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Experimental Stress Analysis

CONJUGATED STRENGTH CRITERION FIT FOR THE FATIGUE LOADING OF MATERIAL

Frantisek Fojtik[1] & Jan Fuxa[2]

Abstract: This article describes experimental results under combined loading of specimens manufactured from construction steel 11523 melting T31052. Specimens were loaded by amplitude of the torque, then by combination of torque and tension prestress. The last set of specimens was loaded in combination of torque and inner overpressure in the conditions of closed vessel. To obtain the required input values the stress-strain analysis of specimens was performed by finite element method in software Ansys. Fuxa's criterion [1] was applied for evaluation of the results. The experiments were performed on modified testing machine equipped by overpressure chamber.

1. Introduction

To verify multi-axial Fuxa´s conjugated strength criterion the new jig was developed which generalizes the possibilities of reconstructed testing machine SHENCK type PWXN [2]. In this case the testing machine was equipped by overpressure chamber which makes possible to load the specimen by constant inner overpressure (in the range 0 to 70 MPa) in the conditions of closed vessel in addition with torque amplitude. It is also possible to add the axial constant tension/pressure prestress into the system.

The performed experiments and their results embody a good agreement with bellow mentioned conjugated strength criterion.

2. Testing Machine

The testing device SHENCK type PWXN was reconstructed for material testing under combined loading in the region of high-cycle fatigue [2, 3]. The new conception of the testing device changes the loading character of the specimen from deformation-controlled to force-controlled (Figure 1).

The base of the testing machine is the frame 1 with gear box. Required torque is gained by the acceleration of driven part and by balance wheel located in measuring box 2. Axial tension/pressure force is gained by straight-line hydro motor 3 connected with hydraulic aggregate. Newly is the testing machine equipped by overpressure chamber 4 which is connected with multiplier and hydraulic aggregate. The measured values of the torque, axial force, number of cycles and inner overpressure are captured by computer 5 measuring cards and consequently evaluated in software LabVIEW [4].

The new conception of the testing device is patented as: Experimental Device for combined loading of specimens, Nr. 17286 (2007) and Experimental Device for combined loading of specimens, Nr. 18921 (2008).

[1] Ing. Frantisek Fojtik, Ph.D.; VŠB-Technical University of Ostrava, Department of Mechanics of Material; 17. listopadu 15, 70833 Ostrava-Poruba, Czech Republic, frantisek.fojtik@vsb.cz
[2] Prof. Ing. Jan Fuxa, CSc.; VŠB-Technical University of Ostrava, Department of Mechanics of Material; 17. listopadu 15, 70833 Ostrava-Poruba, Czech Republic, jan.fuxa@vsb.cz

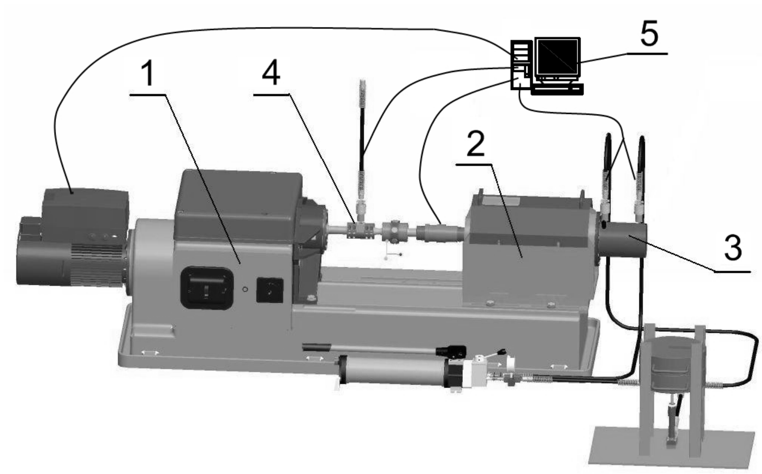

Figure 1: *Testing machine.*

3. Alternating Torsion – First Experiment

Specimens manufactured from steel 11523, melting nr. T31052 whose parameters are see on Figure 2 were loaded by amplitude of torque in the condition of alternating cycle and testing frequency of 25 *Hz*. The amplitude of torque was gradually decreased until the limit 10^7 cycles was reached. In the Figure 2 can be seen measured values and Fuxa's approximation curve (1) [1]. Point of crack initiation under static torsion [5] was measured by reconstructed testing machine INOVA in the institute lab.

