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Experimental Stress Analysis

COMPARISON OF STATIC AND DYNAMIC MODULUS OF ELASTICITY OF CONCRETE SPECIMENS

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Abstract: The paper presents comparison of results of two methods for elastic modulus evaluation of concrete specimens. The first method used for the dynamic modulus of elasticity evaluation was modal analysis. The major advantage of this method is its nondestructive character. The excitation was done by the impact hammer and the response was measured by piezoelectric acceleration transducers. Natural frequencies were evaluated from resonant peaks of Frequency Response Functions. The dynamic modulus of elasticity was calculated on the basis of natural frequencies using relations for structures with continuously distributed mass. Then we continued with standard destructive compression tests. At the end of the paper, the results of comparison of static and dynamic moduli of elasticity are summarized.

1. Introduction

The static modulus of elasticity of concrete is usually evaluated using standard compression test, where stresses and strains are measured directly during mechanical testing and modulus of elasticity is determined from the slope of the linear region of the stress-strain curve. The dynamic modulus of elasticity of concrete can be determined using ultrasonic or resonant methods. The big advantage of dynamic methods over static methods is its nondestructive character and wide variety of specimen shapes and sizes to use. This paper presents comparison of results of impulse excitation method and of standard compression test, the comparison of dynamic and static modulus of elasticity.

2. Description of specimens

The methods for Young's modulus evaluation were compared on concrete specimens made from the concrete mixture designed to achieve high fluidity and good results in the L-box test. The spill of the mixture during slump test was larger than 45cm (the diameter of a pat), which classify this concrete to the self compacting concretes. The concrete mixture was also designed to achieve good material properties but also very good processibility in the form as self compacting concrete. To increase the concrete strength, the grounded lime stone was added to the concrete mixture to reduce porosity of the concrete mixture. The Portland cement CEM I 42.5 R was used for preparation of the concrete mixture.

From this concrete mixture, the specimens of dimensions 400x100x100 mm were made and cut to two pieces of dimensions about 197x100x100 mm (Fig. 1) for the purpose of this test.

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Figure 1: *The tested concrete specimen with mounted acceleration transducer and impact hammer.*

3. Dynamic Young's modulus evaluation

The first method determines Young's modulus of the specimen based on its fundamental resonant frequency. It is called resonant method using impulse excitation. The test arrangement can be done for flexural, longitudinal or torsional vibration. With respect to the dimensions of the specimens , the tests were done only in longitudinal direction.

The specimen was supported in the middle of its span (Fig. 1), the fundamental longitudinal nodal position. The acceleration transducer Bruel&Kjaer of Type 4513B was placed at the centre of one of the end faces of the specimen (Fig. 1 – the right end face). The end face of the specimen opposite to the face where the transducer was located (Fig. 1 – the left end face) was struck by the impact hammer Bruel&Kjaer of Type 8206. Both signals, the excitation force and the acceleration, were recorded and transformed using Fast Fourier Transform (FFT) to the frequency domain and the Frequency Response Function (FRF) (Fig. 3 and 4) were evaluated from these signals using the vibration control station Bruel&Kjaer Front-end 3560-B-120 and program PULSE 10.5. The test was repeated ten times for each specimen and resultant readings were averaged. From an averaged FRF, the fundamental longitudinal resonant frequency was determined for each specimen. Based on the equation for longitudinal vibration of the beam with continuously distributed mass with free-free boundary condition, the Young's modulus can be determined using the relation

$$E = \frac{4lmf^2}{bt}, \tag{1}$$

where l is the length of the specimen, m is the mass of the specimen, f is the fundamental longitudinal resonant frequency of the specimen, b is the width of the specimen and t is the thickness of the specimen.

4. Static Young's modulus evaluation

The second method for Young's modulus of the specimen determination was the standard compression test. The hydraulic press INOVA of type DSM 2500 was used for these tests (Fig. 2). The loading of the specimens was controlled by strain using two strain indicators INOVA PXA 50 (Fig. 2). From the measured data (Fig. 5 and 6) the strength and the Young's modulus were evaluated for each of the specimens. The Young's modulus was evaluated as the cord modulus of elasticity in one third of the strength of the specimen.

