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EXPERIMENTAL ANALYSIS OF DEFORMATIONS AND STRESSES IN SHEETS WITH ANISOTROPY DURING PLASTIC DEFORMATION

František Šimčák [1], František Trebuňa [2] & Vladislav Berinštet [3]

Abstract: Sheet is a semi-finished product widely used to production of thin-walled supporting members. Typical example is automotive industry, where are produced the main parts of coachwork by stamping from cold rolled sheets that have anisotropic properties in plastic range. The paper is oriented to using of biaxial tensile test for the analysis of deformation and stress states during plastic deformation of cold rolled sheets.

1. Introduction

Thin-walled supporting elements are used for design of structures and their effectivity is influenced mainly by material properties, structural design as well as production technology. Though the fact that recently is steel replaced by other materials (aluminium, high-strength alloys, plastics, composite materials), the steel remains today the most important material for the production of thin-walled elements in machinery. As a semi finished products are often used cold rolled sheets (commonly with thickness to 2 mm) that are during production of final products cold formed. Typical example is automotive industry, where were recently realized number of innovations with the aim to increase their carrying capacity and decrease of mass [14]. One possibility to reach this aim is introducing new steels that on one side increase strength properties of products and on the other side they permit good processing properties of sheets during cold forming.

For the processing of material by cold forming and also for loading of thin-walled elements is necessary to know the behaviour of material during transition from elastic to plastic area. It is necessary to know the conditions under which the material of sheet comes to plastic state as well as analysis of deformations and stresses during evolution of plastic deformations. It is known that the cold rolled sheets during plastic deformation demonstrate plastic anisotropy. It is invoked texture sheet anisotropy that is caused by production technology [6]. The conditions of material plasticity with plastic anisotropy are defined analytically by many authors [1,8]. For determination of deformations and stresses in elastic and plastic areas are used numerical methods, mainly the finite element method. It is considered simplified model of isotropic material or an anisotropic model with application of suitable plastic theories. For experimental evaluation of sheet behaviour in plane stress case in plastic area is often used biaxial tensile test [9,11].

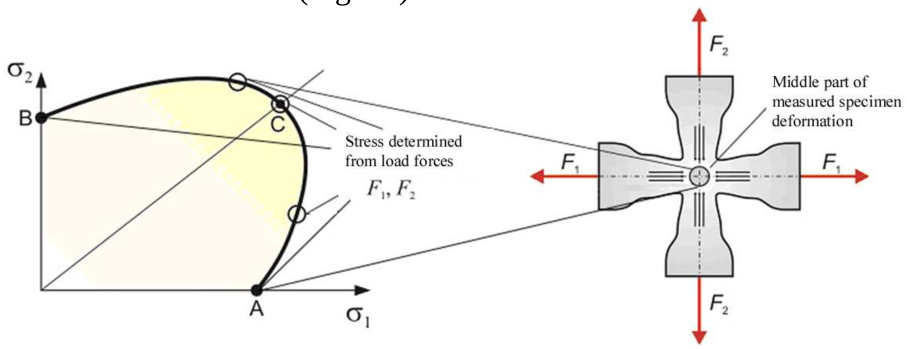
[1] Prof. Ing. František Šimčák, CSc.; Technical University in Košice, Department of Applied Mechanics and Mechatronics, Letná 9, 04082 Košice, Slovakia, frantisek.simcak@tuke.sk
[2] Dr.h.c. mult. Prof. Ing. František Trebuňa, CSc.; Technical University in Košice, Department of Applied Mechanics and Mechatronics, Letná 9, 04082 Košice, Slovakia, frantisek.trebuna@tuke.sk
[3] Ing. Vladislav Berinštet, PhD.; Technical University in Košice, Department of Applied Mechanics and Mechatronics, Letná 9, 04082 Košice, Slovakia, vladislav.berinstet@tuke.sk

On the workplace of authors was built experimental laboratory for realization of biaxial tensile tests. The laboratory was scaled up and modernized which allows providing tests under various loading conditions with using new measurement procedures [3,13].