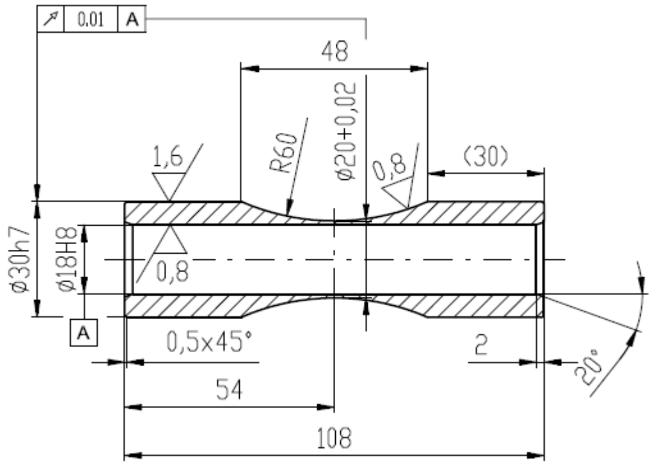

Figure 2: *Testing specimen.*

Fuxa's approximation:

$$\tau_{aF} = (\tau_f + \tau_C)/2 + (\tau_f - \tau_C)/2 \cdot \cos\left\{\pi \cdot \left[\log(4 \cdot N_f)/\log(4 \cdot N_C)\right]^{a_1}\right\}, \qquad (1)$$

for N_f in interval [*1/4; N*$_C$] and τ_{aF} in interval [τ_f ; τ_C],

where:

τ_f is a value of real shear strength, τ_C is the stress at the fatigue limit, N_C is number of cycles at the fatigue limit, a_1 is constant, τ_{aF} is the limit stress amplitude under alternating torsion and N_f is the limit number of cycles (until crack initiation). The mentioned values were obtained by nonlinear regression methods.

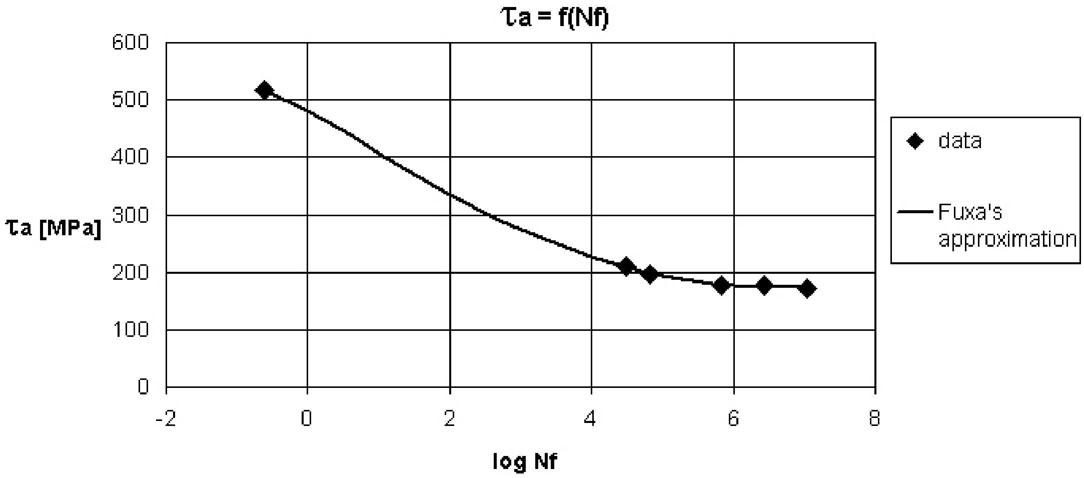

Figure 3: *S-N curve for alternating torsion.*

4. Alternating Torsion and Tension Prestress – Second Experiment

For this way of testing the same specimen as in the previous case were used. Every series of specimens was loaded by different constant axial tension stress. For given tension stress value was chosen the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when was the specimen able to endure 10^7 of cycles. The testing frequency was also 25 *Hz*.

The results of those experiments are mentioned in Figure 4. The mentioned resulting tension stress was in the case of very complicated specimen shape obtained on the base of finite element method in the software ANSYS. The experimental results are also approximated by lower described Fuxa's approximation (2, 3, 4, 5) which takes the influence of mean stress into account. Particular approximations are based on measured number of cycles (Figure 4).

