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Experimental Stress Analysis

RESIDUAL STRESS EVALUATION ACCORDING ASTM E 837-08 REVISION

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Abstract: The contribution deals with measurement of residual stresses on large shaft forgings with respect to the stress evaluation along the depth. The procedure for residual stress evaluation according revision of ASTM E837-08 is described in more detailed way, which enables to evaluate the residual stress along the depth. Integral method according Schajer and smoothing of the residual stress profile using the regularization method according Tichonov is used in this standard. Comparison of some methods for evaluation of the residual stress profile using hole drilling method is provided on an example.

1. Introduction

Identification of residual stresses in the structure is very important for estimation of the structure service or residual service life. The hole-drilling method is an effective technique for solving of this task. In-plane residual stresses can be identified near the measured surface of the workpiece material using this method. In many cases the determination of surface residual stresses is used indirectly for checking of the core residual stresses of large forgings induced by heat treatment process. The best solution, how to avoid the influence of parasitic residual stresses induced often by machining, is to evaluate the stresses under the surface, where these stresses are negligible. The other solution - measurement after surface stress lowering with find stress-free machining - is very expensive procedure for the producer.

Support of service tests with some standard is very important. Traditionally the hole-drilling method was used for measuring only uniform residual stresses with existing standard ASTM E837-01. However, all has been changed with standard revision in 2008 [1], which involves the integral method according Schajer [2] where the evaluated residual stress profile is smoothed using the regularization method according Tichonov [3].

2. Theory of integral method

In the hole-drilling method the residual stresses are calculated from the strains $\varepsilon(h)$ relaxed on the surface at drilling depth h, which are proportional to the integral of residual stresses at the depth H weighted by means of influence functions $\widehat{A}(H,h)$ (uniform bi-axial stress) and $\widehat{B}(H,h)$ (pure shear stress), determined by numerical methods (1) (here the equation is given for biaxial uniform stress).

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In practice, the relaxation response is measured at hole depths $h_i = 1, 2,..,n$, thus (1) can be approximated in discrete form (2) [2].

$$\varepsilon(h)=\frac{1+\nu}{E}\int_0^h \hat{A}(H,h)\sigma(H)dH \qquad 0\le H\le h \tag{1}$$

$$\varepsilon(h_i)=\frac{1+\nu}{E}\sum_{j=1}^{j=i}\bar{a}_{ij}\sigma_j \quad \bar{a}_{ij}=\int_{H_{j-i}}^{H_j}\hat{A}(H,h_i)dH=\hat{A}(H_j,h_i)-\hat{A}(H_{j-1},h_i) \tag{2}$$

For calculation with general non-uniform stress field, there is proposed in [2] to work with transformed strain and stress values according to following relations (3)

$$p=(\varepsilon_3+\varepsilon_1)/2 \qquad q=(\varepsilon_3-\varepsilon_1)/2 \qquad t=(\varepsilon_3+\varepsilon_1-2\varepsilon_2)/2 \tag{3}$$

$$P=(\sigma_3+\sigma_1)/2 \qquad Q=(\sigma_3-\sigma_1)/2 \quad T=\tau_{13} \quad , \tag{4}$$

where ε_1, ε_2, ε_3 are relaxed strains and σ_1, σ_2 , τ_{13} are normal and shear stresses at rectangular rosette directions. The solution for stress components (4) within each depth increment can be expressed in following matrix notation (5)

$$\bar{\mathbf{a}}\ \mathbf{P}=E/(1+\nu)\ \mathbf{p} \qquad \bar{\mathbf{b}}\ \mathbf{Q}=E\ \mathbf{q} \qquad \bar{\mathbf{b}}\ \mathbf{T}=E\ \mathbf{t} \quad , \tag{5}$$

where $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$ are triangular matrices of coefficients a_{ij} and b_{ij}, calculated according (2) from tabulated discrete values of influence functions, given in [2] according used non-dimensional hole depth and mean rosette grid diameter; p, q, t and P, Q, T are strain and stress vectors, including values for all hole depths. Evaluation of principal stresses and their direction β is performed according following relations (6)

$$\sigma_{\max},\sigma_{\min}=P\pm\sqrt{Q^2+T^2} \qquad \beta=\frac{1}{2}\arctan\left(\frac{-T}{-Q}\right) \tag{6}$$