In the contribution is given the knowledge that was gained during evaluation of plastic behaviours of cold rolled steels in plane stress state. For the analysis were used steels for automotive industry produced by US STEEL Košice.

2. Principle of biaxial tensile test

For experimental determination of plastic curves with coordinates $\sigma_1 \geq 0$, $\sigma_2 \geq 0$ (Fig. 1a) can be used biaxial tensile test (Fig. 1b).



a) points on the plastic curve b) cruciform specimen

Figure 1: *Experimental determination of plastic curves by biaxial tensile test.*

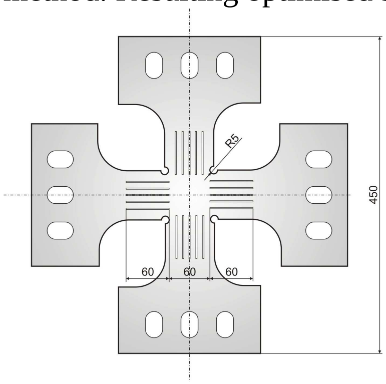
Biaxial tensile test was developed with the aim to model stress states in whole first quadrant ($\sigma_1 \geq 0$, $\sigma_2 \geq 0$). Cruciform specimen is loaded by tension in two perpendicular directions (Fig. 1b) and the plastic deformation is measured in middle part of cruciform specimen. In various states of plane stress (different ratios of tension stresses) which are ensured by change of load forces can be determined points of plastic curve in the whole first quadrant. Advantage of the test is that it is universal for all sheet types and it is possible to determine strengthening of material.

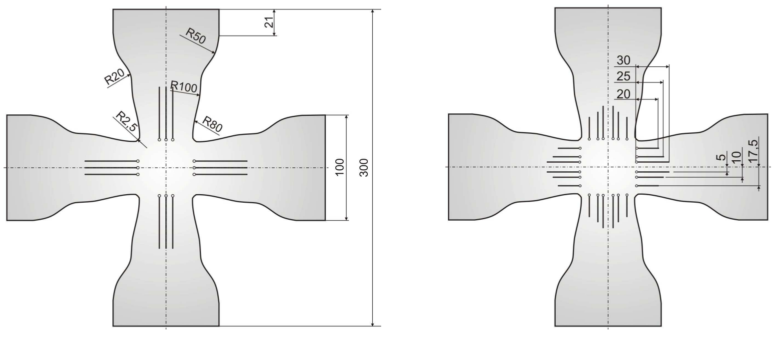
The geometry of specimen, stress and strain distribution in location of deformation measurement as well as method of stress computation in specimen from the loading forces has great impact on results of cross test and their representation. Because at present do not exist any norm (or directive) for determination of the shape and dimensions of cruciform specimens, most laboratories create their own conditions. Geometry of specimen has to be chosen with the aim to reach maximal area of homogeneous stress in middle part of specimen, minimal influence of shear stress invoked by load, uniform stress and strain distribution in middle part of specimen as well as minimal stress differences in middle part in comparison to nominal stresses gained by dividing of load forces by cross-section area in the most narrow part of specimen's arm.

Fulfilling of above-mentioned demands is realized by reducing of middle specimen's part in comparison to arm thickness (it ensures load transfer without any problems), or by creating of notches in location of transition from middle part to arms, which on the other hand causes stress concentration and increases risk of premature specimen damage. Longitudinal notches in arms decrease undesirable loadings in middle part caused by shear and bending and they improve homogeneity of stress distribution [9]. Computation of stresses in the middle (measured) part from the load forces differs for various authors. Some authors use for determination of stresses the finite element methods, the other define effective cross-sections which depend on loading or on ratio of stresses σ_1, σ_2, or they consider only cross-section area of arms [4,5,10,12]. For the specimens with longitudinal notches in arms can be used for stress computations the cross-section area of specimen's middle part [9].