Fuxa's approximation with influence of mean stress:

$$\tau_{aF2} = \left(\tau_f^{\,*} + \tau_C^{\,*}\right)/2 + \left(\tau_f^{\,*} - \tau_C^{\,*}\right)/2 \cdot \cos\left\{ \pi \cdot \left[\log\left(4 \cdot N_f\right)/\log\left(4 \cdot N_C\right)\right]^{a_1} \right\}, \tag{2}$$

$$\tau_f^{\,*} = 1/\sqrt{3} \cdot \left((\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_O \cdot \sigma_t / 3 + \sigma_t^{\,2} \cdot B_O^{\,2} / 9 - \sigma_t^{\,2}\right)^{1/2}, \tag{3}$$

where (4) is the static strength condition for $N_f = 1/4$ and constant B_O is equal to:

$$B_O = 3 \cdot \left(\sqrt{3} \cdot \tau_f / \sigma_f - 1\right), \tag{4}$$

$$\tau_C^{\,*} = \tau_C / 2 \cdot \left\{ 1 + \cos\left[\pi \cdot \left(\sigma_t / \sigma_f\right)^B\right] \right\}, \text{ is the strength condition for } N_f = N_C. \tag{5}$$

In the relations (2, 3, 4, 5) the σ_f is the real tension strength value, τ_f is a value of real shear strength, τ_C is the stress at the fatigue limit, N_C is number of cycles at the fatigue limit, a_1 and B are constants, τ_{aF2} is the limit amplitude of shear stress, σ_t is the constant tension stress and N_f marks the (limit) number of cycles until crack initiation.

The absolute mean relative error value of used approximation is 3,82 % and can be determined according to following formula:

$$CH_F = ABS\left(\tau_{ai} - \tau_{aFi}\right)/\tau_{ai} \cdot 100\,\%, \tag{6}$$

τ_{ai} are the measured stress amplitude values (Figure 4),

τ_{aFi} are the values calculated according to the Fuxa's approximation (2).

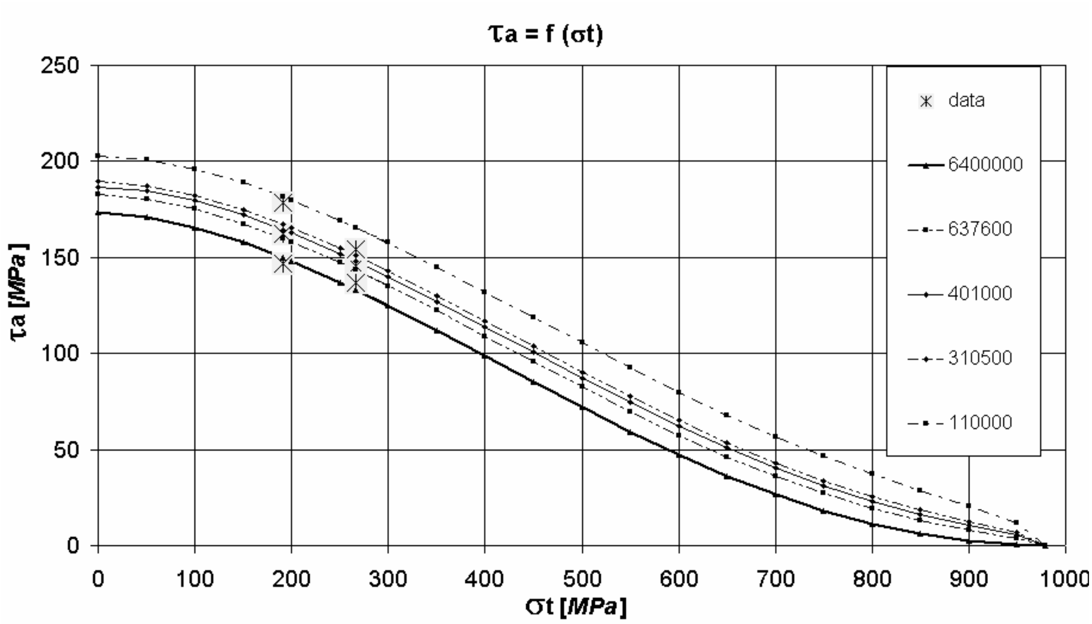

Figure 4: *Fuxa's approximation for combined torsion – tension loading.*

5. Alternating Torsion and Inner Overpressure – Third Experiment

For this way of loading was the above mentioned testing machine equipped by a new type of specimen fixation which makes possible to use the overpressure chamber. This chamber is connected with multiplier and with hydraulic aggregate which serves for gaining of constant inner overpressure in the conditions of closed vessel in the range 0 - 70 *MPa* . For this way of testing the same specimen as in previous case were used. Every series of specimens was loaded by different constant overpressure (three series – 10 *MPa*, 15 *MPa* and 20 *MPa*). For given constant inner overpressure value was chosen the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when the specimen was able to endure 10^7 of cycles. The testing frequency was also 25 *Hz*. The results of those experiments are mentioned in Figure 6.