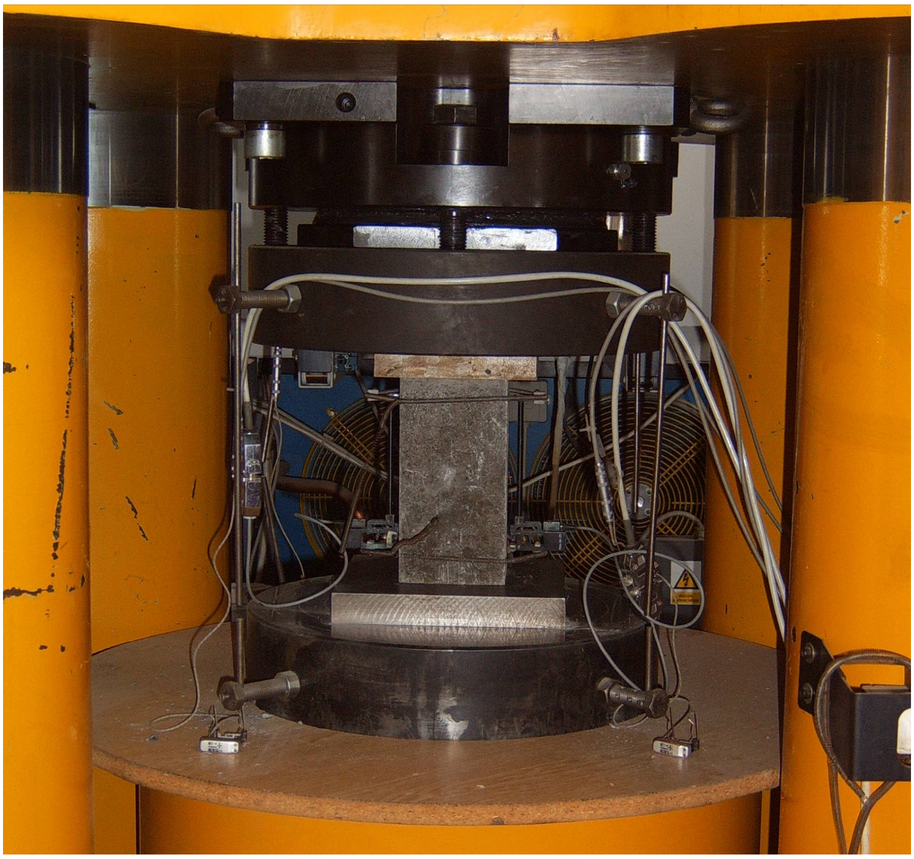

Figure 2: *The standard compression test of the specimen made using the hydraulic press INOVA of type DSM 2500.*

5. Results

Eight concrete specimens were tested using both methods. At first all dimensions (l, t, b) of the specimens were measured using slide calliper and the specimens were weighed. Based on these properties and resonant frequencies evaluated from FRFs of these specimens using equation (1) dynamic Young's moduli were evaluated (Tab. 1). After nondestructive dynamic Young's modulus evaluation we continued with destructive static Young's modulus evaluation of these eight concrete specimens. Evaluated results are summarized in the Table 1.