3. Design proposals of cruciform specimens by the finite element method

The initial shape for cruciform specimen was proposed according to Grandlund [7], (Fig.2). Shape of this specimen ensured appropriate width of arm for its safe fastening into the jaws as well as appropriate functional cross section of specimen. The authors have loading equipment for such experiments and so it was necessary to propose the dimensions of specimen in order to use it for experimental evaluation of sheet plastic properties (Fig.3a). Longitudinal notches in arms improve uniformity of stress distribution and they define effective cross-section in the middle part. In order to unequivocally determine stress magnitudes in the middle area of specimen from the magnitudes of loading forces, it was necessary to determine number and positions of notches along arm width. Optimisation of magnitude and position of notches in the arm of specimen was accomplished by finite element method. Resulting optimised shape of cruciform specimen is given in Fig. 3b.





a) b)

Figure 2: *Cruciform specimen proposed by Grandlund.*

Figure 3: *Chosen shape of cruciform specimen. a) initial shape of notches, b) optimised shape of notches.*

Distribution of stresses was determined in the middle area of specimen with diameter 30 mm for force ratios $F_x{:}F_y=1$, $F_x{:}F_y=2$ and $F_y{:}F_x=2$ in arms of specimen for plastic strains ε_S^p=0,002, 0,005, 0,01, 0,02 and 0,03 in the middle of specimen.

Distribution of stresses σ_x, σ_y and plastic strains ε_x^p, ε_y^p in the middle part of specimen was determined by coefficients k_x, k_y, g_x, g_y according to relations

$$k_x = \frac{\sigma_x}{\sigma_{x0}}, \quad k_y = \frac{\sigma_y}{\sigma_{y0}}, \qquad \sigma_{x0} = \frac{F_x}{A_0} = \frac{F_x}{b \cdot t}, \sigma_{y0} = \frac{F_y}{A_0} = \frac{F_y}{b \cdot t}, \tag{1}$$

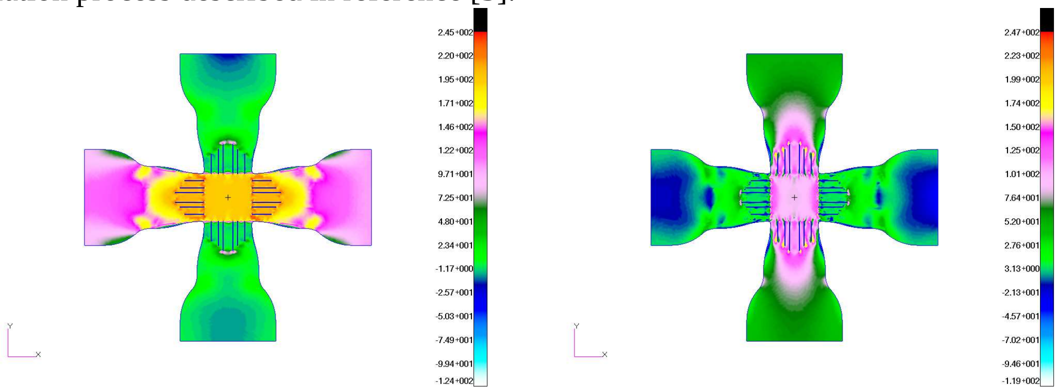
where b is minimum width of arm of cruciform specimen,
t - width of cruciform specimen.

$$g_x = \frac{\varepsilon_x^p}{\varepsilon_{Sx}{}^p}, \quad g_y = \frac{\varepsilon_y^p}{\varepsilon_{Sy}{}^p} \tag{2}$$

and $\varepsilon_{Sx}{}^p$, $\varepsilon_{Sy}{}^p$ are plastic strains in the middle of cruciform specimen.

