak of the cross-correlation matrix is found using a second order polynomial surface. The peak position is used as coordinates of a new reference control point. The procedure as described is repeated for all images analyzed step by step. Each template has to cover a distinguishable structure pattern. Standard used DIC approach using Fast Fourier transformation works well for material pattern with very high contrast and low noise presented. Contrary DIC methodology [8] used in our work is much more robust.

Control points grid is defined as square and orthogonal with constant pitch p in our work, oriented parallel to the Cartesian coordinate system used. A displacement fields obtained from control points tracing are utilized for a consequent calculation of the fields of the strains ε_x, ε_y and γ_{xy} using Matlab function gradient. These strain components are employed for calculation of principal true strains ε_1 and ε_2. First principal strain ε_1 is identical with opening strain at crack tip (it is close to ε_y).

Only first two principal strains are needed for calculation of the plastic strain intensity using incremental theory of the plasticity [9] under plane strain state. First two principal strains are also used for calculation of the principal stress components and elastic strain density w. The deformation theory of the plasticity should be used for strict fulfillment of the J integral definition. Unfortunately third strain component has to be known for deformation theory application. On other hand the deformation theory is not right tool for characterizing of the high ductile specimen with the stress concentrator especially if crack grow is presented. It is reason that discrepancy between both theories for plastic strain calculation was neglected and incremental theory was used.

Let's suppose rectangle integration path for J integral calculation. Consequently the total J integral path G will be spited into four sub integration paths: vertical G_1, G_2 and horizontal G_3, G_4. Anticlockwise integration path has to be outside of the plastic zone and it has to be ended on the crack faces behind crack tip. Crack path formally crossing crack in our case has no influence on the results.

We are with principal stresses and strains in our work. Moreover stress-strain field is calculated in discrete points (similarly as by all other optical experimental methods except of photoelasticimetry). Consequently equation (1) has to be adapted into numerical integration form, as follows:

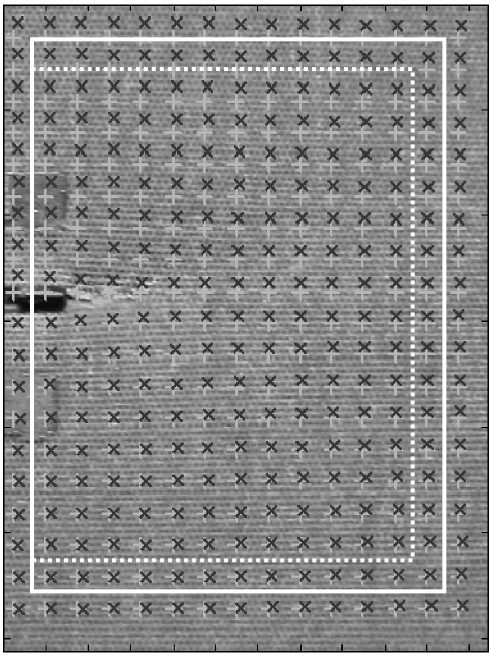
$$J = \sum_{G1,G3} \begin{pmatrix} w\Delta p_y - \sigma_1 \sin(\arctan\gamma_{12})\varepsilon_1 \Delta p_y \cos(\arctan\gamma_{12}) \\ -\sigma_2 \cos(\arctan\gamma_{12})\Delta p_y \cos(\arctan\gamma_{12}) \end{pmatrix} + \sum_{G2,G4} \begin{pmatrix} w\Delta p_x \sin(\arctan\gamma_{12}) - \sigma_1 \sin(\arctan\gamma_{12})\varepsilon_1 \Delta p_x \cos(\arctan\gamma_{12}) \\ -\sigma_2 \cos(\arctan\gamma_{12})\gamma_{12}\Delta p_x \cos(\arctan\gamma_{12}) \end{pmatrix} \quad (3)$$