The residual stress matrices are numerically ill conditioned and leads to unstable residual stress solution. The distribution of calculation steps depend on the strain error sensitivity that grows as the depth from the surface increases and abruptly when using more than 8 calculation steps. Frequently, the experimental hole depth differ from the numerical ones, so that the influence coefficient calculation requires a bivariate interpolation technique, which is the source of errors in the computed stresses.

Several methods have been proposed for reducing these errors. Zuccarello [4] has calculated the optimum distribution of drilling depth based on the fact to have the coefficients of influence function of comparable sizes for each depth increment. Petrucci and Zuccarello [5] have proposed an improved spline method, in which as the influence function as the residual stress field are approximated by polynomial splines. Schajer [1], [3] used Tichonov regularization to smooth the residual stress profile calculated with integral method.

3. Integral method with residual stress smoothing

The ASTM E837-08 standard revision has been expanded in paragraph 10 for computation of non-uniform residual stresses using last mentioned Schajer’s method. Here, the ill conditioning of the coefficient matrices is ameliorated using Tichonov regularization, which is commonly used for stabilization of inverse calculations. It involves applying penalty function using curvature as a target. This procedure modifies equations (5) to

$$\left(\bar{\mathbf{a}}^{\mathrm{T}}\bar{\mathbf{a}}+\alpha_P\mathbf{c}^{\mathrm{T}}\mathbf{c}\right)\mathbf{P}=\frac{E}{1+\nu}\bar{\mathbf{a}}^{\mathrm{T}}\mathbf{p} \qquad \left(\bar{\mathbf{b}}^{\mathrm{T}}\bar{\mathbf{b}}+\alpha_Q\mathbf{c}^{\mathrm{T}}\mathbf{c}\right)\mathbf{Q}=E\bar{\mathbf{b}}^{\mathrm{T}}\mathbf{q} \qquad \left(\bar{\mathbf{b}}^{\mathrm{T}}\bar{\mathbf{b}}+\alpha_T\mathbf{c}^{\mathrm{T}}\mathbf{c}\right)\mathbf{T}=E\bar{\mathbf{b}}^{\mathrm{T}}\mathbf{t} \ , \tag{7}$$

where **c** is tri-diagonal “curvature” matrix, in which the number of rows equals the number of hole depth steps. All except first and last row have [-1 2 -1] centred along the diagonal. The factors α_P, α_Q, α_T control amount of applied regularization, with zero values the equations are un-regularized, increasing positive factor values the smoothing effect is increased; typical values are in the range 10^{-6}.÷ 10^{-2}. For choosing the optimum factor values Morozov criterion is used according following iteration procedure. First, regularized stresses are initially calculated according (7) with estimated factors value equal 10^{-5}. Then these stresses are substituted back to the un-regularized equations (5), giving strains $\mathbf{p}^*$, $\mathbf{q}^*$, $\mathbf{t}^*$, which differ from the original measured strains **p**, **q**, **t**. The difference between each pair of strain vectors is misfit, which is acceptable when laying within the experimental errors in the strain measurement. Here applied Morozov criterion requires the root mean square of the misfit to be equal to the standard error in the strain measurement. A numerical procedure is used in the standard for estimation of these errors based on approximation of each four neighbourhood strain points by parabolic function just determining the noise as the deviation of parabolic line, which procedure is not presented here.

The root mean square of misfit should be within 5% of standard errors of strain measurement. To ensure this condition an iteration procedure is proposed in the standard.

There are some changes in ASTM E837-08 in comparison with original integral method, described by Schajer [2]. In original paper [2], the influence functions $\hat{A}(H,h)$ and $\hat{B}(H,h)$ were tabulated in triangular matrices for ten dimensionless depths h and H and for three dimensionless rosette grid mean radiuses. An approximation procedure has been proposed for estimating the non-tabulated coefficients here, too. Coefficients a_{ij} and b_{ij} were then calculated as the difference of influence function values according (2).