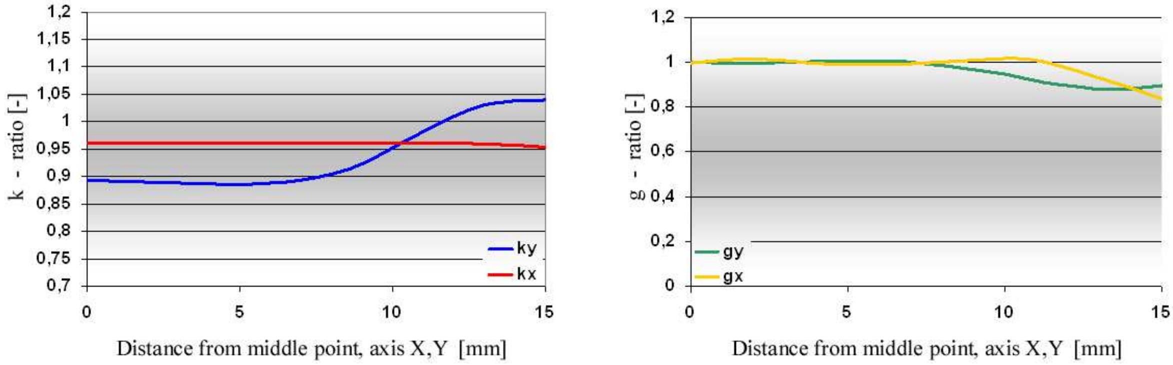
In Fig. 4a are illustrated typical stress distribution σ_x, σ_y in specimen according to Fig. 3b for forces ratio $F_x{:}F_y=2$ and using of Barlat condition of plasticity developed in 1991 [2] for plastic strain $\varepsilon_s^p = 0{,}01$ in middle of specimen.

In Fig. 4b are also shown charts of quantities k_x, k_y, g_x, g_y in directions of axes x, y in middle part of specimen with diameter 30 mm. As result from the charts, maximal deviations of stresses and deformations in monitored area do not cross 15%. To more details is the optimization process described in reference [3].



F_x:F_y=2

a)



b)

Figure 4: *Distribution of stresses σ_x, σ_y in specimen according to Fig. 3b with considering Barlat plastic condition.*

4. Methodology of experiments, reached results

For experimental determination of plastic curves by biaxial tensile test was proposed full experimental chain [3] that consists of hydraulic equipment for biaxial loading of cruciform specimen with a possibility to measure loading forces and also a system for scanning deformation in middle part of specimen during its biaxial tension.

Hydraulic loading equipment (Fig.7) consists of four hydraulic cylinders placed in pairs against each other. On the connecting rods of hydraulic cylinders are fixing jaws placed in slides.

In order to ensure the uniformity of displacements and accordingly constant velocity of connecting rods is into loading equipment included pantograph that ensures the same displacement of cylinders along axis and fixating of specimen’s middle part. In connecting rods of hydraulic cylinders are placed dynamometers for measuring of forces F_x, F_y in directions of cruciform specimen axis.

For the measurement of deformations in middle part of cruciform specimen can be used

a) Electrical residence strain gages applied directly in middle part of cruciform specimen

For the measurement is very appropriate to use strain gage rosette for finite deformations (e.g. type EP-08-062TT-120 - Vishay). Gauges are suitable for the measurement of deformations in ranges ±10% for temperature from -269° to 230°C [15].

b) Contact biaxial extensometer

For the application of contact measurement method of strain in the middle part of cruciform specimen was suggested and designed contact biaxial extensometer. Extensometer consists of two perpendicular elastic members with a points. Elastic members are pushed down into the sheet's surface by points so that during deformation there is a relative shift of points. It causes deformation of elastic members with strain gages. On every elastic member are applied four strain gages connected into Wheatston bridge with exclusion of tension and temperature. Measured length of extensometer *l=30 mm* is determined by distances of points and was proposed such a way that for a given type of cruciform specimen extensometer reads deformations in the area of their homogeneous distribution. Detailed description of contact extensometer is in literature [3,13].

c) Method of digital image correlation

In the case of classical image correlation the deformations of object are determined by CCD camera. In the process of digital image correlation are determined displacements, rotations and bending of small elements, so-called facets that are determined in reference image. Correlation algorithms can determine maximal displacement of point with precision to 1/100 pixel. Such a treatment allows determining deformation of object in a plane parallel to a plane of camera's image. For deformation analysis in 3D are used two cameras. If the object is observed from two different directions, the position of every point is focused to individual pixel in the plane of camera in question. If are known the relative positions of both cameras, magnification factor of objective and all parameters of image, we are able to compute absolute three-dimensional coordinates of every point on the object's surface in space (Fig.5).