Where Δp_x and Δp_y have meaning of the actual grid pitch in x and y directory. Equation (3) can be simplified as $\tan(\gamma_{12}) \cong \cos(\gamma_{12}) \cong \gamma_{12}$ for low angles typical for this integral path definition as follows:

$$J = \sum_{G1,G3} \left(w\Delta p_y - \sigma_1 \sin(\gamma_{12})\varepsilon_1 \Delta p_y \gamma_{12} - \sigma_2 \Delta p_y \gamma_{12}^{\ 2} \right) + \sum_{G2,G4} \left(w\Delta p_x \sin(\gamma_{12}) - \sigma_1 \sin(\gamma_{12})\varepsilon_1 \Delta p_x \gamma_{12} - \sigma_2 \Delta p_x \gamma_{12}^{\ 2} \right) \quad (4)$$

4. Experimental

The high ductile Al-alloy flat, 5 mm thick specimen with premachined slit was used for our experiment. Photoresist structure was prepared on the specimen surface for DIC purpose. The specimen was loaded in tension by compact loading device [10] under condition of constant displacement velocity. Surface deformations were recorded by the 6 MPixel still Canon camera. Measuring DIC grid with the 150 pixels pitch was generated and displacement/strain fields were calculated in a center of each cell defined by four corner grid points. Photography of the specimen surface with 2 mm length crack grew is depicted in Fig. 1. Actual grid point's positions are plotted in black. Original point's positions are labeled by gray color.



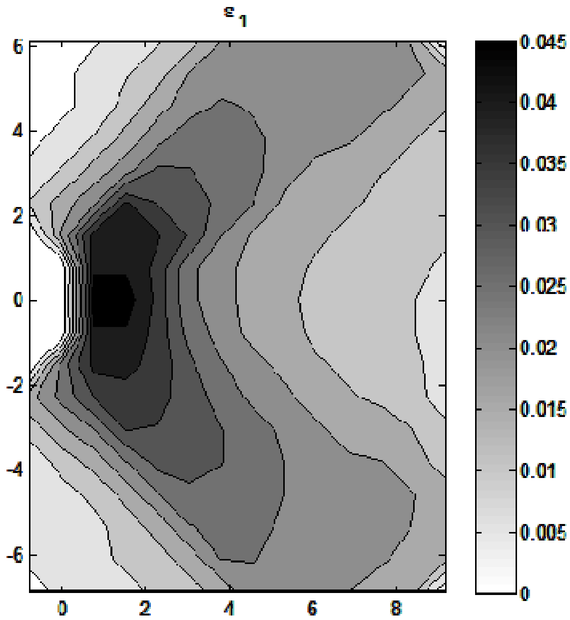


Figure 1: *Specimen surface with DIC grid* ***Figure 2:*** *First principal strain* ε_1.

The principal strains calculated for image 30 are imaged in Figure 2 and 3, first very short crack advancing was observed and loading force reached its maximum at this loading level. Axes have mm scale, coordinates [0,0] are at the crack (slit) tip. Corresponding strain energy density is depicted in Figure 4 and plastic strain intensity at Figure 5.

Development of the *J* integrals during increasing loading for two integration paths are plotted at Figure 6. First integration path was chosen to be going through centers of boundary cells (full line). Second path was one grid pitch closer to the crack tip. These paths are plotted in Figure 1. Corresponding two *J* integrals are slightly different; it means that condition of the path independence was lost. Reason for this fact is very intensive plastic strain intensity and large plastic stain area; See Fig. 5. It was impossible to avoid plasticized areas during *J* integral calculation thanks to relatively small specimen surface area recorded. This *J* integral measured does not fulfill basic conditions supposed by the theory. It is not path independent; consequently it will be necessary to observe bigger area in the next experiment to avoid area plasticized. Nevertheless last five points can be qualitatively taken as JR curve measurement because stable crack growth was observed for these points.

