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SEPARATION OF STRESS IN TWO-DIMENSIONAL PHOTOELASTICITY USING A METHOD OF BOUNDARY ELEMENTS.

Jan Cernosek

Summary
Boundary element method is used for stress separation in two-dimensional photoelasticity.

Key words: Boundary elements, photoelasticity, separation of stress.

Introduction.

Two-dimensional photoelasticity has been traditionally, but falsely, presented as a whole field method of experimental stress analysis. It actually can provide complete information related to stress only at the boundary of the model, where one of principal stress is known. For general interior point photoelasticity provides only two pieces of information related to three components of the stress tensor and an additional information is needed in order to fully determine the state of stress.
The most commonly used method of stress separation is a numerical integration of an equation of equilibrium. The results of this numerical integration are very sensitive to errors in isoclinic parameter measurement and its obvious disadvantage is the accumulation of errors because of the summation procedure.
In order to improve an accuracy of the method and "to spread" the errors over a certain area some researchers have employed the overdetermined approach [1], while others have tried to develop methods which would skip an isoclinic angle measurement altogether[2]. Because photoelasticity is a boundary value method (it provides a complete solution on the boundary of the domain only) it seems to be natural to try to use "the boundary elements approach" which has been successfully applied for solving a great variety of engineering problems.

Boundary Elements Method.

The sum of normal stresses, in the absence of body forces, is a function which satisfies a harmonic differential equation supplemented with boundary conditions

$$S=\bar{S}, s\in\Gamma_1; \quad (1a) \qquad \frac{\partial S}{\partial n} = G = \bar{G}, \ s \in \Gamma_2 \quad (1b)$$

where $\Gamma = \Gamma_1 + \Gamma_2$ is the boundary of the domain and "n" is a normal to the boundary (see Fig. 1).
The "boundary element method" statement for potential problem can be written as [3]

$$\int_\Omega (\nabla^2 S) S^* d\Omega = \int_{\Gamma_2} (G - \bar{G}) S^* d\Gamma_2 - \int_{\Gamma_1} (S - \bar{S}) G^* d\Gamma_1 \qquad (2)$$

where "S^*" is so called "fundamental" solution of a governing equation

$$\nabla^2 (S^*) - \Delta^{(i)} = 0 \qquad (3)$$

The unit "charge" is applied in the point "i" under consideration and $\Delta^{(i)}$ is the Dirac delta function with the property that

$$\int_\Omega S \Delta^{(i)} d\Omega = S^{(i)} \qquad (4)$$

The basic feature of the "fundamental" solution is that

$$\int_\Omega S(\nabla^2 S^* + \Delta^{(i)}) d\Omega = \int_\Omega S(\nabla^2 S^*) d\Omega + S^{(i)} = 0 \qquad (5)$$

Using this equation, Eq. (2) can be rewritten into the form

$$S^{(i)} + \int_\Gamma S G^* d\Gamma = \int_\Gamma G S^* d\Gamma \qquad (6)$$

where $G = \frac{\partial S}{\partial n}$ and $G^* = \frac{\partial S^*}{\partial n}$

with the "fundamental" solution for two-dimensional domain being [3]

$$S^* = \frac{1}{2\pi} \ln\left(\frac{1}{r}\right) \qquad (7)$$

"r" is a distance from the point "i" to the point at the boundary where the conditions (1a,b) are defined. These boundary conditions can be supplied by photoelasticity. At the free boundary, one of the principal stress (say σ_2) equals to zero. Thus,

$$S = \bar{S} = \sigma_1 \qquad (8)$$

and free boundary is an "isostatic" line "s_1", while a normal to this boundary is an "isostatic" line "s_2". The differential equation of equilibrium can be written in a curvilinear coordinate system formed by these two sets of "isostatic" lines in the form

$$\frac{d\sigma_2}{ds_2} + \frac{\sigma_2 - \sigma_1}{R_1} = 0 \qquad (9)$$

where "R_1" is the radius of curvature of "s_1". Noting that

$$\sigma_1 = 0.5(S + D) \quad ; \quad \sigma_2 = 0.5(S - D) \qquad (10)$$

where

$$S = \sigma_1 + \sigma_2 \quad ; \quad D = \sigma_1 - \sigma_2 \qquad (11)$$

we have

$$\frac{dS}{ds_2} = \frac{dD}{ds_2} + \frac{2D}{R_1} = -G \qquad (12)$$

An outside isolated force applied at the boundary of two-dimensional photoelastic model introduces singularity into boundary conditions. This singularity can be avoided by modifying the boundary of the model (for calculation purposes only) by drawing a small circle around the point of application of the force (see Fig 2) and by employing the well known Boussinesq's solution [4].

Stress Separation Procedure.

For the stress separation the boundary of the model is discretized into "N" elements and value of "S" and "G" is assumed to be constant on each of the element. Equation (6) is then written in a discrete form

$$S^{(i)} = \sum_{j=1}^{N} G_j \int_{\Gamma_j} S^* d\Gamma_j + \sum_{j=1}^{N} S_j \int_{\Gamma_j} G^* d\Gamma_j \tag{13}$$

and the integrals in this equation are evaluated using standard Gauss quadrature rule. It is obvious that the accuracy of this approach depends on the number of "boundary elements" and furthermore it suffers from the singularity of the "fundamental" solution (the results in the area close to the boundary are of lower accuracy than the results in points which are in a sufficient distance from the boundary).