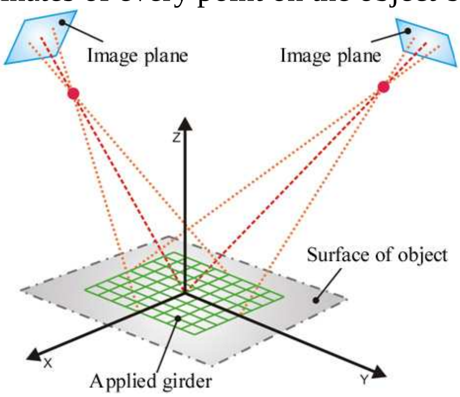


Figure 5: *Principle of 3D pattern correlation with two cameras.*

From the known coordinates of every point on the surface we can determine position of surface in 3D in all areas observed by two cameras. The structure of the surface has to have satisfactorily good quality in order to be able to use correlation algorithms for identical points from both cameras. It is the reason why the surface is sprayed with black and white grain structure (Fig.6). Measurement was accomplished by optical system Q-450 from DANTEC Dynamics (Fig.7). It is a system that allows measurement of three-dimensional displacements and deformations almost all material types and parts [15].

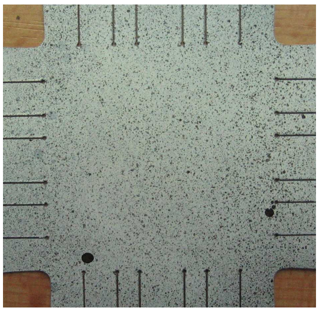


Figure 6: *Black and white grain structure on object's surface.*

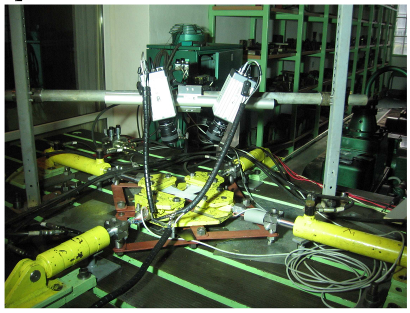


Figure 7: *Hydraulic load equipment with system Q-450.*

Magnitudes of load forces acting on arms of cruciform specimen were measured by strain gage dynamometers installed on piston rods of both perpendicular hydraulic valves. From the forces $F_x=F_1=F_0$, $F_y=F_2=F_{90}$ were the stresses $\sigma_x=\sigma_1=\sigma_0$, $\sigma_y=\sigma_2=\sigma_{90}$ determined by relations

$$\sigma_x(t)=\sigma_1(t)=\frac{F_1(t)}{A_0}\bar{k}_x\ , \quad \sigma_y(t)=\sigma_2(t)=\frac{F_2(t)}{A_0}\bar{k}_y\ , \tag{3}$$

where $\bar{k}_x$, $\bar{k}_y$ are average values of coefficients k_x, k_y in the observed area of specimen according to (1),

$A_0 = b \cdot t$ is a cross-section area of specimen.

Work of experimental chain is automated by a control system. Control system allows creating controlled experiment by registration of signals from measured forces in connecting rods of hydraulic valves and from sensors of deformation in middle part of specimen (in case of strain sensing by electric residence strain gages or contact biaxial extensometer). With the controlled system is possible to provide biaxial tensile tests with prescribed stress or deformation footmark.