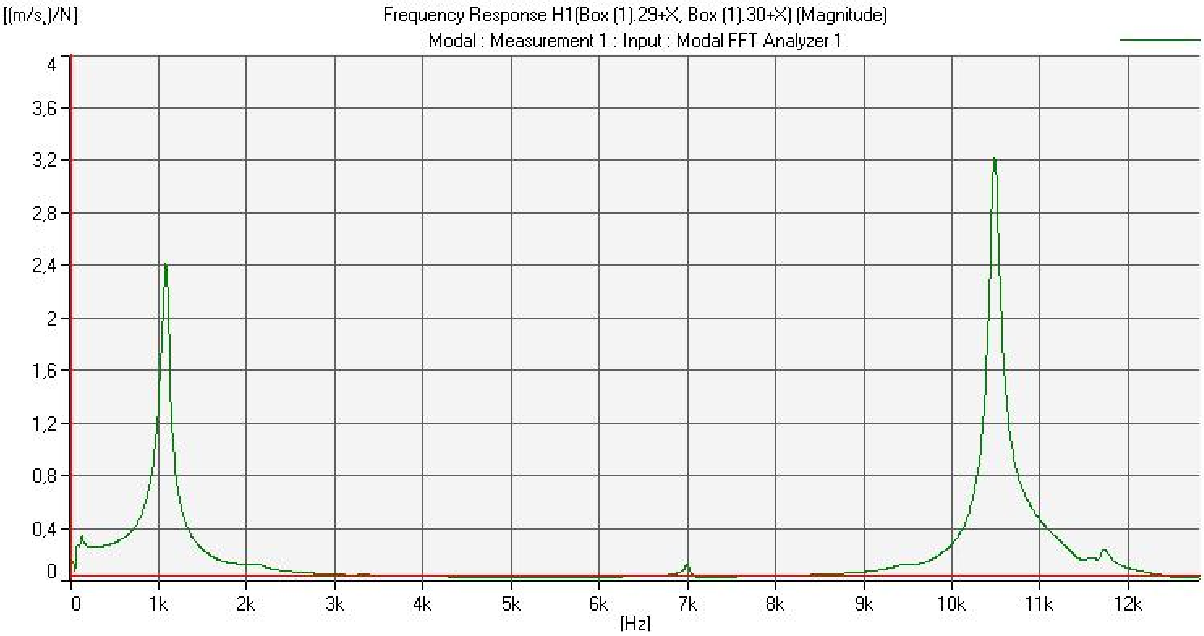

Figure 3: *The Frequency Response Function of the specimen No. 1 with the resonant frequency of the longitudinal vibration f=10,47 kHz.*

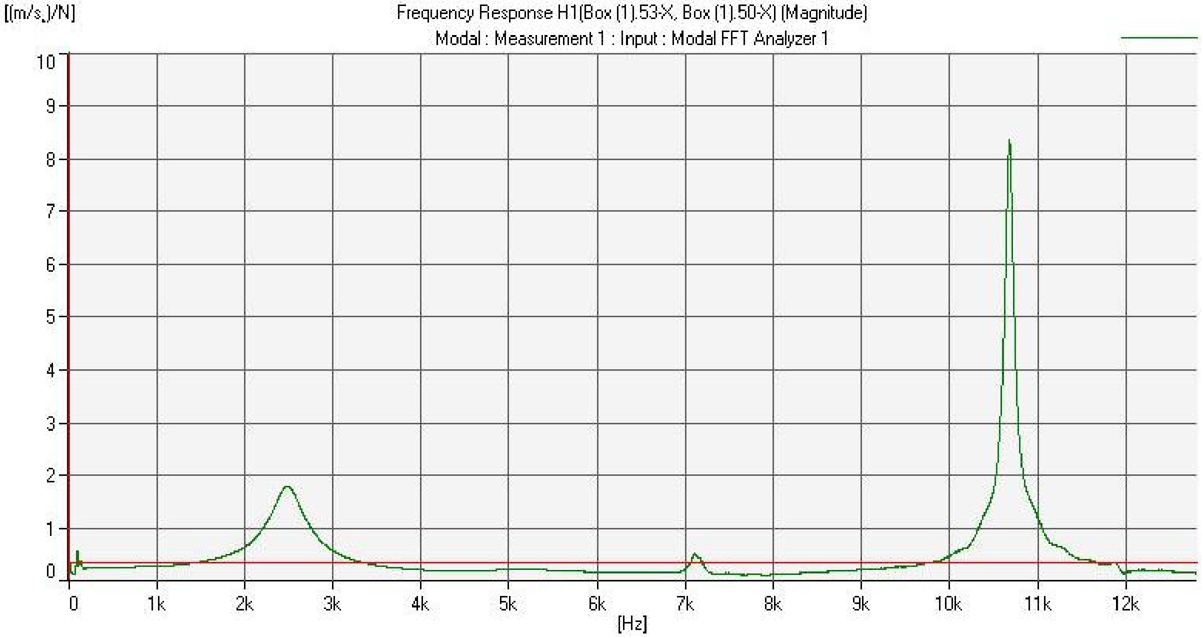

Figure 4: *The Frequency Response Function of the specimen No. 4 with the resonant frequency of the longitudinal vibration f=10,67 kHz.*

