Experimentální Analýza Napětí 2007

APPROXIMATION OF VISCOELASTIC BEHAVIOR OF G/VE COMPOSITES

Miroslav J. Černý[1]

Abstract: *In the paper the approximation of creep functions of standard linear solid is described. The single-integral representations of linear viscoelastic materials is used. The creep curves of composite with vinylester matrix and C glass fibres have been obtained from tests at the constant stress in tension. The parametres of creep function are calculated by collocation method.*

Keywords: *composites, viscoelastic, approximation*

1. Introduction

Composites consisting of vinyl-ester (VE) and glass (G) fibres are frequently used in corrosive environment. Long-term behavior is of great importance for structures, where G/VE composites are used. Investigation of long-term behavior of composites is important for a modeling of environmental effects on composites [1], [2].
Composites with polymer matrices exhibit mechanical properties which come somewhere between the two ideal cases, i.e. perfectly elastic and perfectly viscous, and hence they are termed viscoelastic. In a viscoelastic material the stress is a function of strain and time. In the general case, this type of response is referred to as nonlinear viscoelastic. However, since nonlinear viscoelastic behaviour is not amenable to simple analysis, in many experimental and theoretical investigations this behaviour is often reduced to the linear viscoelastic case where the stress is directly proportional to the strain.
Most recent efforts are concentrated on single-integral representations of linear viscoelastic materials because of their simplicity of application and the accuracy of the results [3].

2. Theoretical background

At low stress levels, the creep strain $\varepsilon(t)$ is related to the applied stress σ_0 by the so-called creep compliance which is defined as

$$D(t) = \frac{\varepsilon(t)}{\sigma_0} \quad , \tag{1}$$

[1] Ing. Miroslav J. Černý, CSc., Klokner Institute, Czech Technical University, Šolínova 7, 166 08 Praha 6, tel.: +420 224353584, e-mail: cerny@klok.cvut.cz

In general, the Boltzmann's superposition principle provides adequate representation of the material time-dependent response in the linear region i.e. at low stress levels. This principle can be formulated as

$$\varepsilon(t) = \int_0^t \bar{D}\ (t-\tau)\sigma(\tau)d\tau\ , \tag{2}$$

$$\sigma(t) = \int_0^t \bar{R}\ (t-\tau)\varepsilon(\tau)d\tau\ , \tag{3}$$

The integral in the right side of eq. (2), (3) is the so-called hereditary integral and the above expression shows that the strain at any given time generally depends on the entire stress history. In contrast in an elastic material, strain at any time solely depends on the stress acting at that time.

3. Standard Linear Solid (Three- element model)

Creep kernel is of the form

$$\overline{D}(t) = \frac{1}{E_1}\delta(t) + \frac{1}{K_2} e^{-\frac{E_2}{K_2}.t}\ , \tag{4}$$

where E_1, E_2, K_2 are constants, δ(t) is Dirac function

and if we denote

$$\tau = \frac{K_2}{E_2}\ , \tag{5}$$

after some rearrangement we obtain

$$\overline{D}(t) = \frac{1}{E_1}\delta(t) + \frac{1}{\tau}\frac{1}{E_2} e^{-\frac{t}{\tau}}\ , \tag{6}$$

what expressed by compliances gives

$$\overline{D}(t) = C_1\delta(t) + \frac{C_2}{\tau} e^{-\frac{t}{\tau}} \tag{7}$$

Creep test yields at the constant stress

$$\sigma = \sigma_0\ h(t) \qquad \sigma_0 = \ const.\ , \tag{8}$$

measured strain $\varepsilon_m(t)$.
Function $h(t)$ is Heaviside function. Substituing expression for stress (8) into (2)

$$\varepsilon(t) = \int_0^t \overline{D}(t-\tau)\ \sigma(\tau)\ d\tau\ , \tag{9}$$

we get

$$\varepsilon(t) = \sigma_0 h(t) \int_0^t \overline{D}(\tau)\, d\tau , \qquad (10)$$

or

$$\varepsilon(t) = \sigma_0 h(t)\, D(t) , \qquad (11)$$

where

$$D(t) = \int_0^t \overline{D}(\tau)\, d\tau , \qquad (12)$$

and D(t) is a creep function (compliance).

4. Experimental procedure

All the tests were conducted in the Centre of Composites of the Czech Technical University, Klokner Institute in Prague.

Composites were prepared in the form of the sheets by a wet lay-up method for creep testing, VE resins from Dow Chemical Derakane 411-45 were used with reinforcement from C glass veil mat.

Specimens were cut from these plates in the desired dimensions, using a diamond wheel saw. Every specimen was polished on two types of abrasive paper. Test specimens were of size for creep tests 15x 150 mm. The thickness was given by the thickness of the sheet (average 3-4 mm).