On the base of stress state evaluation of the specimen loaded by inner overpressure (in the conditions of closed vessel) the significant circumferential σ_{t1} and axial σ_a stress can be observed on the surface. It is not easy to determine those two stresses analytically due to the complicated shape of specimen and face of acting inner overpressure. Hence this stress state had to be determined by finite element method in software ANSYS.

The static analysis was performed, where ¼ of specimen and ¼ of pistons of overpressure chamber (closed vessel) was modelled as one body, the SOLID element 186 was used, the material parameters was obtained on the base of tensile test. The boundary conditions are chosen so that the resting ¾ of specimen is compensated by symmetry and further one point of specimen face is fixed in three directions (x, y, z). Opposite end of the specimen is free. On the relevant length the inner overpressure was applied. This length results from the dimensions of testing jig. Results of circumferential and axial stresses in *MPa*, obtained from FEM analysis for inner overpressure 10 *MPa*, are on Figure 5.

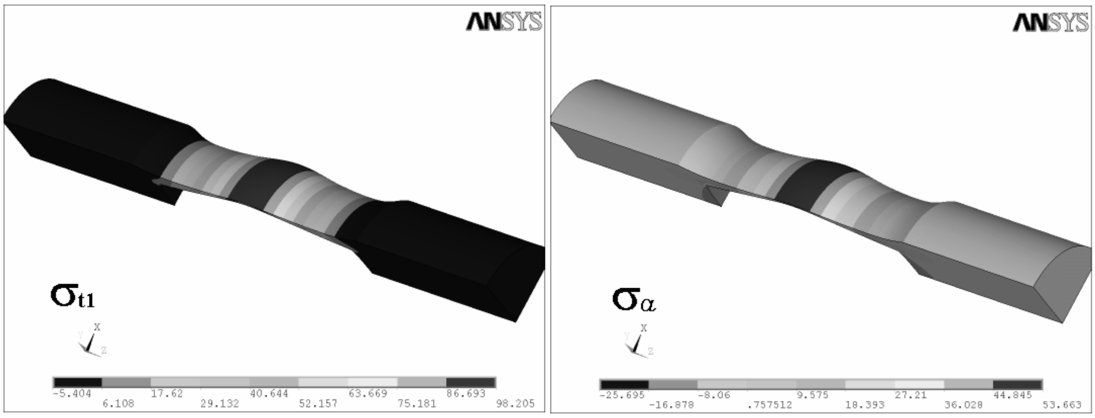

Figure 5: *Circumferential and axial stresses for inner overpressure 10 MPa.*

Results obtained from performed experiments and computation are on Figure. 6. Those results are approximated by already mentioned Fuxa's approximation (2, 3, 4, 5) which takes into account the influence of mean stress. For from inner overpressure obtained stress state it is necessary to adjust the relations (3) mentioned equation can be written as follows:

$$\tau_f^{\,*} = 1/\sqrt{3}\cdot\begin{pmatrix}(\sqrt{3}\cdot\tau_f)^2 - 2\cdot\sqrt{3}\cdot\tau_f\cdot B_O\cdot(\sigma_{t1}+\sigma_a)/3 \\ +(\sigma_{t1}+\sigma_a)^2\cdot B_O^{\,2}/9-\sigma_{t1}^{\,2}+\sigma_{t1}\cdot\sigma_a+\sigma_a^{\,2}\end{pmatrix}^{1/2}. \qquad (7)$$

This equation is based on reference stresses [1] and on the stress state analysis for given loading case. Particular approximations result from measured number of cycles (Figure 6).

In this case the same constants as in the case of tension/torsion combination are used for the approximation (Figure 4). The curves at the fatigue limit are in the case of both described problems equivalent.

The absolute value of mean relative error CH_F (6) is here 6,9 %.