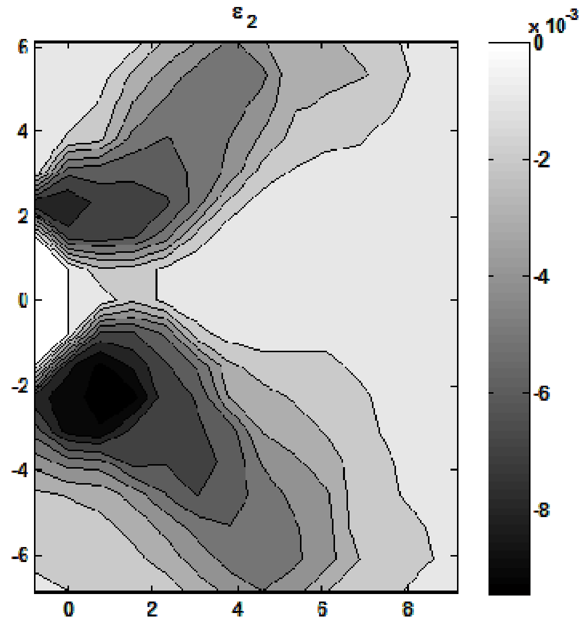


Figure 3: *Second principal strain ε_1.*

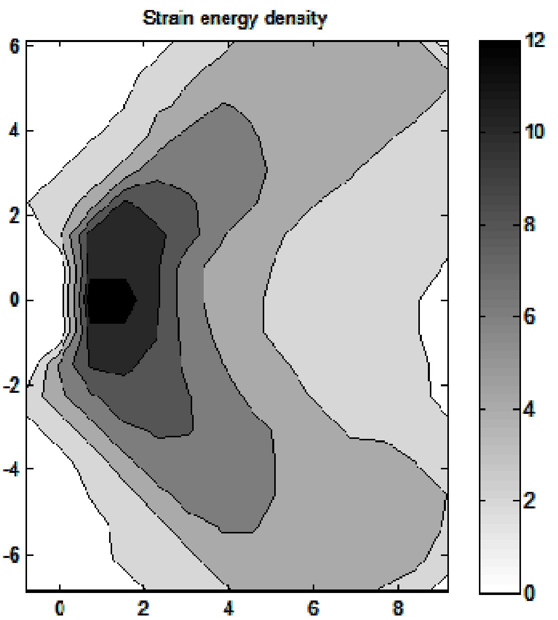


Figure 4: *Strain energy density*

During standardized measurement of *J* integral on thick CT specimen under plane strain condition no stable crack growth was observed and in contradiction with the experiment presented no intensive plasticization occurred. J_c was measured as to be 65 Jmm^{-1} by the standardized measurement. *J* integral has value 30 Jmm^{-1} in our experiment for the first very short stable crack advance. Similar *J* integral value as for normalized measurement was obtained for new 2 mm crack advanced. The crack growing is preceded by the damage zone development as it was radiographically documented [10].