First, the tensile tests have been conducted on specimens. The rate of loading was 1 $mm.min^{-1}$ what corresponds to deformation rate 1%.min^{-1}.

Furthermore, the specimens have been subjected to constant tensile load at a special set-up constructed at Klokner Institute, corresponding to stress levels 17.0 resp. 25.0 MPa. At the Fig.1 some results of creep tests on specimens are shown.

The lateral displacements were measured by LVDT sensors. The tests show significant changes in deformations already after 2 h loading.

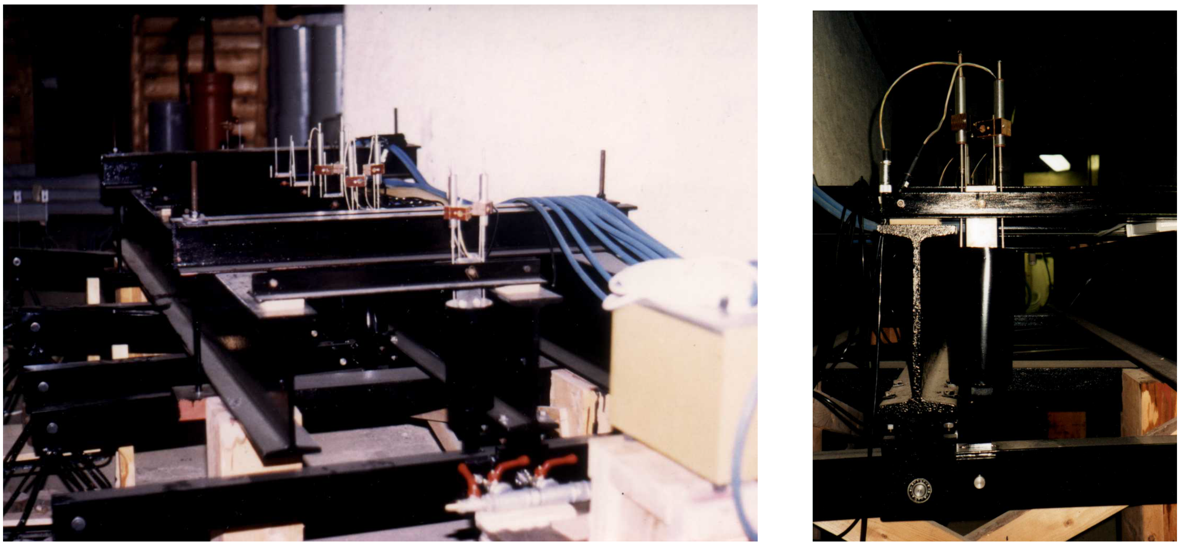

Fig.1 Experimental set-up

5. Approximation of $D_m(t)$ by creep function $D(t)$

The creep curves obtained from experiments $D_m(t)$ should be substituted by creep function $D(t)$ from three- element model (Standard Linear Solid). In the present case it would be reasonable to substitute function $\varepsilon_m(t)$ [4]

$$\varepsilon_m(t) = \sigma_0 \, D_m(t) = D_m(t)\,, \tag{13}$$

then it is not nesessary to transform $\varepsilon_m(t)$ to unit stress level. Functions $\varepsilon_m(t)$ have been obtained from creep tests for stress level σ_0.
Three- element model yields form eq. (7) and (12):

$$D(t) = C_1 + C_2 - C_2 \, e^{-\frac{t}{\tau}}\,, \tag{14}$$

where three parameters C_1 , C_2 and τ should be found. The parameters will be calculated by collocation method

$$\varepsilon_m(0) = \sigma_0 \, D_m(t) = \sigma_0 \, C_0\,, \tag{15}$$

$$\varepsilon_m(+\infty) = \sigma_0 \, C_\infty\,, \tag{16}$$

Similarly we obtain the expression for compliance time

$$\tau = -\frac{t_1}{l\,n \dfrac{\sigma_0}{\sigma_0} \dfrac{C_\infty - D_m(t_1)}{C_\infty - C_0}}\,, \tag{17}$$

or after some rearrangement

$$\tau = -\frac{t_1}{l\,n \dfrac{\sigma_0 C_\infty - \varepsilon_m(t_1)}{\sigma_0 C_\infty - \sigma_0 C_0}}\,, \tag{18}$$

Eq. (14) can now be written in the form

$$D(t) = C_\infty - (C_\infty - C_0) e^{-\frac{t}{\tau}} \tag{19}$$

It should be emphasized, that very important is particularly a choice of point of coincidence t_1 for curves $\varepsilon_m(t)$ and $\varepsilon(t)$.

6. Results and discussion

Short-term tensile experiments were executed on 5 specimens for the identification of the ultimate tensile strength and modulus of the material, the mean values are shown at Tab.1.