In the revised standard, the triangular matrices with coefficients a_{ij} and b_{ij} are directly expressed without tabulating the influence functions. Twenty rows and columns of the matrices are related to the absolute value of depth up to 1 mm with 0,05 mm steps. Only one matrix is given for three rosette types. The matrix is valid only for 1/16 in. (5.13 mm) rosette grid mean radius and 0.080 in. (2 mm) hole radius. The rule is given here, how to recalculate the matrices for another rosette size or hole diameter.

It should be also mentioned here, that standard revision requires in comparison with old version to calculate and give to the test report normal *xy*-stresses also for the case of uniform stress, evaluated using power series method.

4. Using standard revision for residual stress measurement in service

4.1. Used tested piece and measuring instrumentation

The standard service test was made on rotor forging, made from 28CrMoNiV59 vacuum degassed steel, quenched and tempered, machined and stress relief annealed (normalizing and tempering), see Figure 1. The surface was turned before measurement in standard way. The relaxed strains were measured using device common to VISHAY RS - 200 with special 2-edge eccentric mill with the diameter 4 mm. Strain gauge rosette HBM 3/120 RY21 with grid mean diameter 13 mm was used.

The drilling was made in 20 equal steps up to 2 mm. Some irregular steps were made up to 5 mm depth.

Procedure for coefficients approximation, evaluating and smoothing residual stresses was made inside MS Excel. This is suitable tool for matrix calculations and iteration procedure using solver for searching of optimum values for regularization factors α_P, α_Q, α_T.

4.2. Data evaluation

The percentage of combination strains *p* and the larger of *q* and *t* related to their values at maximum hole depth have to be plotted before data evaluation according the standard ASTM E837-08 to find out, if the stresses are uniform within the hole depth or not. The deviation between plotted charts and typical chart for uniform stress should be less than 3 %. For uniform stress, the evaluation should be performed according power series method (paragraph 9), otherwise the integral method have to be used (paragraph 10).

Corresponding percentage plot of released strain *p* in comparison with that derived for uniform stress including deviation from uniform stress is made in Figure 2. There is obvious, that the distribution of residual stresses under the surface is strongly affected with the depth and should be evaluated using integral method.

However, according the old standard, there was only possible to evaluate the residual stresses with power series method. If the evaluated values exceed the allowable limit, the surface was always first released from potential machining stresses with fine turning and then the test was repeated. If still the stresses remained high after this procedure, the heat treatment of the shaft had to be repeated.

The same procedure was applied here. The residual stress evaluation using power series method at standardized equal depths is given in Figure 3 before and after fine machining of shaft surface. The residual stresses drop under the allowable value of 60 MPa after fine machining. However, the new standardized integral method enables to avoid the fine machining just evaluating the stress under the surface, which is presented in the next text.

Except of standardized method (data smoothing from 20 steps), the evaluation is also made for six non-equal depths with the original integral method (error optimization according Zuccarello [4]). Following depths were used: <0.3, 0.7, 1.0, 1.5, 2, 3.0> mm (Figure 7 – opt.).

Following regularized factors were obtained applying the standardized integral method for 20 steps depths of 0.1 mm: $\alpha_P = 6\cdot10^{-7}$, $\alpha_Q = 5\cdot10^{-5}$, $\alpha_T = 3\cdot10^{-4}$. The regularized curves of released values *p, q, t* together with non-regularized measured discrete values are given in Figure 4. The same is made for evaluated stress components *P*, *Q* and *T* (Figure 5).

The comparison of evaluated non-regularized and regularized principal stresses for both original and fine grinding surface is given in Figure 6. Finally, the comparison of evaluated principal residual stresses using power-series method, optimized integral method and smoothed integral method is made in Figure 7.

How due to the value of regularization factor the stress is smoothed is shown for the case of residual stress component *T* and α_T in Figure 8.