Example of Separation of Stress.

The diametrally compressed disk (2 inches in diameter) was chosen as an example. Two small circles of 0.174 inches were drawn around the points where the forces were applied. The circumference of the disk was divided into 44 elements with nodes located in the middle of each element. The fringe order was measured at the points at the distance 0.05 inch from the nodal points. These points were located on the normal to the boundary of the disk. Because the fringe order is zero at the boundary of the disk, the gradient of "S" was determined as

$$\frac{dS}{dn} = \Delta_{(0.05)} \frac{1}{0.05}$$

The boundary conditions for nodes located on the circles around the points of force applications were determined using a solution for the isolated force applied to the straight edge of a half space [4]. The simple computer program was written to calculate the values of first scalar invariant in the interior points of the two-dimensional photoelastic model. The inputs into the program are listed in Table 1, while Table 2. lists the results of the calculation together with theoretical values of the first scalar invariant of the stress tensor.

References:

1)Berghause, D. G.,"Stress Analysis Using Elastic Field Equations, Photoelasticity and Least-Squares", 1987 SEM Conference, Houston, Texas.
2)Doyle, J. F.,"Photoelastic Stress Separation Along Lines of Symmetry", Experimental Mechanics, vol. 26, No. 3.
3)Brebbia, C. A.,"The Boundary Element Method for Engineers", Pentech Press, 1978
4)Timoshenko, S., Goodier, J. N.,"Theory of Elasticity", McGraw-Hill, 1970

Element	1	2	3	4	5	6	7	8	9	10	11
X_e	1.0	.98	.94	.87	.77	.64	.50	.34	.17	.15	.08
Y_e	.00	.17	.34	.50	.64	.77	.87	.94	.98	.90	.83
Node	1	2	3	4	5	6	7	8	9	10	11
X_n	.99	.96	.90	.82	.70	.57	.42	.26	.16	.11	.04
Y_n	.09	.26	.42	.57	.70	.82	.90	.96	.94	.86	.83
S	.00	.00	.00	.00	.00	.00	.00	.00	-.86	-2.38	-3.20
G	.60	.80	.80	1.0	1.4	2.2	3.8	9.8	-6.9	-15.4	-20.3

X_e; Y_e coordinates of extremal points of an element

X_n; Y_n coordinates of nodes

S; G first scalar stress invariant and its gradient

Table 1-Inputs into a computer program. Definition of the problem.

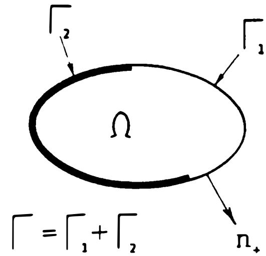

Fig. 1-Definitions

Section "a-a" Point	1	2	3	4	5	6	7	8	9	10
X_p	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
Y^p	0	0	0	0	0	0	0	0	0	0
S^p_{calc}	.68	.67	.63	.56	.48	.39	.29	.20	.11	.02
S_{th}	.63	.62	.59	.53	.46	.38	.30	.21	.14	.06
Section "b-b" Point	1	2	3	4	5	6	7	8	9	10
X_p	.0	.10	.19	.29	.39	.48	.58	.68	.77	.87
Y^p	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25
S^p_{calc}	.74	.73	.69	.62	.53	.44	.34	.25	.16	.08
S_{th}	.72	.70	.65	.58	.49	.40	.31	.22	.14	.06
Section "c-c" Point	1	2	3	4	5	6	7	8	9	10
X_p	.0	.09	.17	.26	.35	.43	.52	.61	.69	.78
Y^p	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
S^p_{calc}	1.13	1.09	.99	.84	.68	.53	.39	.28	.17	.09
S_{th}	1.06	1.02	.91	.77	.62	.48	.35	.24	.14	.07

X_p; Y_p coordinates of the point.

S_{calc} calculated magnitude of first scalar stress invariant.

S_{th} theoretical magnitude of first scalar stress invariant.

Table 2-Calculated values of the first scalar invariant for diametrally compressed disk.

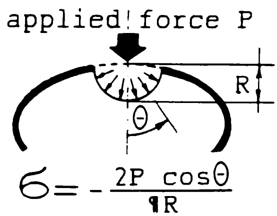

Fig. 2- Boundary condition for an isolated force*.

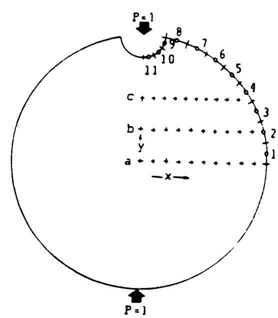

Fig. 3-Separation of stress in a disk 44 elements/44 nodes.

Jan Cernosek, PhotoStrain, 7814 Bryn Mawr, Dallas TX 75225, USA