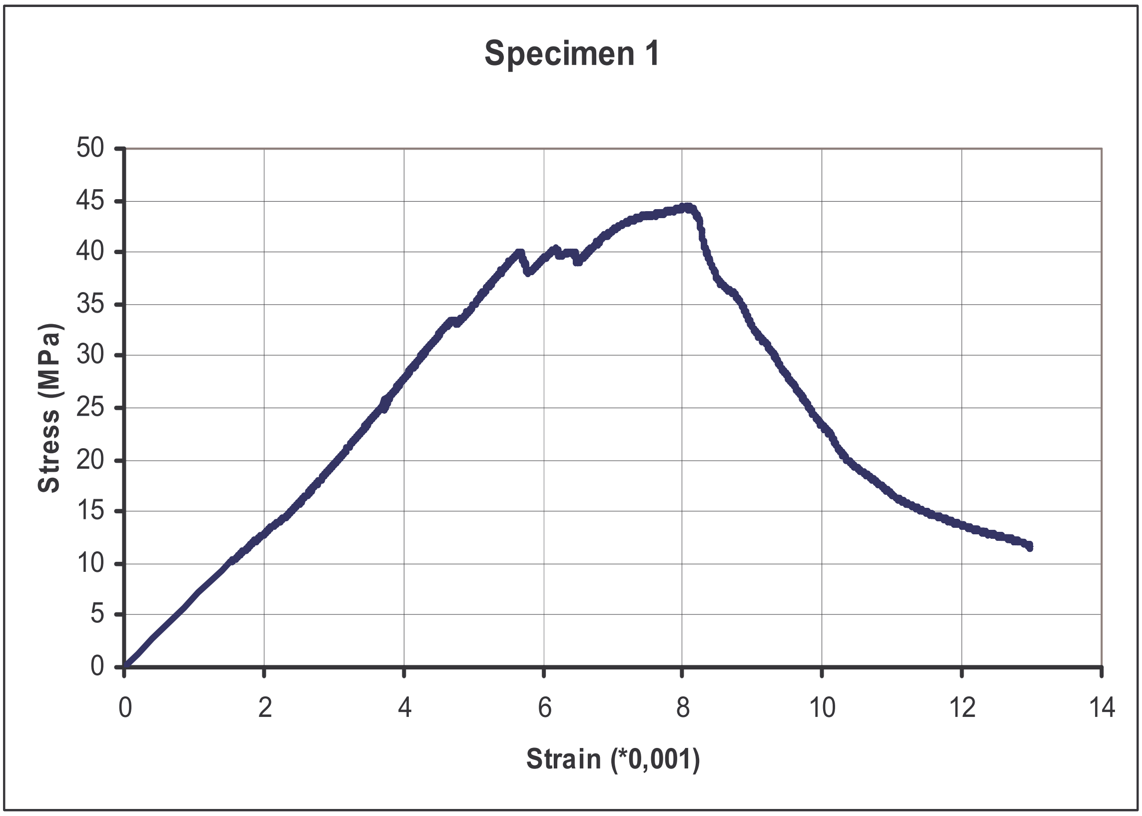

Figure 5: *The stress-strain loading curve of the specimen No. 1*

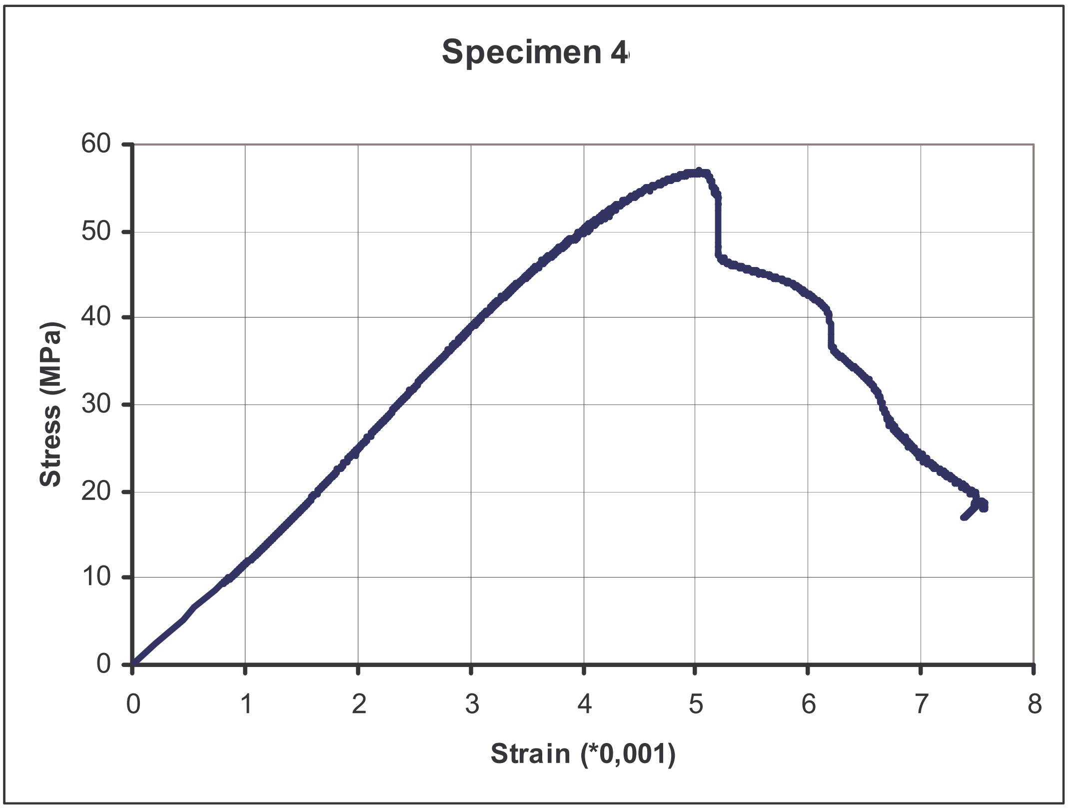

Figure 6: *The stress-strain loading curve of the specimen No. 4*

Table 1: *The dynamic Young's moduli of the tested specimens*

Made	Tested	Specimen	Weight	Length	Width	Thickness	Frequency	dyn. E	static E
		No.	[kg]	[m]	[m]	[m]	[Hz]	[MPa]	[MPa]
12.6.2008	22.10.2008	1	4,4389	0,1976	0,1002	0,1012	10472	37926	23301
12.6.2008	22.10.2008	2	4,5809	0,1976	0,1008	0,1018	10720	40548	28750
12.6.2008	22.10.2008	3	4,4511	0,1974	0,1004	0,1007	10536	38573	19357
12.6.2008	20.3.2009	4	4,4805	0,1982	0,1071	0,0982	10670	38566	29851
12.6.2008	20.3.2009	5	4,5627	0,1973	0,1011	0,0994	10758	41470	27900
12.6.2008	20.3.2009	6	4,4397	0,1970	0,1005	0,1017	10590	38383	28554
12.6.2008	20.3.2009	7	4,5368	0,1986	0,1000	0,0995	10748	41734	32280
12.6.2008	20.3.2009	8	4,4195	0,1975	0,1006	0,0991	10820	41014	29593

6. Conclusion

From the obtained results it is obvious that the static Young's modulus is about 30% lower then dynamic Young's modulus. It is caused mainly by the fact that the dynamic modulus is approximately equal to the initial tangent modulus while the static modulus is equal to the cord modulus. Partially it is caused by small statistical file in which two lowest values of static modulus (specimens No. 1 and 3), which differ from other values very much, considerably affect all statistical data. As it can be seen the big advantage of the dynamic modulus of elasticity determination is its small variation, it is two times less than the standard deviation of the static modulus. The main advantage of the dynamic modulus determination is its nondestructive character which allows testing of the same specimen in different times or different conditions. Other advantage is also that small variations in the shape of the specimen can be considered as an average of the dimensions measured in different places. But if the loaded end faces of the specimen are not parallel (Fig. 2 – upper and lower one), inaccuracies appear in the stress-strain curve during static loading.

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