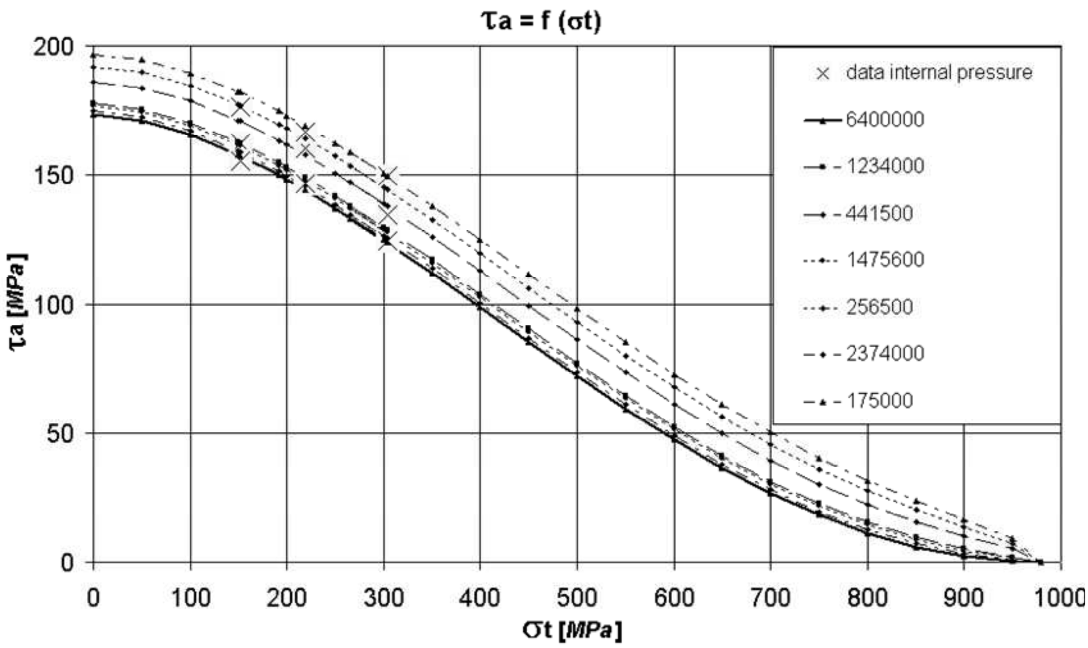

Figure 6: *Fuxa's approximation for combined torsion – inner overpressure loading.*

6. Conclusions

The reconstructed testing machine equipped by overpressure chamber is briefly described in this contribution. Further are mentioned three types of fatigue experiments performed at the specimens made from steel 11523 melting T31052.

First experiment – alternating torsion. Obtained results are approximated by Fuxa's approximation. The Fuxa's approximation embodies a good agreement with experiment (Figure 3).

Second experiment – combined loading by amplitude of torque and by constant axial tension force. The results are successfully approximated by Fuxa's approximation with the influence of mean stress (2, 3, 4, 5). Also here the Fuxa's approximation embodies the good agreement with experiment (Figure 4). The constants of strength criterion were tuned on this experiment.

Third experiment – combined loading by the amplitude of torque and by inner overpressure in the condition of closed vessel. Appropriate circumferential and axial stresses are obtained by finite element method and the modification of Fuxa's approximation is mentioned. The experimental results are approximated by Fuxa's approximation whose constants result from previous experiment. The good agreement can be seen here and hence it is possible to state the appropriate constant tuning for further possible combined loading (Figure 6).

Described Fuxa's approximation is a part of conjugated stress criterion which is based on the conception of reference normal and reference shear stresses, see [1] for more details.

Acknowledgements

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References

[1] Fuxa, J.; Kubala, R.; Fojtík, F.: Idea of Conjugated Strength Criterion, *Acta Mechanica Slovaca*, **Vol.** (1), pp. 125-130, ISSN 1335-2393

[2] Fojtík, F.: The Fatigue Testing Machine and Experiment, In *Applied Mechanics 2008*, Wisla, ISBN 978-83-60102-49-7, Poland, (2008), pp. 11-13

[3] Fuxa, J.; Fojtík, F.; Kubala, R Torque machine fit for high cycle fatigue of material testing, In *Experimental Stress Analysis 2007*, Hotel Výhledy, ISBN 978-80-7043-552-6, Czech Republic, (2007), pp. 37-39

[4] Macura, P.: *Experimentální metody v pružnosti a plasticitě*, script VŠB – TU Ostrava, 2006, 107 pp, ISBN 80-7078-934-4

[5] Fojtík, F.; Fuxa, J.: Examination of the Fatigue Life under Combined Loading of Specimens, *Applied and Computational Mechanics*. 2008, **Vol.** (2) No. 1, pp. 37-44, ISSN 1802-680X.