For experimental evaluation of plastic properties were used cold-rolled steels produced by US Steel Košice. In Tab. 1 are given mechanical properties of cold-rolled zinc-coated sheet HX220PD with higher strength properties from IF steel alloyed by phosphorus. Relative direction was direction of rolling. With respect to observed range of plastic deformations was considered coefficient of normal anisotropy $r_{(5)}$ evaluated for 5% plastic deformation.

Table 1: *Mechanical properties of tested sheets*

Material	***Thickness [mm]***	***Direction***	***$R_{p\,0,2}$ [MPa]***	***R_m [MPa]***	***A_{80} [%]***	***n***	***$r_{(5)}$***
HX220PD	0,70	0°	220	380	39,3	0,218	1,14
		45°	228	364	37,8	0,220	1,80
		90°	241	381	37,8	0,211	1,82

From the measured stress – strain dependences were for given materials determined points of plastic curves for chosen magnitudes of plastic strains. For determination of points on plastic curves was used criterion of maximal magnitude of plastic strain with plastic strains ε_k^p = 0,002, 0,005, 0,01, 0,015 a 0,02 . The experimentally determined points of plasticity were consecutively compared with analytical plasticity conditions according to Von-Mises (HMH) theory

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 = R_e^2 \tag{4}$$

with Hill's plasticity conditions from 1948

$$\sigma_1^2-\frac{2r_0}{1+r_0}\sigma_1\sigma_2+\frac{1+r_{90}}{1+r_0}\frac{r_0}{r_{90}}\sigma_2^2 = R_{e0}^2 \tag{5}$$

and from year 1993

$$\frac{\sigma_x^2}{R_{ex}^2}-\frac{c\sigma_x\sigma_y}{R_{ex}^2R_{ey}^2}+\frac{\sigma_y^2}{R_{ey}^2}+\left[(p+q)-\frac{p\sigma_x+q\sigma_y}{R_{ebi}^2}\right]\frac{\sigma_x\sigma_y}{R_{ex}R_{ey}}=1\,, \tag{6}$$

as well as Gotoh's plasticity condition

$$A_1\sigma_x^4+A_2\sigma_x^3\sigma_y+A_3\sigma_x^2\sigma_y^2+A_4\sigma_x\sigma_y^3+A_5\sigma_y^4+\left(A_6\sigma_x^4+A_7\sigma_x\sigma_y+A_8\sigma_x^2\right)\tau_{xy}^2+A_9\tau_{xy}^4=R_{e0}^4\,. \tag{7}$$

Description of individual plasticity conditions is given in work [1] as well as in original works of their authors. In Fig.8 are shown experimentally gained points of plasticity curves compared with above-mentioned analytical plasticity conditions gained by method of digital image correlation for plastic strains ε_S^p = 0,002; 0,005; 0,01; 0,015; 0,02 a 0,03 in the middle part of specimen.