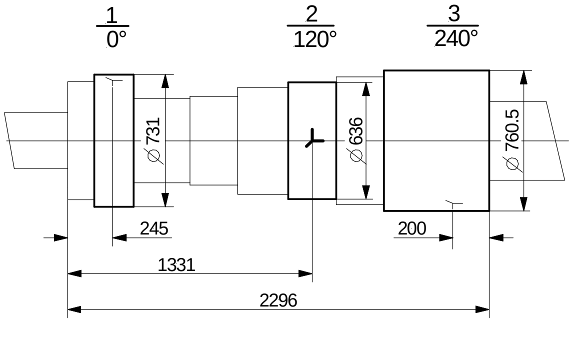

Figure 1*: Schematic view of tested shaft (data presented from position 3).*

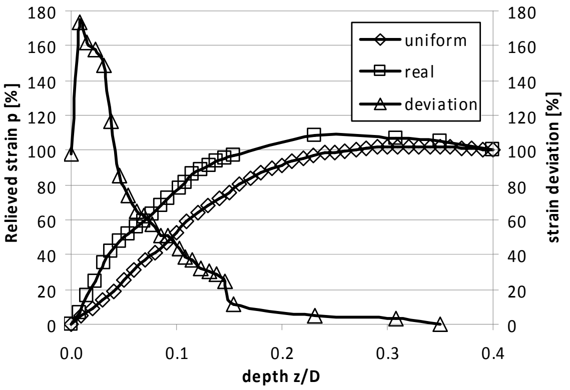

Figure 2: *Percentage relieved strain p and deviation from strain caused by uniform stress.*

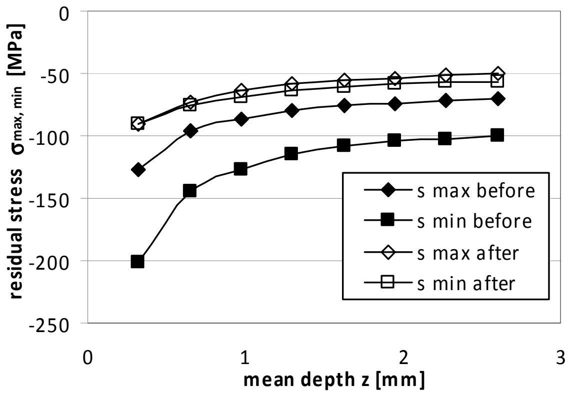

Figure 3: *Residual stress before and after fine machining (power series method).*