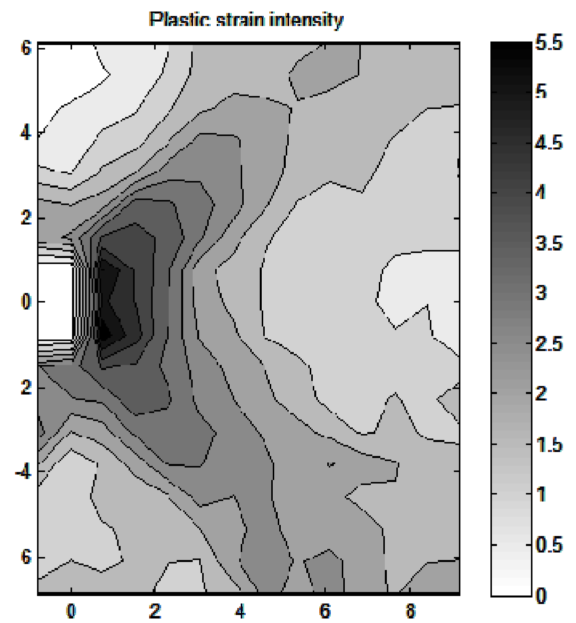


Figure 5: *Plastic strain intensity*

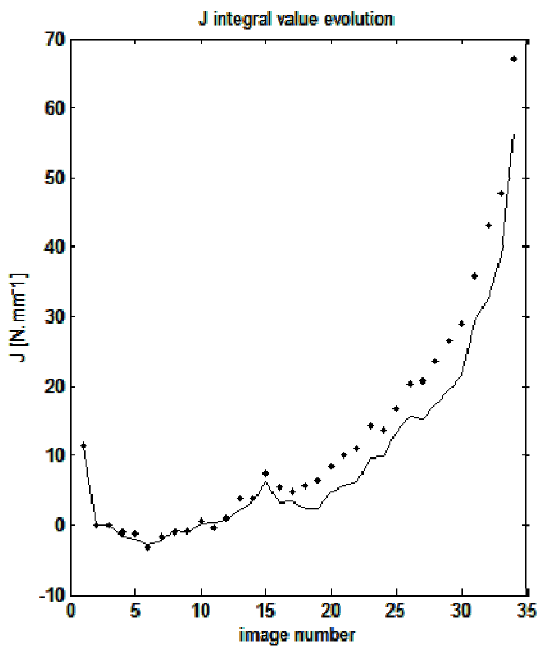


Figure 6: *Comparison of two J integrals calculated through different paths. Integration was done on the boundary for full line, dotted line depict path one grid point closer to the crack tip.*

5. Conclusions

- It was qualitatively shown that measurement of the *J* integral as path integral by its definition is possible and it can be beneficial. Value obtained will be different for different specimen thicknesses although specimen planar geometry remain the same.
- It is possible to measure *JR* curve as well by the method presented.
- Further experiments are needed when bigger area will be observed to avoid plasticized are during path integration.
- Such extended plasticization is not indicated by the standardized measurement. Consequently false conclusions can be obtained if standardized measurement on very thick CT specimen is applied for relatively thin flat specimen.

Acknowledgement

This work has been supported in part by Research Grant No. 103/09/2101 of the Grant Agency of the Czech Republic and by the Ministry of Education, Youth and Sports of the Czech Republic under Research Project AV0Z20710524.

References

[1] Anderson T.L.: *Fracture mechanics: Fundamentals and Application*s, CRC Press, Boca Raton, 1995

[2] Atluri, S.N., "Computational methods in the mechanics of fracture", Vol.2 in Computational Methods in Mechanics, Elsevier Science Publishers, 1986

[3] Broberg, B.: "Cracks and Fractue", Academic Press, London 1999, ISBN 0-12-134130-5

[4] May, G. B., Kobayasihi, A. S.: "Plane stress stable Crack Growth and *J*-integral/HRR field, Int. J. Solids Structures, Vol. 32, No.6/7, 1995, pp.857-881

[5] Read, D.T., "Applied J-Integral in HY130 Tensile Panels and Implications for Fitness for Service Assessment", Report NBSIR 82-1670, National Bureau of Standards, Boulder, CO, 1982

[6] Okada, H., Atluri, S.N., Omori, Y., Kobayashi, A.S.: "Direct evaluation of T*e integral from experimentally measured near tip displacement field, for a plate with stably propagating crack", International Journal of Plasticity, Vol. 15, 1999, pp. 869-897

[7] W.H. Peters and W.F. Ranson, Digital imaging techniques in experimental stress analysis, Opt Eng 21, (1982), pp. 427–431.

[8] Jandejsek, Ivan ; Vavřík, Daniel. Material Characterization using Digital Image Correlation. In Book of Contributions of 46th International Scientific Conference "EXPERIMENTAL STRESS ANALYSIS 2008". Ostrava : VŠB - Technical University of Ostrava, Faculty of Mechanical Engineering, Department of Mechanics of Materials, 2008. S. 107-110. ISBN 978-80-248-1774-3

[9] Vavřík, D.; Zemánková, J. (2004) Crack Instability in Ductile Materials Analyzed by the Method of Interpolated Ellipses, Experimental Mechanics, Vol. 44, pp. 327-335

[10] Vavřík, D.; Jandejsek, I.; Jakůbek, J.; Jakůbek, M.; Holý, T. (2008) Microradiographic Observation of the Strain Field in Vicinity of the Crack Tip. In 17th European Conference on Fracture. ISBN 978-80-214-3692-3, CD-ROM