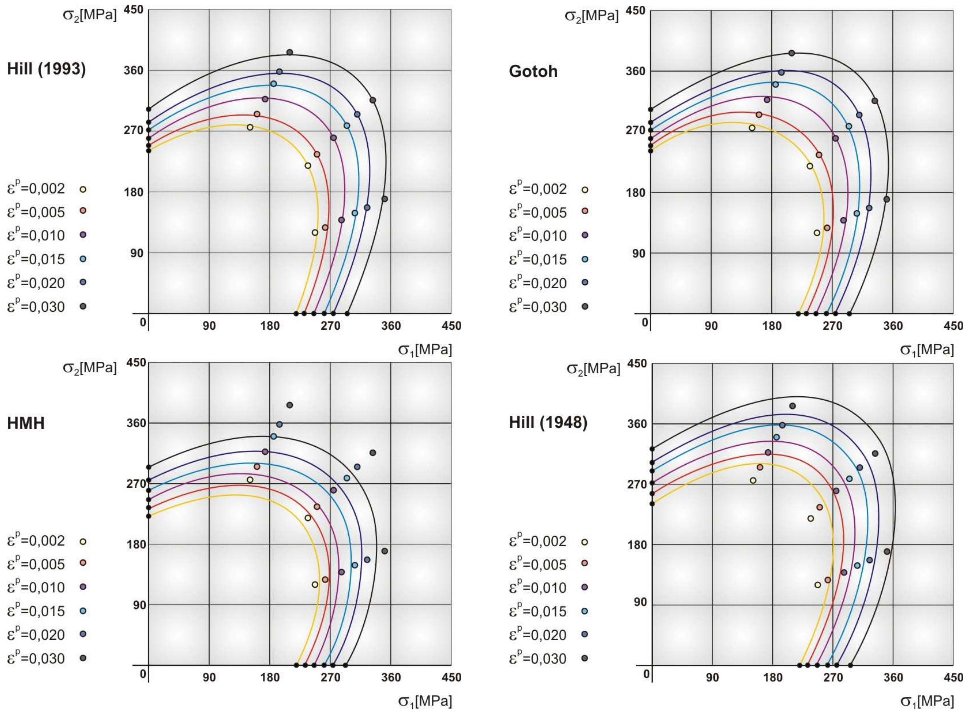


Figure 8: *Experimental determined points of plastic curves for the sheets made of material HX220PD.*

In Fig.9 is shown typical distribution and forces of principal strains in middle area of specimen for $\varepsilon_S^p = 0{,}01$ and 0,02 determined by the method of digital image correlation for force ration specimen's arm 1:1.

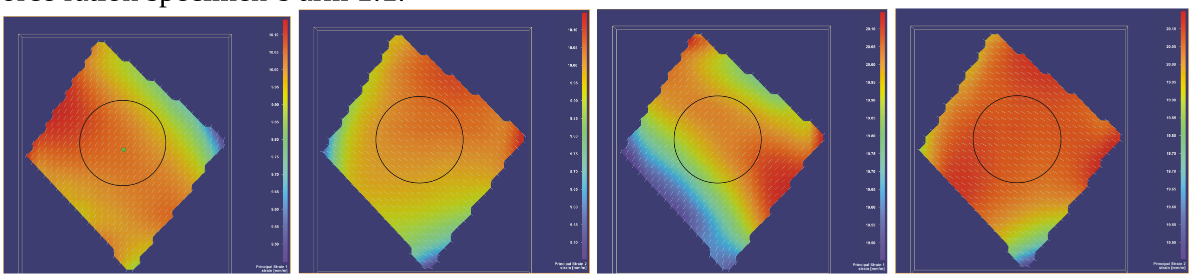


Figure 9: *Distributions and directions of principle strains for force ratio 1:1, for* $\varepsilon_S^p =$ *0,01 and 0,02.*

5. Conclusions

In the paper is described the method for determination of deformations and stresses in sheets with plastic anisotropy in plane stress state modeled by biaxial tensile test. For determination of plastic deformations in middle part of specimen was used optical method which exploits digital image correlation. The results gained from experiments were compared with the plastic curves determined analytically according to various theories. As results from Fig.8, the best agreement of experimental results is with Hill condition of plasticity published in 1993. Also Gotoh's condition of plasticity is in a good agreement with the experiments, but for the force ratio 1:2 and 2:1 for plastic deformation under 0,01 slightly overevaluates stresses. Hill's theory from 1948, which is the most used in numerical modeling of plastic deformations significantly overevaluate stresses mainly closely after beginning of plastic deformation. Difference in comparison to the experimental results is to 10%. On the other hand, Von Mises theory (HMH) of plasticity for isotropic materials has increased differences with increased plastic deformation. The differences with respect to experimental values are caused also by a fact that this theory can not taken into account anisotropy of plastic properties of material in question.

In Fig.9 are the fields of plastic deformations gained with the optical system Q–450 that uses digital image correlation. In the middle part is roughly homogeneous deformation that supports correctness of cruciform specimen design. On the base of first opinions gained from using of this measurement method can be stated that it is suitable for determination of plastic deformations in plane stress state for the strains that cross 0,05%.

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