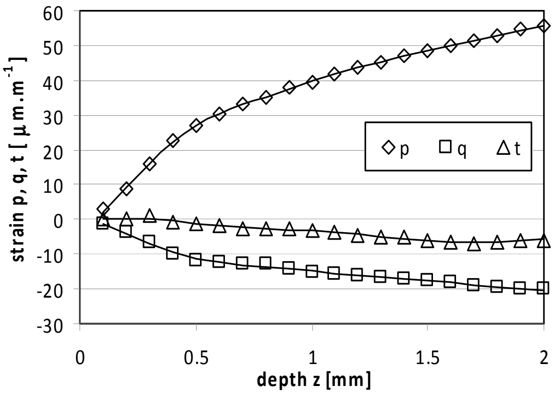

Figure 4: *Relieved original (marks) and regularized (curves) strains p, q, t.*

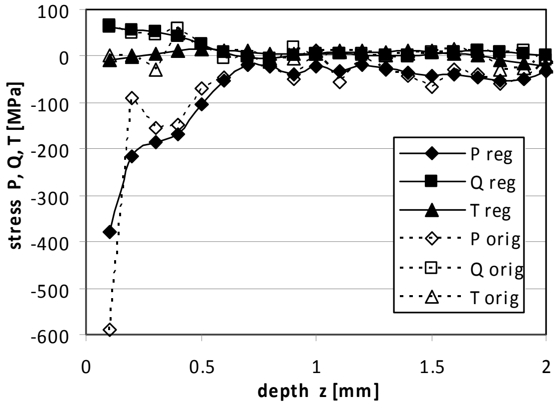

Figure 5: *Original and regularized residual stresses P, Q, T.*

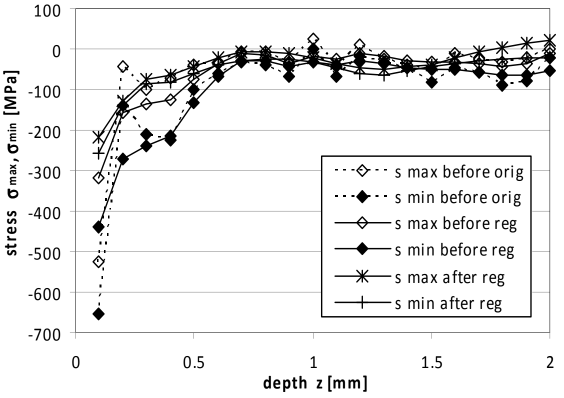

Figure 6: *Original and regularized principal stresses.*

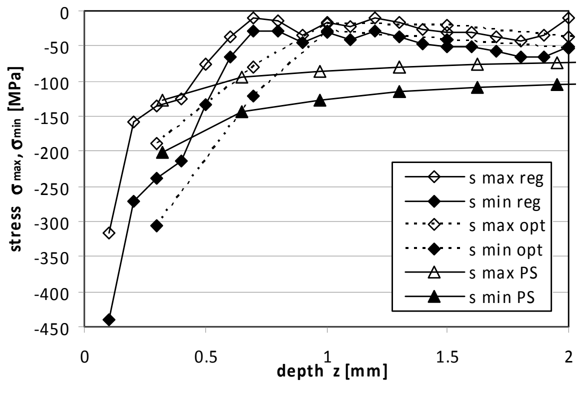

Figure 7: *Original and regularized residual stresses P, Q, T.*

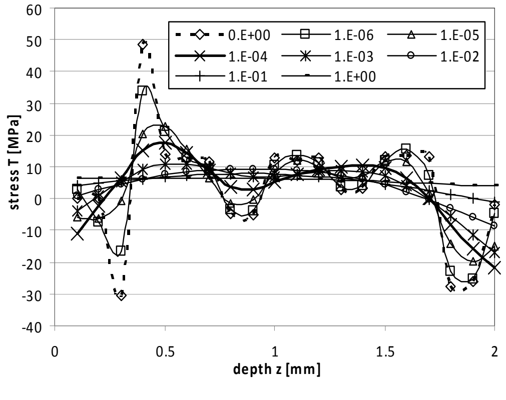

Figure 8: *Original and regularized stresses T for several values of* α_T.

4.3. Discussion

The difference between original strains and those, backwards calculated after regularization, is very low, as it is seen from Figure 4. On the other hand, some difference is seen between original and regulated residual stresses, especially closely to the surface. The main influence for high principal stresses has the stress component P for which the regularization factor is beyond expectation low in comparison with the other factors.

Residual stresses calculated for the identical depths are the same as for the power series method as for the non-regularized integral method. However after regularization, this first depth stress is substantially lowered (see Figure 6).

Very positive founding is, that the regularized residual stresses at the depth around 1 mm and deeper are comparable with those, computed for 6 optimum depths according Zuccarello (Figure 7).

Evaluation of residual stresses using power series method gives unrealistically high stresses.

Using integral method it is possible to determine the residual stresses under the surface, which is obvious from the Figure 6. No fine machining would be necessary to decide, that the residual stresses are under allowable limit of 60 MPa. The influence of the machining goes to 0,75 mm depth, which is evident after comparison of the residual stress profiles before and after surface fine machining.

5. Conclusion

The revised standard ASTM E837-08 seems to be rather complicated for common user, but the method has full functionality for evaluating profile of residual stresses up to 2 mm under the surface with stress smoothing.

The usage of the standard in this area is determined for more steps of drilling and data smoothing. Standard integral method for low steps even if made along optimized non-constant depth steps is not allowed. Nevertheless the last mentioned method gives comparable results, which was presented in this article.

The expected correction of influence elastic-plastic deformation to evaluated residual stresses has again not been included to the revised standard.

Acknowledgments

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