[MPa]	Strain ε [-]					
	0.000	0.002	0.004	0.006	0.008	0.010
Stress	1.82	6.92	12.5	18.2	23.9	29.4
Tang.mod.	2390	2680	2840	2880	2810	2660
Sec.mod.	2390	3460	3120	3030	2990	2940

Tab. 1. Short term behavior of Derakane 411-45 (C glass veil mat)

The creep response of the glass/ vinylester composite for different applied stress levels and durations has been obtained. Behaviour of composite based on resin Derakane 411-45, reinforced by C glass veil mat is illustrated in Fig. 2.

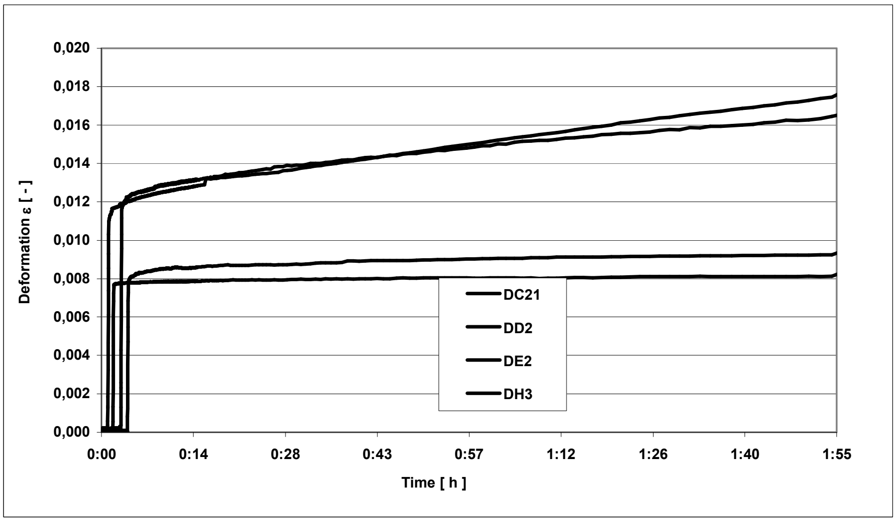

Fig. 2. Creep response of the G/VE composites

The creep data of the 30 and 45% of applied rupture stress, were fitted to mentioned equation, which is valid for stress levels in the linear range, in order to obtain a value of τ and finally expression for $\varepsilon(t)$. The above compliance time was calculated for all creep data. For example, the compliance time was calculated from

$$\sigma_0 C_\infty=0.0176,\ \sigma_0 C_0=0.012,\ t_l=0.5,\ \varepsilon_m(0.5)=0.015 \text{ as } \tau=0.651.$$

Approximation of creep strain can be obtained in the form

$$\varepsilon(t)= 0.0176- (0.0176-0.012)\, e^{-t/0.651}$$

The values of τ and $\varepsilon(t)$ were found and the respective experimental and fitted values of the creep response were calculated.

time	arg	exp	ε(t)
0	0	1	0,012
0,1	-0,15361	0,857607	0,012797
0,2	-0,30722	0,735489	0,013481
0,3	-0,46083	0,63076	0,014068
0,4	-0,61444	0,540944	0,014571
0,5	-0,76805	0,463917	0,015002
0,6	-0,92166	0,397858	0,015372
0,7	-1,07527	0,341206	0,015689
0,8	-1,22888	0,292621	0,015961
0,9	-1,38249	0,250953	0,016195
1	-1,5361	0,215219	0,016395

Tab. 2. Approximated creep of Derakane 411-45 (C glass veil mat)

7. Conclusion

In the contribution an approximation of viscoleastic behavior of a glass fiber reinforced vinyl-ester composite is presented. Tested composite exhibits a significant creep properties. A collocation method for the calculation of standard viscoelastic solid is proposed. The prediction of the creep strain dependency of the time linear viscoelastic behaviour of glass fiber reinforced vinyl-ester resin is shown. The prediction results in a function from which the linear viscoelastic behaviour of the composite can be derived and where all variables included have a clear physical meaning and all can be measured through simple experiments.
Application of the three- element model to a G/VE composite is shown.

Acknowledgements: This research has been supported by project MSM6840770015.

References

[1] Černý, M.J: *Modeling of Degradation Processes in G/VE Composites*. Proceedings, Internat. Conference on Composite Engineering (ICCE/5), Las Vegas, July 1998.

[2] Černý, M.J.: *Environmental Effects on Behavior of Vinyl-ester/ Glass Composites*. Proceedings, Internat. Conference on Composite Materials (ICCM-15), Durban, 2005

[3] Černý M.J.: *Viscoelastic Behaviour of Glass Fibre VE Composites.* Proceedings, 23rd Internat. Conference „Reinforced Plastics“, Karlovy Vary, 2005.

[4] Kovařík V.: *Theory of Viscoelasticity*. Textbook, KI CTU, 1993.